MATHEMATICS

Paper 4024/11
Paper 11

Key Messages

All working should be clearly shown and answers accurately transcribed to the appropriate answer space.
Accuracy with basic number work is essential.
Final answers should be given to the required degree of accuracy.
Care must be taken to form letters and digits clearly and unambiguously.

General comments

Most candidates were able to make a reasonable attempt at this paper and the full range of marks was awarded. The presentation of the scripts was generally good. Time did not appear to be a factor as the majority completed the paper with most attempting the last question. The questions which candidates had most difficulty with were 1**(b)**, 5**(b)**, 9, 14**(b)**, 15**(b)**, 21**(a)(ii)**, 24**(a)**, 24**(b)**, 24**(c)**.

Comments on specific questions

Question 1

(a) This seemed to test the candidates throughout the entire range as many left this part and part **(b)** unanswered. Some filled in more than two squares.

(b) Quite a number of candidates offered the same solution for both parts.

Question 2

(a) Even though BODMAS was frequently quoted, a good number of candidates applied the order of operations incorrectly. Common wrong answers were $(8 – 5) \times (4 + 3) = 3 \times 7$ and $8 – 5 \times 4 + 3 = 8 – 20 + 3 = 8 – 23 = -15$, suggesting that candidates need more confidence when applying BODMAS.

(b) This was generally quite well answered, but there were some candidates who were unable to multiply by 100 successfully.

Answer: **(a)** -9 **(b)** 103

Question 3

(a) The correct answer of 18.75 was seen frequently. The word “estimate” seemed to make some unsure of what was required here.

(b) “Approximately” seemed to confuse some candidates here also. Many candidates showed the arrow pointing directly to either $\frac{3}{4}$ or $\frac{7}{8}$.

Answer: **(a)** 18.75 **(b)** arrow between $\frac{3}{4}$ and $\frac{7}{8}$

Question 4

(a) This was generally well done, although a number of partial factorisations were seen.

(b) Many candidates were able to factorise the expression correctly. However, some candidates continued to solve an equation.

Answer: **(a)** $3x^2(4 - 5x)$ **(b)** $(x - 3)(x + 2)$

Question 5

(a) Many candidates achieved the correct answer. A common wrong answer was 4.

(b) This part proved challenging to even the most able. The use of 6.8 and 4.3, followed by an adjustment, was common. Frequent incorrect answers were 7 and 6.85.

Answer: **(a)** 4.25 **(b)** 2.6

Question 6

This question was well negotiated by some candidates. Not all who found a satisfactory decimal equivalent of $\frac{22}{7}$ reached the expected answer. The main problem for most candidates seemed to be finding $\frac{22}{7}$ to sufficient decimal places to be able to correct their answer to 2 significant figures.

Answer: 0.0013

Question 7

(a) Candidates often correctly answered, but the question was not understood by some.

(b) A mixed response to this part was seen. Calculations could have been made simpler by cancelling.

Answer: **(a)** 48 **(b)** 72

Question 8

Many candidates correctly found *m* and *n*, but this question was not universally understood. The redrawing of the triangles with the angles in the same positions was a successful approach for some.

Answer: $m = 9$, $n = 11$

Question 9

The expected strategy for finding the lowest common multiple (LCM) was used by some who obtained the correct answer. Other candidates either did not know about LCM, or were unable to recognise that this is what the problem required. Some successfully listed the times of the buses until they coincided.

Answer: 14 30

Question 10

Most candidates were successful in applying a correct method to solve the pair of simultaneous equations and obtained the correct solutions. As usual, care is required when eliminating a variable particularly when subtraction is involved, and this caused problems for some.

Answer: $x = 5$ $y = -3$

Question 11

(a) A high success rate was seen in this subtraction of fractions question.

(b) Many correct answers were seen, but some were spoiled by converting $1\frac{2}{3}$ to $\frac{7}{3}$ or $1\frac{3}{4}$ to $\frac{7}{3}$.

Answer: **(a)** $\frac{11}{35}$ **(b)** $\frac{20}{21}$

Question 12

Most candidates achieved full marks in all parts. Less able candidates did not recognise the irrational number.

Answer: **(a)** 2 **(b)** 8 **(c)** $\sqrt{2}$

Question 13

This question required knowledge of some basic facts which were not known by some candidates e.g. 1000m in 1 km and $1\frac{1}{4}$ hours is 1 hour 15 minutes, not 1.15.

(a) This was usually correctly answered.

(b) (i) This was also usually correct.

(ii) This part proved more challenging with many candidates having problems in sorting out the required fraction.

Answer: **(a)** 64 **(b)** (0)9 50 **(c)** 1.28

Question 14

(a) This was usually correctly answered. Some candidates, however, gave $\frac{3}{4} + \frac{3}{5}$ leading to a probability greater than one.

(b) This part proved demanding for many candidates with very few correct answers seen. The majority had difficulty in finding and combining all the probabilities required. For example, $1-\frac{3}{4}$ was seen, but then was followed by inappropriate additions or even multiplications. Again, some answers greater than one were given. A common wrong answer was $\frac{1}{4} + \frac{3}{5} = \frac{17}{20}$.

Answer: **(a)** $\frac{3}{10}$ **(b)** $\frac{11}{20}$

Question 15

(a) Many candidates achieved the correct answer. Some preferred to give $k = 9$.

(b) Some correct answers were seen, but the initial approach by many was to multiply out all the numbers and then attempt to simplify what remained. This inevitably led to errors. Others resorted to some strange regroupings of the numerator with terms such as $3^2 \times 2^{12} = 6^{14}$ appearing.

Answer: **(a)** 2^9 **(b)** 44

Question 16

(a) The correct use of the fraction $\frac{8}{5}$ ensured a good response to this part.

(b) This part was reasonably well done. Some candidates created difficulties for themselves by writing 20 million as 20,000,000 and then tried to find 15% of this number.

Answer: **(a)** 60 **(b)** 20.7

Question 17

(a) A good number of correct answers were seen, but some candidates struggled getting figures for the thousand million.

