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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

8094874271

MATHEMATICS (SYLLABUS D) **4024/22**

Paper 2 **October/November 2013**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

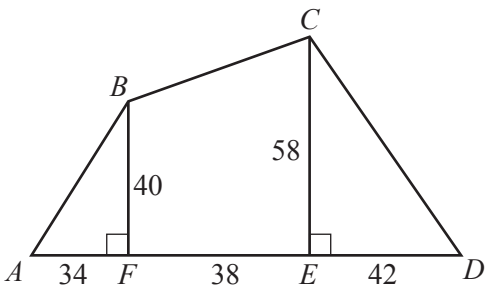
This document consists of **24** printed pages.

Section A [52 marks]

Answer **all** questions in this section.

For Examiner's Use

1



ABCD is a level field.
F and *E* are points on *AD* such that *BF* and *CE* are perpendicular to *AD*.
BF = 40 m and *CE* = 58 m.
AF = 34 m, *FE* = 38 m and *ED* = 42 m.

(a) Calculate the area of the field.

Answer ..... m^2 [3]

(b) Calculate the length of BC.

For Examiner's Use

Answer ..... m [2]

(c) Calculate $C\hat{D}E$.

Answer .....[2]

For Examiner's Use

2 **(a)** The results of a survey of the number of cars owned by 50 families are given in the table below.

Number of cars	0	1	2	3
Number of families	4	35	6	5

(i) Calculate the mean number of cars per family.

Answer ..... [2]

(ii) When the same 50 families were surveyed at a later date, the results were as follows.

Number of cars	0	1	2	3
Number of families	x	37	y	5

The mean number of cars per family stayed the same as before.

Find x and y.

Answer $x =$ .....

$y =$ ..... [2]

For Examiner's Use

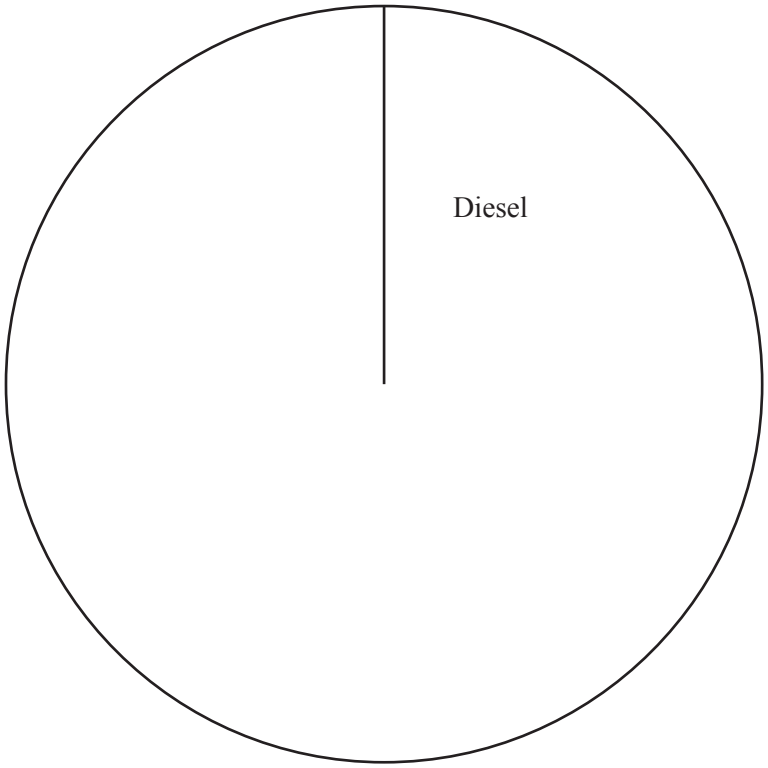
(b) A service station sells diesel, unleaded and super unleaded fuel.
During one week, 13 500 litres of diesel and 36 000 litres of unleaded were sold.
The total number of litres of fuel sold that week was 54 000.

(i) What fraction of the total number of litres sold was super unleaded?
Give your answer in its lowest terms.

