This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO  Correct Working Only often written by a ‘fortuitous’ answer
ISW  Ignore Subsequent Working
MR   Misread
PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.
### 1. \[ \int \left( x^3 + \frac{1}{x^3} \right) \, dx = \frac{x^4}{4} + \frac{x^{-2}}{-2} + c \]

3 \times B1  

[3] Allow unsimplified, 1 mark for each term, including “c”

### 2. \[ (1 - \frac{3}{4}x)^6 \]

**(i)** Term in \( x^2 \)

\[ 6C_2 \times \left( \frac{\pm 3x}{2} \right)^2 = \frac{135x^2}{4} \]

M1  

A1  

Term in \( x^3 \)

\[ 6C_3 \times \left( \frac{\pm 3x}{2} \right)^3 = \frac{-540x^3}{8} \]

A1  

\[ \text{For either unsimplified term co} \]

\[ \text{co (omission or error with “−” can still gain 2 out of 3)} \]

**(ii)** Term in \( x^3 \)

\[ = \frac{270x^3}{4} - \frac{135kx^3}{2} \]

\[ \text{→ } k = 1. \]

M1  

A1  

\[ \text{co (omission or error with “−” can still gain 2 out of 3)} \]

### 3. **(i)** \[ x^2 + px + q = (x + 3)(x - 5) \]

\[ \text{→ } p = -2, \quad q = -15. \]

(any other method ok)

M1  

A1  

\[ \text{Must be } (x + 3) \text{ and } (x - 5). \]

**(ii)** \[ x^2 + px + q + r = 0 \]

Use of \( b^2 - 4ac \)

Uses \( a, b \) and \( c \) correctly

\[ r = 16 \]

or

\( (x + k)^2 \rightarrow 2k = p \, (M1) \]

\[ k^2 = q + r \, (M1) \]

\[ \text{→ } k = -1 \rightarrow r = 16 \, (A1) \]

M1  

DM1  

A1  

\[ \text{Any use of } “b^2 - 4ac” \]

\[ c \text{ must include both } q \text{ and } r. \]

### 4. \[ y = \frac{4}{3x - 4} \]

**(i)** \[ \frac{dy}{dx} = -4(3x - 4)^{-2} \times 3 \]

\[ \text{If } x = 2, \quad m = -3 \]

\[ \text{Eqn of tangent } \quad y - 2 = -3(x - 2) \]

B1 B1  

M1 A1  

[4] Correct without \( \times 3 \). For \( \times 3 \).

**(ii)** \[ \tan \theta = \pm(-3) \]

\[ \text{→ } \theta = \pm108.4^\circ \quad (\text{or } \pm71.6^\circ) \]

\[ \text{or scalar product, } \tan \theta = y\text{-step } \div x\text{-step} \]

\[ \text{or use of } \tan (A - B) \quad \text{M1A1 for each} \]

M1  

A1  

\[ \text{Correct link with } (\pm \text{ his gradient}) \]

\[ \text{co (accept acute or obtuse) or } -71.6^\circ \text{ or radians} \]
5 (i) \[
\begin{align*}
\frac{\cos \theta}{\tan \theta (1 - \sin \theta)} &= \frac{\cos^2 \theta}{\sin \theta (1 - \sin \theta)} \\
&= \frac{1 - \sin^2 \theta}{\sin \theta (1 - \sin \theta)} \\
&= \frac{1 + \sin \theta}{\sin \theta} = 1 + 1
\end{align*}
\]  
M1 Use of \( t = s + c \)

(ii) \[
\begin{align*}
\frac{\cos \theta}{\tan \theta (1 - \sin \theta)} &= 4 \rightarrow \frac{1}{\sin \theta} + 1 = 4 \\
\rightarrow \sin \theta &= \frac{1}{3} \rightarrow \theta = 19.5^\circ, 160.5^\circ
\end{align*}
\]  
M1 Replaces \( \cos^2 \theta \) with \( 1 - \sin^2 \theta \) to form \( f(\sin \theta) \).
A1 AG. Ensure all ok. Must show difference of 2 squares.

6 (i) \( f(x) = \frac{x + 3}{2x - 1} \)

\[ f(f(x)) = \frac{x + 3}{2x - 1} + 3 = \frac{7x + 7}{2x - 1} \]

B1 Replacing “x” twice - must be correct
M1 Correct algebra – clearing \((2x - 1)\)
A1 AG – all correct.