(b) Candidates needed to consider two aspects for this question; converting grams to kilograms and realising that food was required for seven days. The answers were varied with none, one or both aspects considered. A reasonable number of correct answers were seen however.

Answer: **(a)** 4×10^{10} **(b)** 5.6×10^{8}

Question 18

(a) This was usually solved correctly.

(b) Many correct answers were seen. Some candidates omitted the inequality sign in their answers.

(c) The method was generally understood with a high success rate. Some candidates kept a denominator of 12 throughout all their working, whilst others made errors in transposing terms in their equation.

Answer: **(a)** $\frac{3}{5}$ **(b)** $(y) \geq 2$ **(c)** $\frac{7}{10}$

Question 19

(a) Many candidates spotted the patterns.

(b) A good response was seen for the *n*th term.

(c) This was usually correct, but not all candidates were successful with the algebra.

(d) This was often found independently of previous work.

Answer: **(a)** 25 36 **(b)** n^2 **(c)** $n^2 - n$ **(d)** 780
20 30

Question 20

(a) This was usually correct.

(b) Again, this was usually correctly answered.

(c) This part was well done in most cases.

(d) This was generally correctly answered. Occasionally the coordinates were interchanged.

Answer: **(a)** 1 3 **(b)** 1 4 **(c)** 2 **(d)** (-1, 3)

Question 21

(a) (i) Many candidates gave 1.5 students without considering whether this was a sensible answer.

(ii) This required a more testing aspect of histograms and many candidates struggled with this part.

(b) This was usually correctly answered.

Answer: **(a) (i)** 15 **(ii)** 27 **(b)** 54

Question 22

(a) This part was generally well answered. Some candidates, however, gave the answer as $\frac{11}{24}$.

(b) A good number of correct solutions were seen, but the topic of rearrangement of a formula continues to be demanding for some candidates. Here, less able ones usually began with a correct first step but then struggled with the remaining steps. A common incorrect answer was $a = (ab - b^2) / c$.

Answer: **(a)** $\frac{24}{11}$ **(b)** $\frac{bc}{c-b}$

Question 23

(a) A good response to this part was seen. Many candidates included a negative sign.

(b) This part caused problems for many candidates. Many made no attempt to draw a tangent. Some just gave the answer as 10. The few candidates familiar with this topic often scored both marks.

(c) A good number of correct solutions were seen, and clearly the association of distance and area was mostly well understood. Some numerical errors were made within their calculations. A common incorrect answer was 645 from 43×15.

Answer: **(a)** 1.5 **(b)** 0.7 to 1 **(c)** 570

Question 24

(a) Many candidates struggled with this part and few stated two pairs of equal angles, together with reasons. There were very few who managed to show that the triangles were similar. Often confusion with congruence was apparent. Many concentrated entirely on parallel lines.

(b) This part also proved demanding for most candidates. A few spotted the required ratio but were unable to proceed any further. Some attempted to apply Pythagoras' theorem.

(c) For the majority of candidates there was just too much choice of similar triangles for them to focus on the task in hand and very few correct answers were seen.

Answer: **(b)** 10.5 **(c)** 3

Question 25

(a) (i) This part was mostly correct. 24 – 2 + 3 = 24 – 1 = 23 was a common wrong answer.

(ii) This was also mostly correctly answered.

(iii) A good number of correct solutions were seen to this part.

(b) A good response was seen from candidates with many scoring both marks. After a correct substitution, some were unable to evaluate $(a + 1)^2$ correctly, usually resulting in $a^2 + 1$. For others, the omission of brackets at the initial stage was the main reason for the lack of any marks.

Answer: **(a) (i)** 25 **(ii)** 10 **(iii)** $\frac{2}{3}$, $-\frac{1}{2}$ **(b)** $6a^2 + 11a + 8$

MATHEMATICS

Paper 4024/12
Paper 12

Key messages

All working should be clearly shown and answers accurately transcribed to the appropriate answer space.
Accuracy with basic number work is essential.
Final answers should be given to the required degree of accuracy.
Care must be taken to form letters and digits clearly and unambiguously.

General comments

There were many well presented scripts of a good standard that showed that candidates had been well prepared to deal with questions on all aspects of the syllabus.

Candidates of all abilities continue to do well in questions involving standard algebraic procedures, such as factorisation and the solution of simultaneous equations. Also, candidates continue to do well in questions involving fractions and the usual arithmetic operations. This year, the question dealing with upper and lower bounds seemed to suggest a significant improvement in this topic. The question on statistics showed that, for many candidates, there was uncertainty concerning frequency polygons.

Some questions require intermediate calculations involving basic number work. Many candidates lose marks for inaccuracies at such stages. This is an area where many candidates could make a significant improvement.

It is important that Examiners can see all the working that is required for the solution of a given problem. It is also important that the working is clearly and neatly set out. Candidates should be encouraged to develop a clear style in setting out written and numerical work. In some scripts, where initial work in pencil was inked over, the final version of a candidate's work was not always clear.

Comments on specific questions

Question 1

(a) Most candidates were able to start from $\frac{72}{100}$ and cancel accurately. Occasionally, the final answer was left as $\frac{72}{100}$.

(b) This part was generally well answered. Sometimes $\frac{2}{5}$ was stated twice. It seemed that some candidates thought that two fractions adding up to $\frac{4}{10}$ were required. "Fractions" such as $\frac{0.8}{2}$ were penalised.

Answer: **(a)** $\frac{18}{25}$ **(b)** $\frac{2k_1}{5k_1}$ and $\frac{2k_2}{5k_2}$

Question 2

(a) This was well answered. Candidates seemed confident with the idea of *difference.* A common error was 24 – 18.

(b) This part was also mostly correct. Here, the idea of *warmer* had to be translated by the candidate into mathematical terms. Answers of –40, from –18 – 22, and –4 were also seen.

Answer: **(a)** 42 **(b)** 4

Question 3

(a) Generally, the implications of the mathematical conditions were appreciated, leading to a correct shape. There were some parallelograms. There were also some triangles and some shapes with more than 4 sides. Candidates should use rulers in such situations.