Answer ..... [1]

(ii) Complete the pie chart to represent the amounts of fuel sold.

Answer



[3]

For Examiner's Use

3 **(a)** Find the value of $\dfrac{a+\sqrt{a^2+b^2}}{a^2-2ab}$ when $a=-4$ and $b=-3$.

Give your answer as a fraction.

Answer ..... [2]

(b) Expand the brackets and simplify $(3x^2-1)(2x+3)-x(9x-2)$.

Answer ..... [2]

(c) **(i)** Factorise $9x^2+5x-4$.

Answer ..... [1]

(ii) Use your answer to **part (c)(i)** to solve the equation $9x^2 + 5x - 4 = 0$.

For Examiner's Use

Answer $x =$ ..... or ..... [1]

(d) The sum of three consecutive integers is 84.

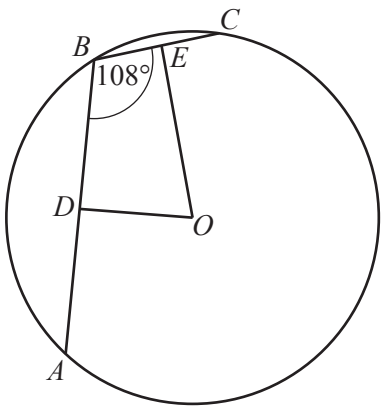
Find these three integers.

Answer ..... , ... , ... [2]

For Examiner's Use

4 **(a)** AB and BC are chords of a circle centre O.
D is the midpoint of AB and E is the midpoint of BC.
$A\hat{B}C = 108°$.

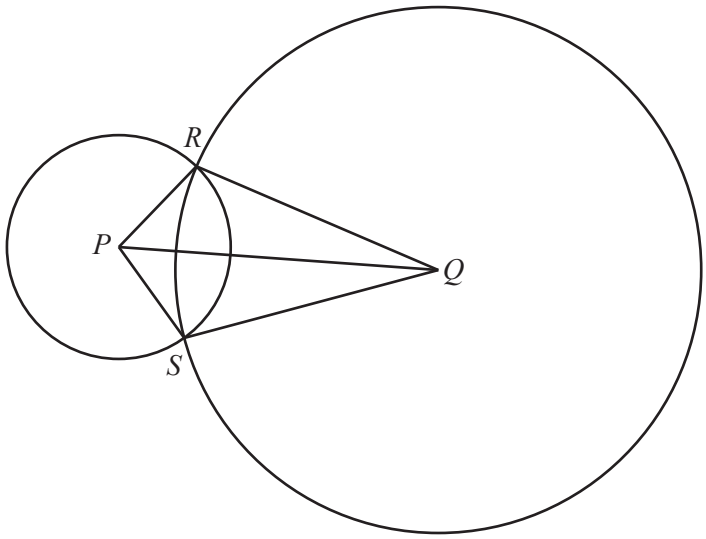
Find $D\hat{O}E$ giving your reasons.



Answer $D\hat{O}E$ = ... because .....

..... [2]

(b)

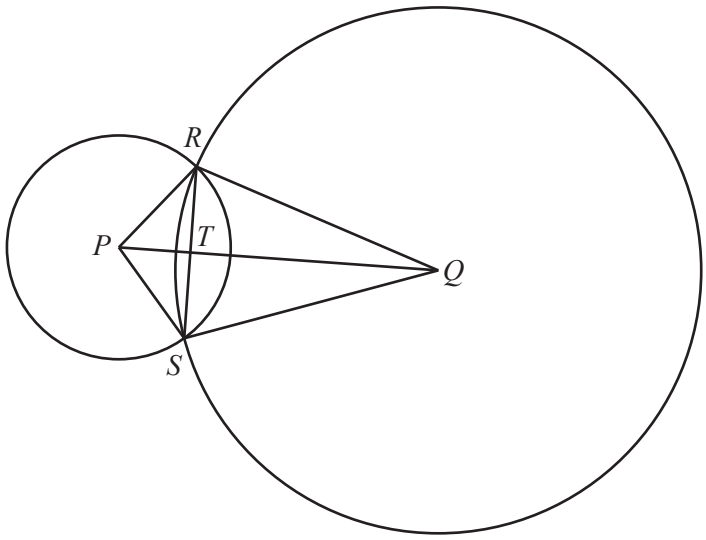


A circle centre P and a circle centre Q intersect at R and S.