(ii) \( y = \frac{x + 3}{2x - 1} \)

\[ \rightarrow 2xy - y = x + 3 \]

M1 Attempt to make x the subject and complete method
\[ \rightarrow x(2y - 1) = y + 3 \]

\[ \rightarrow f^{-1}(x) = \frac{x + 3}{2x - 1} \]

A1 (M1, A1)

\textbf{or} since \( f(f(x)) = x \),

\[ f^{-1}(x) = f(x) = \frac{x + 3}{2x - 1} \]

\[ \text{(M1, A1)} \]

7 (i) \((2, 5)\) to \((10, 9)\) gradient = \( \frac{1}{2} \)

Equation of \( L_2 \)

\[ y = \frac{1}{2} x \]

Gradient of perpendicular = \(-2\)

Eqn of Perp

\[ y - 5 = -2(x - 2) \]

Sim Eqns \( \rightarrow C(3.6, 1.8) \)

B1 co
B1√ \( \sqrt{\text{on gradient of } L_1} \)
M1 Use of \( m_1m_2 = -1 \)
M1 Correct form of line eqn
A1 co

(ii) \( d^2 = 1.6^2 + 3.2^2 \rightarrow d = 3.58 \)

M1 Correct method for \( AC \)
A1 co (accept with \( \sqrt{5} \) in answer)
### 8 (i) \( \overrightarrow{BA} \cdot \overrightarrow{BC} \) or \( \overrightarrow{AB} \cdot \overrightarrow{CB} \)

\[
\overrightarrow{BA} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \quad \overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}
\]

\[
\overrightarrow{BA} \cdot \overrightarrow{BC} = -8 = 3 \times 7 \times \cos \theta
\]

\[
\rightarrow \quad \theta = 112.4^\circ \text{ or } 1.96 \text{ radians}
\]

(ii) \( \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC} \)

\[
= \begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}
\]

| Correct two vectors for angle \( ABC \). | B1 |
| Correct method for one of the sides. | M1 |
| Correct use for any pair of vectors. | M1 |
| Correct method for moduli. | M1 |
| All linked correctly. co | A1 |
| (67.6° usually gets 4/6) | |
| Correct method. (allow for \( d = a + b - c \) or for \( d = a + c - b \) or for \( d = b + c - a \)) | M1 |
| A1√ for his \( \overrightarrow{BC} \). | |

### 9 (i) (a) \( f(x) = 3 - 4\cos^2 x \).

One limit is \(-1\)

Other limit is \(3\)

(b) \( 3 - 4\cos^2 x = 1 \rightarrow \cos^2 x = \frac{1}{2} \)

\[
\rightarrow \quad \cos x = \pm \frac{1}{\sqrt{2}}
\]

\[
\rightarrow \quad x = \frac{1}{4} \pi \text{ or } \frac{3}{4} \pi
\]

(ii) (a)

(b) \( f \) has an inverse since it is \(1:1\) or increasing or no turning points.

| Correct irrespective of inequalities | B1 |
| Correct irrespective of inequalities | B1 |
| Makes \( \cos x \) the subject. co (radians). √ for “\( \pi - (1^{st} \text{answer}) \)” (“exact” means that decimal answers only earn A0 A1√) | M1 |
| Joins \((0, -1)\) to \((\pi, 7)\), providing increasing function | B1 |
| Not a line, flattens at extremities-needs inflexion. | B1 |
| co independent of part (i) | B1 |
| 10 (a) | $a + 5d = 4a$ or $\frac{(a + 4a)}{2} \times 6$ | B1 | co |
|        | $\frac{6}{2}(2a + 5d)$ or $\frac{(a + 4a)}{2} \times 6 = 360$ | M1 | A1 | Correct left-hand side. All correct. |
|        | Sim Eqns $a = 24^\circ$ or $\frac{2\pi}{15}$ rads | A1 | Either answer. |
|        | Arc length $= 5\theta$ | M1 | Correct use of arc length with $\theta$ in rads. |
|        | Perimeter $= 12.1$. | A1 | co |

| 10 (b) | $\frac{k + 6}{2k + 3} = \frac{k}{k + 6}$ | M1 | A1 | Correct eqn for $k$. |
|        | $\rightarrow k^2 - 9k - 36 = 0 \rightarrow k = 12$ | A1 | Co condone inclusion of $k = -3$. |
|        | (NB stating $a, ar, ar^2$ as $f(k)$ gets M1) | 3 |
|        | $r = \frac{2}{3}$, $a = 27$ | M1 | A1 | Correct formula for $S_\infty$ must have $-1 \leq r \leq 1$. co. |
|        | $\rightarrow S_\infty = 27 \div \frac{2}{3} = 81$. | 2 |

| 11 | $y = 4\sqrt{x} - x$. | B1 | co – independent of working. |
|     | (i) At $A$, $4\sqrt{x} - x = 0 \rightarrow A(16, 0)$ | B1 | B1 |
|     | $\frac{dy}{dx} = 2x^{-\frac{1}{2}} - 1$ | B1 | B1 for each part. |
|     | $= 0$ when $x = 4 \rightarrow (4, 4)$ | M1 | A1 | Sets to 0 and solves his eqn. co |

|     | (ii) Vol $= \pi \int y^2 \, dx =$ | M1 | Use of correct formula + attempt at integration |
|     | $\pi \int (16x + x^2 - 8x^{\frac{3}{2}}) \, dx$ | A3,2,1 | One mark for each term – unsimplified |
|     | $\pi \left[ 8x^2 + \frac{x^3}{3} - 8x^{\frac{5}{2}} \right]$ | DM1 | Correct use of his limits. |
|     | Limits $0$ to $16 \rightarrow 136.5\pi$. (or $137\pi$) | A1 | co – (429 ok) |