(b) There were many good answers. The order of rotational symmetry was more often correct than the number of lines of symmetry. A common wrong answer was 4 and 1. Candidates who drew parallelograms in part **(a)** tended to quote the numbers, 0 and 1, given in that part.

Answer: **(a)** Drawing of kite or isosceles trapezium **(b)** 2, 0

Question 4

(a) This ratio question was more challenging than it appeared. 72÷11 was seen and a common incorrect answer 8 came from 24÷3. There were also some fractional answers given.

(b) Candidates were much more confident with this part. Perhaps the context of money was more familiar. There were some candidates who divided 360 by 3.

Answer: **(a)** 9 **(b)** 144

Question 5

This question was well answered and seemed to indicate a significant improvement in this topic. Candidates at all levels seemed well equipped to gain marks here. The errors seen included inaccurate substitution once 72 had been established, and the use of direct proportion with x^2 or $\sqrt{x}$ or inverse proportion with $\frac{1}{x}$.

Answer: 18

Question 6

This question was often well answered. The demands of this type of question are not yet appreciated by many candidates. In a non-calculator paper, elaborate calculations using numerical values of π are not expected in such situations. Some candidates were unsure of the formula for the area of a circle, using such as $2\pi r$, $2\pi r^2$, or even $4\pi r^2$. Some used 6 as the radius, and some gave the answer 8 - 6π.

Answer: 64 - 9π

Question 7

(a) This was generally well answered. Candidates should be aware that the relevant inequality sign is an integral part of the answer in questions such as this. $x \le 3$ was a common incorrect answer.

(b) The majority of candidates understood that a list of integers was required. A good many achieved the correct answer. The inclusion of 2 was common. Some lists included rational numbers. Expressing the answer as a correct double inequality had not completed the problem.

Answer: **(a)** $x \le 4$ **(b)** - 1, 0, 1

Question 8

(a) There were many correct answers seen indicating a significant improvement in this topic. A common error was 0.9 + 0.05 = 0.905. Misconceptions included 0.9 – 0.6 and subtracting 0.05 from 0.9.

(b) Many candidates went on to complete this problem successfully. 1.4 was seen from candidates who considered only half the perimeter. A common misconception was 2×0.9 + 2×0.6 with 3 rounded down to 2.5.

Answer: **(a)** 0.95 **(b)** 2.8

Question 9

(a) This addition of fractions question was generally well done.

(b) This multiplication of fractions was quite well done. There were some answers left as $3\frac{9}{12}$ or $\frac{15}{4}$.

Most candidates found both $\frac{5}{3}$ and $\frac{9}{4}$.

Answer: **(a)** $\frac{31}{40}$ **(b)** $3\frac{3}{4}$

Question 10

(a) This was generally well answered.

(b) Many candidates rounded correctly and concluded successfully. Here, it was necessary that the correct answer was supported by seeing 20, 9 and 0.6 in the working. Some errors in basic number work were apparent after this stage had been reached. Common errors in rounding were 19, 10 and 1. Long methods without rounding gained no marks.

Answer: **(a)** 22 **(b)** 300

Question 11

(a) This was mostly correctly answered. A common incorrect answer was – 4.

(b) The most popular approach here was to multiply both sides by *a*. This was done successfully by the majority of candidates. The subsequent algebra was not always accurate. A number of candidates thought that this, like part **(a)**, was a numerical stage.

Answer: **(a)** - 3 **(b)** $a = \frac{b^2}{b-c}$

Question 12

(a) This was well answered. The notation of column vectors was generally understood and used well. The main problem lay in the handling of the negative numbers.

(b) A lot of correct answers were seen. Most candidates achieved a correct, initial equation, but the resulting work sometimes ended in errors.

Answer: **(a)** $\begin{pmatrix} 5 \\ -10 \end{pmatrix}$ **(b)** $s = 5$, $t = 2$

Question 13

(a) This was often well answered.

(b) Again, this was often well answered. Some candidates enlarged from (5,3), but increased the scale factor. Some drew a triangle of the correct size using (5,3) as one of the vertices. There were a number of omissions of this part.

Answer: **(a)** $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

Question 14

(a) This was usually well answered.

(b) The positive responses to this question showed that the equation $y = mx + c$ was generally well understood. The work was well organised, finding the gradient first and then completing the equation. Care was needed in the working with, for example, $\frac{-3}{-6}$. Sometimes the answer was stated as c = 4. Some candidates worked with the formula for the length of a line segment.

Answer: **(a)** (-3, 2.5) **(b)** $y = \frac{1}{2}x + 4$

Question 15

Usually, when candidates realised that Pythagoras' theorem was required, at least two of the three available marks were earned, and often, all three. Some candidates made an estimate for $\sqrt{65}$, such as 8 or 8.1, before applying Pythagoras' theorem, but many dealt with $(\sqrt{65})^2$ successfully. There were a number of incorrect efforts that attempted to find the area directly from the data in the question.

Answer: 28

Question 16

(a) This was generally well answered. Candidates beginning with (12 – 2) x 180 often completed successfully. Candidates beginning with 360 ÷ 12 often left 30 as their answer.

(b) The full name for a regular triangle was required. There were relatively few mentions of *equilateral*. Where candidates expanded on merely mentioning triangle, the usual response was *isosceles*. A variety of polygons were given as the answer.

Answer: **(a)** 150° **(b)** Equilateral triangle

Question 17

(a) This part was well answered. Answers with figures 185 suggested that conversion of units was the problem.

(b) (i) This was also well answered. The misreading of London time 4.5 hours ahead of Chennai time led to the common wrong answer of 19 15.

(ii) Many correct solutions were seen to this part. Quite a number of candidates achieved a partial solution. Most attempts seemed to be part mental arithmetic with some written work. Very few showed a complete written method. The main problem, generally, seemed to be that both the 4 00 and the 4 30 were not always included in the computation.

Answer: **(a)** 1.85 **(b)(i)** 10 15 **(ii)** 10 hours 5 minutes

Question 18

(a) (i) This was well answered. The answer was not 11^2 as some candidates stated. There was some confusion with answers such as $\frac{1}{3}$.