(i) Show that triangle PRQ is congruent to triangle PSQ.

[3]

(ii)



RS and PQ intersect at T.

(a) State the name of the special quadrilateral $PRQS$.

Answer ..... [1]

(b) Find $P\hat{T}R$.

Answer ..... [1]

For Examiner's Use

5 **(a)** $\mathscr{E} = \{x : x \text{ is an integer and } 2 \leqslant x \leqslant 12\}$
$M = \{x : x \text{ is a multiple of } 3\}$
$P = \{x : x \text{ is a prime number}\}$

(i) $a \in M \cap P$

Find a.

Answer ..... [1]

(ii) Find $(M \cup P)'$.

Answer ..... [1]

(b) In a survey, 90 people were asked "Do you own a car?" and "Do you own a bicycle?".
A total of 27 people said they owned a bicycle.
Of these, 13 owned **only** a bicycle.
11 people owned neither a car nor a bicycle.

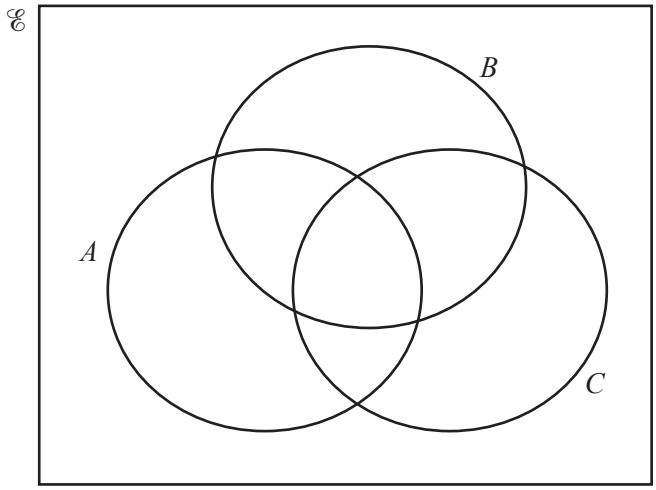
By drawing a Venn diagram, or otherwise, find how many people said that they owned a car.

Answer ..... [2]

(c) The Venn diagrams show a Universal set, $\mathscr{E}$, and subsets A, B and C.

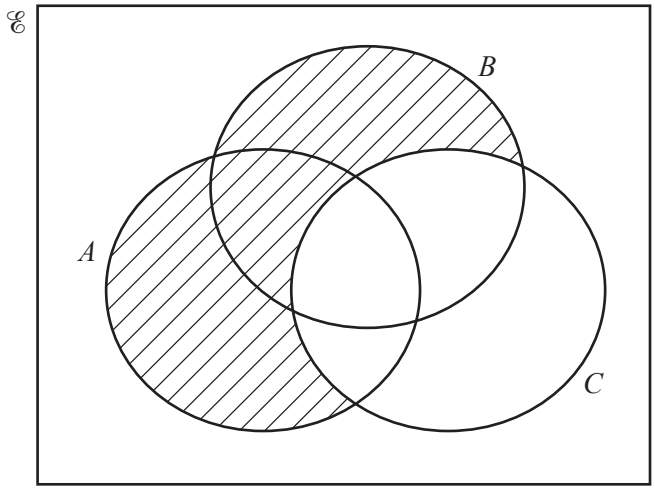
For Examiner's Use

(i) Shade the set $(A \cup C)' \cap B$.



[1]

(ii) Express in set notation the subset shaded in this diagram.



Answer ..... [1]

For Examiner's Use

6 **(a)** **(i)** The cost price of bicycle A is $620.
The shopkeeper sells it and makes a profit of 45%.

Calculate the selling price.

Answer $..... [1]

(ii) In a sale, the price of bicycle B is reduced from $2400 to $1596.