(ii) A number of candidates stated that this was impossible. Otherwise, this part was done quite well.

(b) A lot of success was seen in this question. For candidates using the value for each term, rather than the terms themselves as given in the question, it was expected that these would be correct, so that – 5 for 5^{-1} was not accepted.

(c) Again, this part was often well answered. Candidates should fully evaluate the terms in questions of this nature, so 2^6 was not given any credit.

.

Answer: **(a)(i)** 11 **(ii)** - 3 **(b)** 5^{-1}, 4^0, 2^3, 3^2 **(c)** 64

Question 19

(a) (i) This was well answered. Common incorrect answers seen were 4, -4 and 12.

(ii) Many good solutions were seen to this part. Sometimes the inverse function appeared as $x – 4$, and sometimes with y.

(b) The complete solution needed much algebraic skill and few candidates reached the correct answer. Most candidates successfully gained the part mark, showing that the basic idea of the function notation was understood.

Answer: **(a)(i)** -12 **(ii)** $\sqrt[3]{x+4}$ **(b)** $a^2 - 7a + 11$

Question 20

(a) This was generally well done although some did not correct to two significant figures.

(b) Many correct answers were seen, with some guess-work from a minority.

(c) This was also well answered. Sometimes 10^7 was omitted in the subtraction.

Answer: **(a)** 1.1×10^8 **(b)** Senegal, South Korea **(c)** 3.4×10^7

Question 21

(a) Some good work was evident here. Some candidates thought ahead, and gave the probabilities in their lowest terms. Occasionally, there was some confusion with class size, the denominator 55 appearing for Class A, and denominators such as 54, 29 and 24 appearing for class B. Sometimes whole numbers were given for these probabilities, such as the numerators of the expected probabilities.

(b) After correct work in part **(a)**, generally the correct answer was obtained. Care is always needed with basic arithmetic processes. After incorrect work in the first part, there was evident understanding of the process required to obtain the probability asked for. Occasionally the operations of x and + in the correct method were interchanged.

Answer: **(a)** $\frac{10}{25}, \frac{15}{25}, \frac{20}{30}, \frac{10}{30}$ **(b)** $\frac{8}{15}$

Question 22

(a) The table was generally completed correctly.

(b) A variety of expressions were given in this part. A common incorrect one was $n + 3$.

(c) Evidence of a correct method following on from the previous part was seen occasionally, such as $n + 3 = 83$. The answer here was often arrived at independently however.

Answer: 11, 14, 17 **(b)** $3n + 2$ **(c)** 27

Question 23

(a) Not all candidates are familiar with the idea of a frequency polygon. There were some accurate responses, and a number where the frequencies were plotted at the upper end of the intervals instead of the middle. Sometimes the frequency polygon appeared as well as a block diagram. Quite often, the response consisted of a block diagram by itself. The occasional cumulative frequency curve was seen.

(b) This was well answered. A common wrong answer was $8 < t \le 12$.

(c) This was also well answered. A common wrong answer was 12.

(d) There were a good number of expressions of the correct idea. A significant number of candidates were swayed by irrelevant factors such as the modal class, the fact that the frequency polygon stopped at 18, or even the possibility that there may have been people who took longer than 20 minutes but the information had not been recorded.

Answer: **(b)** $4 < t \le 8$ **(c)** 13

Question 24

(a) Many correct answers were seen. The calculation $\frac{30}{100} \times 350$ was generally appreciated, but candidates needed to remember that it was the sale price that was required. Sometimes $\frac{350}{30} \times 10$ was seen.

(b) This was also well answered. A common error was $275 \div 0.8$. With the correct method, care was needed in placing the final decimal point.

(c) With attention to basic arithmetic, many candidates scored well in this part. The idea of taking 25% seemed to be understood, but care was needed in the long multiplication required. Some candidates subtracted this from 4500. Again, \$320 ×12 needed care, and in some cases only \$320 was added on. A number of candidates did not subtract the final \$4500. Some candidates thought that this question required the calculation of simple interest.

Answer **(a)** 245 **(b)** 220 **(c)** 465

Question 25

(a) (i) Many candidates were able to factorise the given quadratic.

(ii) This part was also well answered.

(b) Many correct solutions were seen. Many candidates presented their answer in its lowest terms even though not asked to do so.

(c) This topic continues to be successful for many candidates. Usually, sensible multiples of the equations were used for elimination. When using this approach, the main problem for candidates is being consistent when subtraction between equations is required. When using substitution, correct equations were seen after reduction to one variable. The problem here was in removing brackets. It was easy to end up with a simple equation in one variable that had no correct numerical terms. In this type of question, candidates should be encouraged to simplify their answers.

Answer **(a)(i)** $(x+4)(x-3)$ **(ii)** $(5x+2y)(5x-2y)$ **(b)** $\frac{3}{2p}$ **(c)** $x = 4$, $y = -2$

MATHEMATICS

Paper 4024/21
Paper 21

Key messages

All working should be clearly shown and answers accurately transcribed to the appropriate answer space.
Accuracy with basic number work is essential.
Final answers should be given to the required degree of accuracy.
Care must be taken to form letters and digits clearly and unambiguously.

General comments

Many candidates produced well reasoned and well presented scripts, although a few were unprepared for some of the topics tested.
Candidates should be advised to work with at least four significant figures in their working and then give their final answer to three significant figures.
All drawings, constructions and graphs should be done in pencil.

Comments on specific questions

Section A

Question 1

(a) (i) Some candidates were unfamiliar with the notation for "number of elements in a set". Many wrote a list of the elements, which was not answering the question.

(ii)(a) Most candidates found this part challenging but many used their Venn diagram well and picked out the required elements.

(ii)(b) Those achieving success in the previous part recognised the multiples of 4.

(b) Many candidates could manage this problem, either by using their Venn diagram to good effect or by using an equation such as $28 – x + 17 – x + x + 12 = 50$, but this latter method often ended in $x = 7$ which is not quite the answer. A few argued that 45 musicians from 38 children meant 7 too many, so 7 did both, hence 21 for piano only, which was a neat and efficient method.

Answers: **(a)(i)** 11 **(ii)(a)** 4, 8, 12, 16 **(ii)(b)** a multiple of 4 **(b)** 21

Question 2

(a) The key to this question is to see that the \$48 charge was a one-off charge each time it was hired and did not depend on how many days the digger is used. On this understanding many calculated both costs correctly to get the saving using Option 2. An efficient argument used by some was:- "Option 2 saves \$48 and costs an extra \$39", better by \$9. Many thought the \$48 was payable per day.

(b) This was a simple interest question but it would have been better for the candidates if the standard formula I = PRT/100 had been abandoned. We need to see that the amount, \$2781, is 103% of the principle. This could of course come from P + PRT/100 = A.

Answers: **(a)** Option 2 by \$9 **(b)** \$2700

Question 3

(a) Most candidates recognised the difference of two squares and many were successful.

(b) Most candidates knew what a product was and scored the mark for producing the correct equation. However the subsequent manipulation and factorisation was not always free from error.

(c) (i) Many good responses were seen to this question. The majority simplified the left hand side and then multiplied by the denominator.

(ii) This was a straightforward test of the quadratic formula. Many candidates handled it well, though some had difficulty rounding 5.89897 to one decimal place.

Answers: **(a)** $(3x - 8y)(3x + 8y)$ **(b)** 2.5, −5.5 **(c)(ii)** 5.9, −3.9

Question 4

(a) Candidates needed to find the areas of two equal trapezia and one parallelogram and some managed this very efficiently. Some dissected the diagram into rectangles and triangles and this introduced more chances to make errors. Many candidates made no attempt at this question.

(b) The information tells the candidate that the area of the outer circle minus the area of the inner circle is 1206 but many could not see this. $\pi r^2 = 1206$ was very common. Many who successfully got to $\pi R^2 - 707 = 1206$ transposed to $\pi R^2 = 1206 - 707$. Some used $(15 + x)$ instead of R for the radius of the outer circle, which caused great problems when they expanded $(15 + x)^2$.

(c) (i) Just a few candidates correctly applied the factor $\frac{10}{15}$ to the length 50.

(ii) Only the most able candidates knew that the area factor was $(\frac{10}{15})^2$. Most offered no response here.

Answers: **(a)** 1660 **(b)** 24.7 **(c)(i)** 33.3 **(ii)** $\frac{4}{9}$

Question 5

The orientation of this diagram added a degree of difficulty to these calculations.

(a) The right angle *ODF* was not seen by some candidates.

(b) Many candidates did not see the alternate angles at *D* and *C*.

(c) (i) This was a straightforward application of the theorem on "opposite angles of a cyclic quadrilateral" and candidates generally were successful.

(ii) Again, many candidates were successful, some on a follow through basis.

(iii) Candidates used various methods to calculate angle *AOD* successfully.

(iv) Again, candidates found angle *BAO* by various methods using the known angles.

Answers: **(a)** 32° **(c)(i)** 94° **(ii)** 28° **(iii)** 56° **(iv)** 60°

Question 6

(a) The more able candidates were successful but some could not cope with 1 – (−1) and 2 – (−2). Less able candidates offered no response.

(b) A few candidates gave the correct inequalities, but many offered wrong ones or none at all.

(c) This part proved challenging for many candidates. Some did not know where the line $y = -2$ was, some did not understand a reflection and some offered no response.

(d) Just a few able candidates could do this work. The majority had limited understanding of what a stretch was.

Answers: **(a)** $\frac{1}{2}$ **(b)** $y \le \frac{x}{2}$, $y \ge -1$ **(d)(i)** 2 **(ii)** (8, −1) **(iii)** 12

Question 7

(a) (i) Most candidates correctly divided 360 by 6. However some produced answers ranging from −120 to 720.

(ii) It is not always easy to explain what is obviously true and many just said they had the same length. Many said they were opposite sides of a parallelogram. What was needed was the recognition that triangle *AOB* was equilateral so that *AO* must equal *AB*, which equals *BC*.

(b) A few able candidates had mastered vector algebra and got all parts right. Some got part **(i)** correct but could not produce the other vectors. Many candidates left all answer spaces blank or filled them with random numbers/letters.

(i) This part was correctly done by many candidates. Common incorrect responses were **a** + **b** and **a** – **b**.

(ii) If the candidate wrote the route $\overrightarrow{FB} = \overrightarrow{FA} + \overrightarrow{AB}$ then success usually followed, although few candidates did this.

(iii) Some thought that $\overrightarrow{AG}$ was $\frac{1}{3}$ of $\overrightarrow{AB}$ rather than $\frac{1}{4}$ of $\overrightarrow{AB}$.

(iv) If candidates could see that $\overrightarrow{OH}$ was $\overrightarrow{OB} + \overrightarrow{BH}$, they were successful in answering this part.

(v) Only the most able candidates managed to answer this part correctly.

Answers: **(a)(i)** 60° **(b)(i)** $\mathbf{b} - \mathbf{a}$ **(ii)** $2\mathbf{b} - \mathbf{a}$ **(iii)** $\frac{1}{4}(3\mathbf{a} + \mathbf{b})$ **(iv)** $\mathbf{b} - \frac{1}{2}\mathbf{a}$ **(v)** $\frac{1}{4}(3\mathbf{b} - 5\mathbf{a})$

Section B

Question 8

Many candidates were well prepared for this question.

(a) (i) Many candidates produced the correct bearing.

(ii) Many candidates found the correct positions for *B* and *C* but some could not deal with the bearing of 127°.

(iii) With a good diagram for part **(ii)** many candidates found this bearing correctly and with good accuracy.

(b) (i) Using the correct fraction of a full circle of radius 7, many were successful.

(ii) Many candidates used the correct fraction of the full circumference and many remembered to add the two radii. Some candidates were unsure of the word "perimeter", judging by the wrong working seen.

(iii) Few candidates understood where the locus of points 5 cm from *JM* was. For the other locus many produced the correct angle bisector.