Calculate the percentage reduction given.

Answer .....% [2]

(iii) Tax on the original price of bicycle C is charged at 20% of the original price.
After tax has been included, Matthew pays $1080 for this bicycle.

Calculate the original price.

Answer $..... [2]

(b) Ada invests $600 in an account that earns simple interest.
At the end of 3 years, the investment is worth $681.

Calculate the rate of simple interest per year.

Answer .....% [3]

For Examiner's Use

7 **(a)** Express as a single matrix $5\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 4\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$.

Answer [2]

(b) Express as a single matrix $\begin{pmatrix} 7 & -1 & 3 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$.

Answer [2]

(c) $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix}$

(i) Find $\mathbf{A}^{-1}$.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) $\mathbf{B} + 3\mathbf{I} = \mathbf{A}$ where $\mathbf{I}$ is the 2×2 identity matrix.

Find $\mathbf{B}$.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

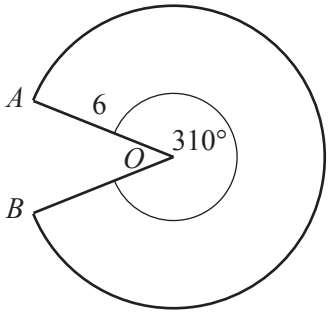
Section B [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

For Examiner's Use

8



The diagram shows a sector AOB of a circle with centre O and radius 6 cm.
The angle of the sector is 310°.

(a) Calculate the total perimeter of the sector.

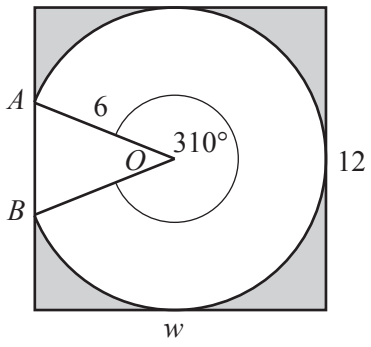
Answer ..... cm [3]

(b) Calculate the area of the sector.

Answer ..... cm^2 [2]

(c) This sector is cut from a rectangular piece of card of height 12 cm and width w cm.

For Examiner's Use



One edge of the rectangular piece of card passes through A and B.
The other edges are tangents to the circle.

(i) Calculate the value of w.

Answer ..... [3]

(ii) When the sector is cut out, the triangle AOB is retained.
The rest of the rectangular piece of card, shown shaded, is discarded as waste.

Calculate the percentage of the rectangular piece of card that is discarded as waste.

Answer .....% [4]