(iv) Only a few candidates with good understanding of part **(iii)** and the conditions in this part scored here.

Answers: **(a)(i)** 307° **(iii)** 074° **(b)(i)** 30.8 **(ii)** 22.8

Question 9

(a) Finding the mean of this grouped frequency distribution was dealt with very well by those who knew the method.

(b) The cumulative frequency table was correctly completed by many.

(c) The graph was well drawn. Some did not notice that a "smooth cumulative frequency curve" had been requested, so a ruled graph was not appropriate. Some plotted all points 1 cm to the left of their correct positions, which lost them marks.

(d) (i) The median's position on the graph was well known.

(ii) The inter-quartile range was calculated correctly by those familiar with the use of these graphs.

(iii) The reading-off of the number who took less than 75 minutes was not always accurate and then many did not convert to "more than 75 minutes". Some candidates used $\frac{75}{120}$ and many left this part blank.

Answers: **(a)** 54.5 **(b)** 50, 68, 77 **(d)(i)** 50 to 55 **(ii)** 31 **(iii)** $\frac{16}{80}$

Question 10

(a) The information given to find that the side of the square was $(10 - x)$ was often not understood and this part could not be done. However this did not prevent the rest of the question being tackled.

(b) The missing values in the table were invariably calculated correctly and the graph was plotted and well drawn with the exception of the part near the maximum. Candidates were very reticent to take the curve above (3, 147).

(c) (i) The maximum was nearly always said to be 147, because that is the highest value in the table; very few indeed let their curve stray above (3, 147).

(ii) This part was well understood and the two values were often correct.

(d) Many candidates substituted into $\frac{4}{3}\pi r^3$ but some could not simplify to $\frac{1}{2}x^3$. There were very few graphs drawn.

Answers: **(b)(i)** 63, 32 **(c)(i)** 148 **(ii)** 1.8, 5.2 **(d)** 5.9

Question 11

(a) (i) This was a simple test of Pythagoras's Theorem. Only a few candidates made errors or did not recognise the method.

(ii) Some candidates used the cosine rule successfully; some quoted one or other form of the cosine rule but made errors in the subsequent work. Some used various trigonometric statements, but invariably not leading to the correct answer. Many who obviously knew some trigonometry from their work in later parts left this part blank.

(b) (i) Nearly all candidates could calculate the one missing angle in the horizontal triangle *EBF*.

(ii) This was a test of the use of the sine rule and many candidates did this successfully.

(iii) The majority of candidates were unfamiliar with the phrase "angle of depression". Those who did know the meaning had no trouble finding its value; its complement appeared a few times for one mark.

Answers: **(a)(i)** 18.6 **(ii)** 11.2 **(b)(i)** 50° **(ii)** 11.8 **(iii)** 51.9

Question 12

(a) (i) Most attempts used the correct methods but slips in the calculations were sometimes made.

(ii) A full range of responses were seen here. Many knew the rules for transposing the elements of matrix **A** and many correctly worked out the determinant and divided by it.

(b) (i) Many candidates could not interpret the information correctly to complete the matrix **Q**. Many did not attempt this part.

(ii) Just a few candidates could find **PQ**. Many left their response as a 3 × 2 matrix.

(iii) There were some able candidates who produced the difference 3 and knew it was the extra distance achieved by Luke. However not many candidates had enough information from the previous parts to enable them to say anything sensible about the meaning of **PQ**.

(c)

(i) This was well answered by most candidates although some had trouble reading the times from the graph.

(ii) This was a straightforward test of speed = distance/time and was successful for many candidates.

(iii) This part was quite often done correctly.

(iv) The candidate had to notice that the line from the stadium to the café is steeper than the other two lines showing movement. Many gave a choice of two movements, misinterpreting the "and" in the answer space.

For this part **(c)** there were many non-attempts. Perhaps earlier work in the paper had cost too much time.

Answers: **(a)(i)** $\begin{pmatrix} -5 & 6 \\ 0 & -2 \end{pmatrix}$ **(ii)** $\frac{1}{6}\begin{pmatrix} 2 & -6 \\ 2 & -3 \end{pmatrix}$ **(b)(i)** 1.5, 2 **(ii)** $\begin{pmatrix} 112 \\ 115 \end{pmatrix}$ **(iii)** 3

(c)(i) 138 **(ii)** 44 **(iii)** 28 **(iv)** Stadium, café

MATHEMATICS

Paper 4024/22
Paper 22

Key messages

All working should be clearly shown and answers accurately transcribed to the appropriate answer space.
Accuracy with basic number work is essential.
Final answers should be given to the required degree of accuracy.
Care must be taken to form letters and digits clearly and unambiguously.

General comments

There were several straightforward questions giving the opportunity for all candidates to show their potential, along with some difficult parts to test the most able.

All five Section B questions produced a full range of marks although **Question 12** was less popular than the others, possibly because it was the last question. Most candidates appeared to have sufficient time to complete as much of the paper as they were able.

The presentation was generally good with almost all candidates answering the questions in the space provided. Construction work was generally well done, although candidates should be advised to leave any construction arcs that they have used.

The tendency for candidates to make early approximations was an issue and caused the loss of many marks. Candidates should be aware that they must work with at least four significant figures and then give their final answer to three. Better use of their calculators, and in particular, the memory facility, would be of great benefit to many candidates.

Comments on specific questions

Section A

Question 1

(a) There were many correct descriptions of the shape. Occasionally the shaded area was split into smaller shapes and their descriptions were listed.

(b) Again, many correct answers were seen although $x \le 5$ was sometimes confused with $y \le 5$, and $y \le 6 - x$ was given as $y \le x + 6$ or $y \le x - 6$. A small number of candidates gave equalities or reversed inequalities.

(c) A few candidates drew lines parallel to one of the other lines on the diagram. A few used (0, 5) rather than (5, 0) but most had the right idea and drew lines of sufficient accuracy to gain the mark.

(d) This proved to be the most challenging part of the question, with many giving the answer + 1, the gradient of the given line $y = x + 2$, rather than that of the perpendicular.

Answers: **(a)** pentagon **(b)** $x \le 5$, $y \le 6 - x$ **(c)** line through (5, 0) and (8, 3) **(d)** – 1

Question 2

(a) Most candidates attempted to use a common denominator, but many made errors here or when they attempted to multiply through. A number got as far as 5*x* = 3 but continued with $x = \frac{5}{3}$.

(b) There were very many correct answers here, although a few left the factors ($y + 9$) and ($y - 9$) and same gave 9 and $\sqrt{81}$.

(c) (i) Most candidates attempted to find an expression for the area of the parallelogram, although many split the figure into two triangles or a rectangle and two triangles, making the working more difficult.

(ii) This part was very well answered with the majority using the quadratic formula rather than factorising. A number used +24 instead of –24 and finished with answers with wrong signs and a small number did not extend the fraction line under the whole of the numerator and evaluated $-24 \pm \frac{52}{8}$.

(iii) Most candidates realised that they had to add 6 to their positive value in the previous part, although a few multiplied by 6.

Answers: **(a)** $\frac{3}{5}$ **(b)** 9 or – 9 **(c)(ii)** 3.5 and – 9.5 **(iii)** 9.5

Question 3

(a) There were many correct answers but 30 was seen frequently, probably the time taken to get to Juan's house and 35 also occurred regularly.

(b) Most candidates realised that they had to divide distance by time but many used minutes and gave 0.083 as their answer.

(c) Those candidates who realised that \$4.42 was 85% of the full price almost always went on to give the correct answer, but many added 15% on to \$4.42. A few worked with 75% rather than 85%.

(d) Almost all candidates had the horizontal line for two hours, but many were unable to get the correct gradient for the return journey.

(e) Many correct answers were seen here, even from those who had drawn the graph incorrectly. The most common wrong answer was the time when they arrived at Simon's house, this usually being 21 20.

Answers: **(a)** 36 **(b)** 5 **(c)** 5.20 **(e)** 20 30

Question 4

(a) A small number of candidates gave a linear measurement but the majority quoted an angle. Some misread their protractor, some did not use the correct North South line and others did not understand that they should measure from the North clockwise, producing answers of 80, 260 , 100 or 190.

(b) This was well answered, with just a small number omitting to show their construction arcs and a few positioning *Y* to the north of *WX*.

(c) Occasionally, *Z* was on an incorrect bearing (often 18°) from *W* and sometimes the requirement 'due North of *X*' was ignored, but the most common error was to give the distance from *Z* to *W* or *Y* rather than *Z* to *X*.

Answers: **(a)** 280° **(c)** 28

Question 5

(a) (i) A significant number of candidates omitted this part and a few simply read off values at $7\frac{1}{2}$ and $8\frac{1}{2}$ or 8 and 9 but there were many correct answers.

(ii) The majority of candidates read off the value of the median correctly. Occasionally 7.5 was given and a small number gave their answer as the interval $7 < \ell \le 8$.

(iii) Again, this was well answered, although a number used 45–15 = 30 and either left their answer as 30, or gave 7.2.

(iv) A number read off 38 correctly, but then gave $\frac{38}{60} = \frac{19}{30}$ as their answer, but many understood what was required and gained both marks

(b) (i) Candidates generally demonstrated a good understanding of how to find the mean of a grouped frequency distribution and the majority arrived at the correct answer. Arithmetical slips were fairly unusual and the most common errors were to use the end points rather than the mid-points or to divide by 6 rather than 60.

(ii) Most candidates correctly added 13, 5 and 3 to obtain 21 leaves but this was not always correctly changed to a percentage.

Answers: **(i)** 25, 9 **(ii)** 7.2 **(iii)** 1.2 **(iv)** $\frac{11}{30}$ **(b) (i)** 5.65 **(ii)** 35

Question 6

(a) (i) This part was well answered, although many candidates added an extra element, usually 2.

(ii) Some candidates did not understand the notation and gave the answer 3 or {3}.

(iii) Many candidates correctly deduced that *C* could contain the elements 2, 5, 7, 11, 13 but did not realise that it must only have 3 elements in it. A significant number of candidates included 3 in their answer with {3, 5, 7} seen frequently.

(b) The shading of almost all candidates was within *P* and *R*, suggesting that they had interpreted ($P \cup R$) correctly. The most common errors were to also shade one or more of the intersections of this with *Q*.

(c) This was very well answered with the majority using a Venn diagram. A small number with the right idea lost a mark with a careless arithmetical error.

Answers: **(a)(i)** 4, 8, 10, 14 **(ii)** 1 **(iii)** 3 out of 2, 5, 7, 11, 13 **(c)** 16

Question 7

Many candidates still attempt to use basic mathematical procedures such as division, squaring, square root, algebraic multiplication…. when manipulating vectors. Writing down a path to follow would assist many candidates i.e. $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$.

(a) (i) A straightforward start, with many correct answers seen, although **a** – **b** was seen frequently.

(ii) Candidates found this part challenging for a relatively standard type of question. Relatively few realised that the first step was to find $\overrightarrow{OD}$.

(iii) Very few realised that the vector $\overrightarrow{OE}$ and $\overrightarrow{ED}$ were equal and that they could use the easy step from their answer to the previous part.

(b) Many candidates omitted some or all of these three parts and only the most able candidates made any real progress.

Answers: **(a)(i)** **b** – **a** **(ii)** $\frac{1}{2}$ (**b** + **c**) **(iii)** $\frac{1}{4}$**b** + $\frac{1}{2}$ **c** **(b)(i)** $\frac{2}{5}$ (**b** – **a**) **(ii)** 2 : 3 **(iii)** $\frac{3}{5}$ **a** – $\frac{7}{20}$ **b** – **c**

Section B

Question 8

This proved to be a popular question and able candidates, choosing appropriate methods and avoiding the temptation to approximate prematurely, were able to gain good marks.

Many candidates made incorrect assumptions, such as, in part **(a)** that *AC* bisected the right angle. Some candidates gained little credit in part **(a)**, but then gained all, or most, of the marks in part **(b)**.