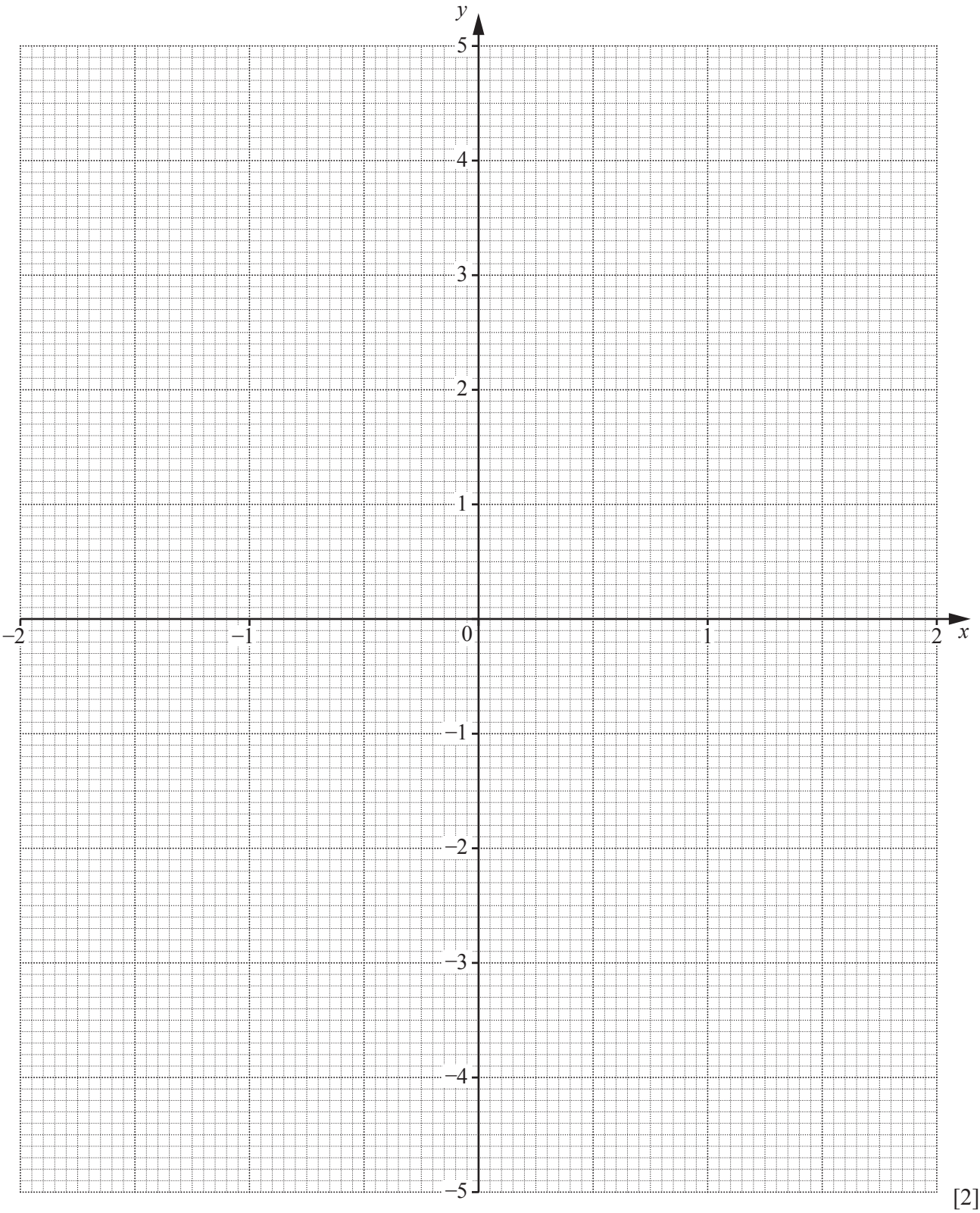
For Examiner's Use

9 The variables x and y are connected by the equation $y = x + \frac{1}{x}$.

The table below shows some values of x and the corresponding values of y.
The values of y are correct to 2 decimal places where appropriate.

x	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	4.25	2.5	2.08	2	2.05	2.17	2.32	2.5

(a) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

For Examiner's Use

(b) By drawing a tangent, estimate the gradient of the curve when $x = 0.75$.

Answer ..... [2]

(c) Let $f(x) = x + \frac{1}{x}$.

(i) Given that $f(a) = b$, find $f(-a)$ in terms of b.

Answer ..... [1]

(ii) Hence, or otherwise, complete the table below for $y = x + \frac{1}{x}$.

x	–2	–1.75	–1.5	–1.25	–1	–0.75	–0.5	–0.25
y					–2			

[1]

(iii) On the grid opposite, draw the graph of $y = x + \frac{1}{x}$ for $-2 \leqslant x \leqslant -0.25$. [1]

(iv) Write down an estimate for the gradient of the curve when $x = -0.75$.

Answer ..... [1]

(d) **(i)** On the grid opposite, draw the graph of the straight line $y = 4 - x$. [1]

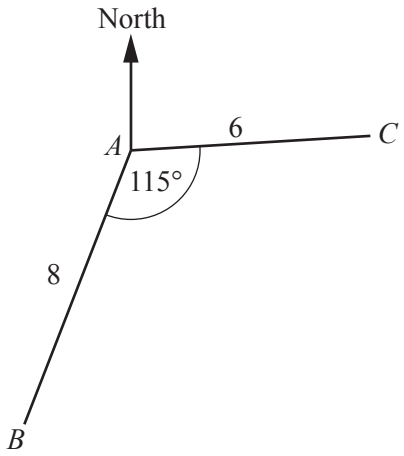
(ii) Write down the x-coordinate of each of the points where the graphs of $y = 4 - x$ and $y = x + \frac{1}{x}$ intersect.