(a) (i) Most candidates used the cosine rule to find angle *ABC* directly.
Some stated $21.3^2 = 20^2 + 2^2 – 2 \times 20 \times 2 \cos B$ but then reduced the right hand side to 324 cos *B* or else made a sign error to obtain *B* = 51.6°, even though the diagram clearly showed that it must be obtuse. Others used the cosine rule to find either *CAB* or *BCA* and then the sine rule to find *ABC*, but this inevitably produced the acute angle (51.6) rather than the obtuse one.

(ii) Many different methods were used here, but very many started with angle *ACD* = 45° and gained no marks. Others correctly found *DAB* to be 51.6° but then took it to be *DAC*.

(b) (i) This was well answered by most candidates, although a small number gave the answer 209°, either confusing with bearings or simply subtracting from 360° instead of 180°.

(ii) A number of unnecessarily long methods were seen, but most candidates used the sine rule correctly in triangle *CDE*. Apart from those who used premature approximation, most candidates gained all 3 marks.

(iii) The less able candidates did not realise the 3D nature of this part of the question and attempted trigonometrical and/or triangle calculations involving 118°. Most of the remaining candidates knew they were working in the right angled triangle *BCE* but a good proportion found the angle *CBE*.

Answers: **(a)(i)** 128.4° **(ii)** 14.4 **(b)(i)** 29° **(ii)** 9.66 , **(iii)** 11.7°

Question 9

(a) (i) Candidates generally had no problem with the scale multiplication of matrix **B**, but errors were often made in the subsequent adding and subtracting of the directed numbers with –3 – (–4) becoming –7 and 0 – (–2) becoming –2.

(ii) This is a popular tried and tested procedure and outcomes showed that most candidates knew what was required. A few made mistakes in calculating the determinant, –6 being the most common wrong value. The adjoint was usually correct.

(b) (i) There were some arithmetical errors seen but most answers were given as 2 × 1 matrices.

(ii) A small number of candidates wrote about 'the areas of carpet and underlay', but most realised they were dealing with costs.

(c) (i) This proved to be quite challenging for many candidates. Very often the figure drawn was a rotation of 180° about the origin. Many who correctly drew the line $y = -x$ were still unable to find all three points correctly.

(ii) Candidates were more successful with this transformation, although a number produced a translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

(iii) The ideas involved with the stretch transformation were largely misunderstood and relatively few candidates gave the correct answers.

Answers: **(i)** $\begin{pmatrix} -5 & 0 \\ 1 & 2 \end{pmatrix}$ **(ii)** $\frac{1}{6}\begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix}$ **(b)(i)** $\begin{pmatrix} 974 \\ 328 \end{pmatrix}$ **(c)(iii)(a)** 4 (or –4) , **(b)** $m = 1$, $n = 4$

Question 10

Even though the diagram showed the major segment and the stem stated this fact there was much confusion in the minds of very many candidates between segment and sector.

(a) (i) Many candidates gained two marks by finding $\frac{5}{6}$ of the area of the circle, but relatively few found the area of the triangle. Some of those who did, subtracted it from the area of the sector.

(ii) Many correctly found the major arc length *AB*, but the majority then added 30 (and sometimes 45), confirming the segment/sector confusion.

(b) Candidates had more success here. Wrong answers usually involved a wrong length for *AB*, sometimes 25 and sometimes 15.7, the length of the minor arc *AB*.

(c) (i) Almost all candidates realised that the reduced radius was one fifth of 15.

(ii) Relatively few candidates used the idea of similar figures to divide their answer to part **(a)(i)** + 248 by 25. Most attempted to find the three separate areas with their reduced dimensions, but the arithmetic defeated most.

Answers: **(i)** 686 **(ii)** 93.5 **(b)** 12.4 **(c)(i)** 3 **(ii)** 37.4

Question 11

(a) The first three parts were very often correct, candidates applying angle properties of circles and triangles successfully. Less success in the next two parts often resulted from not appreciating that angles *BAC* and *DCA* were alternate angles or from assuming that *OCA* and *DAC* were alternate angles when they are not.

(b) (i) Many candidates realised that angle *QTR* was 32 because they were alternate angles, but few gave *SPQ* to be 116 with acceptable terms or reasoning.

(ii) Many gave the correct arc of 5cm, but rather fewer could give the locus of points 4cm from *PS*. A significant number measured 4cm along *PQ* and/or along *SR*.

(iii) Most of the candidates who showed good understanding of the constructions in part **(ii)** were able to achieve the mark for the shaded region.

Answers: **(a)(i)** 56° **(ii)** 34° **(iii)** 62° **(iv)** 42° **(v)** 110° **(b)(i)(a)** 32°, alternate
(b) 116° , allied or interior or adjacent

Question 12

(a) There were very few convincing explanations, with many candidates not recognising the difference between a 'cuboid' and a 'cube' or assuming that the height of the cuboid had to be greater than the length of the base.

(b) This proved to be the most challenging part of the whole paper. Some candidates gave the correct expression for volume of the sphere, although most omitted the brackets and gave $\frac{4}{3}\times 3\times\frac{x^3}{2}$, and a few gave the correct expression for the cuboid x^2 $(8 - x)$, but it was rare to see both.

(c) Most candidates who attempted this question gained marks in this part. Almost all achieved the mark for 58.5 and there were many excellent graphs drawn, although the position of (7,220.5) was sometimes misplaced with candidates apparently thinking that the curve should look like a cumulative frequency curve. Many candidates read off the value of x when the volume of the solid was 120 cm^2 correctly (4.6) but most gave this as their answer, not realising that the height required was $8 - x$.

(d) This part again proved to be challenging. A few did get as far as $27x$ and many of these did plot some points which they attempted to join freehand, apparently not realising that it was a straight line which should be drawn using a ruler. Most of those who did draw the line (freehand or with ruler) managed to give an acceptable value of x, although a number rounded their answer to 5.

Answers: **(c)(i)** 58.5 **(iii)** 3.4 **(d)** 4.8