Answer $x =$ ..... and ..... [1]

(iii) Find the equation for which these x values are the solutions.
Give your equation in the form $Ax^2 + Bx + C = 0$.

Answer ..... [2]

10 (a)



Two boats sail from A. One boat sails to B, and the other boat sails to C.
$AB = 8$ km, $AC = 6$ km and $B\hat{A}C = 115°$.

(i) Calculate the distance, BC, between the boats.

Answer ..... km [4]

(ii) The bearing of B from A is 200°.

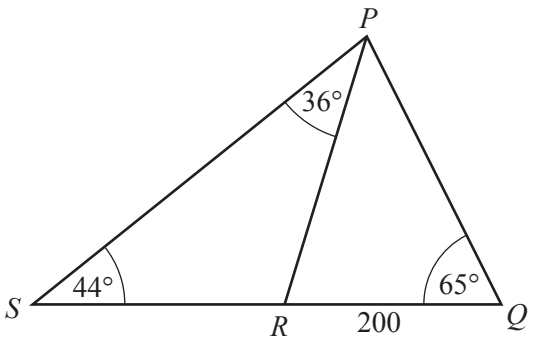
Find the bearing of A from C.

Answer ..... [2]

For Examiner's Use

(b)

For Examiner's Use



In triangle PQS, $S\hat{Q}P = 65°$ and $Q\hat{S}P = 44°$.
R is the point on QS such that $QR = 200$ m and $R\hat{P}S = 36°$.

(i) In triangle PQR, by using the sine rule, show that $PR = \dfrac{200 \sin 65}{\sin 35}$.

[2]

(ii) Hence show that $SR = \dfrac{200 \sin 65 \sin 36}{\sin 35 \sin 44}$.

[2]

(iii) Hence find the length of SR.

Answer ..... m [1]

(iv) Hence evaluate $\dfrac{\text{area of triangle } SPQ}{\text{area of triangle } PQR}$.

Answer ..... [1]

For Examiner's Use

11 **(a)** Express as a single fraction, in its simplest form, $\frac{7}{p+2} - \frac{4}{2p-3}$.

Answer ..... [3]

(b) The distance between London and York is 320 km .
A train takes x hours to travel between London and York.

(i) Write down an expression, in terms of x, for the average speed of the train.

Answer ..... km/h [1]

(ii) A car takes $2\frac{1}{2}$ hours longer than a train to travel between London and York.
The average speed of the train is 80 km/h greater than the average speed of the car.

Form an equation in x and show that it simplifies to $2x^2 + 5x - 20 = 0$.

[3]

(iii) Solve the equation $2x^2 + 5x - 20 = 0$, giving your answers correct to 2 decimal places.

For Examiner's Use

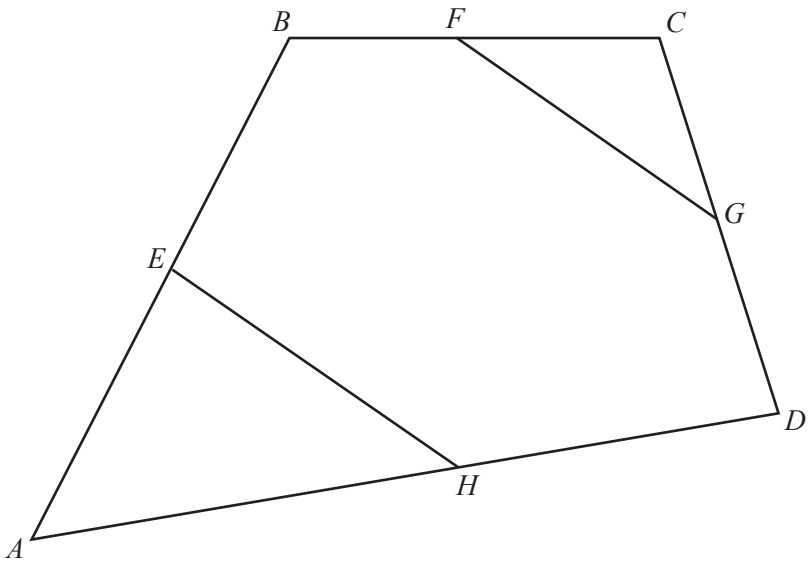
Answer $x =$ ..... or ..... [3]

(iv) Hence find the average speed of the car correct to the nearest km/h.

Answer .....km/h [2]

For Examiner's Use

12 (a)



(i) $\overrightarrow{AD} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

Calculate $|\overrightarrow{AD}|$.

Answer ..... [1]

(ii) $\overrightarrow{AE} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

H is the midpoint of AD.

Find $\overrightarrow{EH}$.

Answer $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

For Examiner's Use

(iii) $\overrightarrow{BF} = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ $\overrightarrow{CG} = \begin{pmatrix} 0.5 \\ -1.5 \end{pmatrix}$

F is the midpoint of BC.

Find $\overrightarrow{FG}$.

Answer $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

(iv) Use your answers to **parts (ii)** and **(iii)** to complete the following statement.

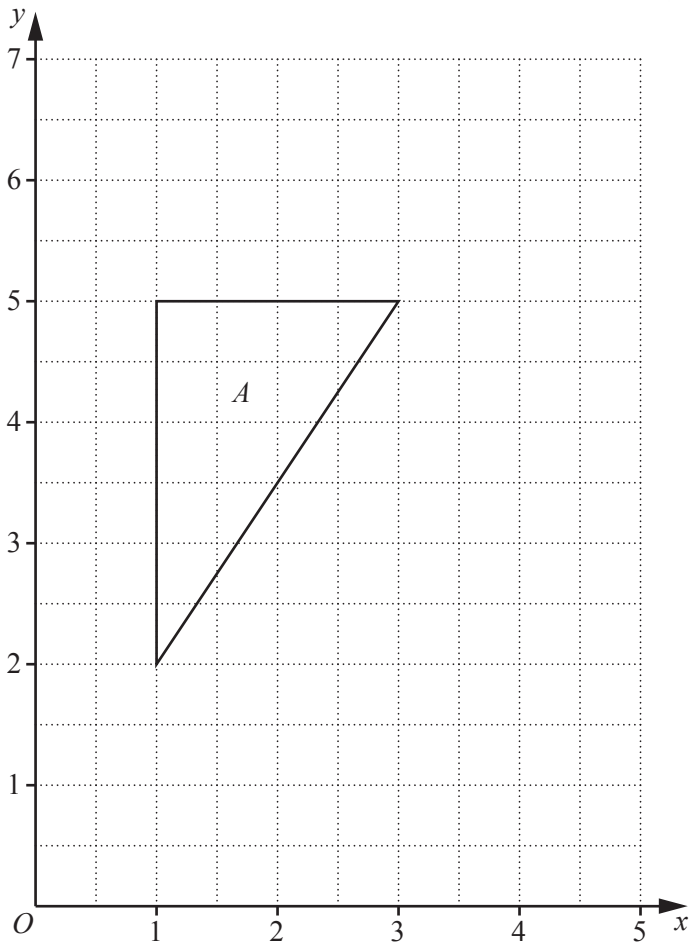
The lines EH and FG are ..... and ..... [1]

(v) Given that E is the midpoint of AB, show that G is the midpoint of CD.

[2]

TURN OVER FOR THE REST OF THIS QUESTION

(b)



For Examiner's Use

Triangle A has vertices (1, 2), (1, 5) and (3, 5).

(i) An enlargement, centre (1, 2), scale factor 1.5, maps triangle A onto triangle B.

Draw triangle B. [2]

(ii) An enlargement, centre (1, 2), scale factor -0.5, maps triangle A onto triangle C.

Draw triangle C. [2]

(iii) Find the ratio area of triangle C : area of triangle B.

Answer ..... : ..... [1]