MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/31

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.



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The following abbreviations may be used in a mark scheme or used on the scripts:

- AEF Any Equivalent Form (of answer is equally acceptable)
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- CWO Correct Working Only – often written by a 'fortuitous' answer
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{2}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.



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1	Either:	Obtain $1 + \frac{1}{3}kx$, where $k = \pm 6$ or ± 1	M1	
		Obtain $1-2x$	A1	
		Obtain $-4x^2$	A1	
		Obtain $-\frac{40}{3}x^3$ or equivalent	A1	
	<u>Or</u> :	Differentiate expression to obtain form $k(1-6x)^{-\frac{2}{3}}$ and evaluate $f(0)$ and $f'(0)$	M1	
		Obtain $f'(x) = -2(1-6x)^{-\frac{2}{3}}$ and hence the correct first two terms $1-2x$	A1	
		Obtain $f''(x) = -8(1-6x)^{-\frac{5}{3}}$ and hence $-4x^2$	A1	
		Obtain $f'''(x) = -80(1-6x)^{-\frac{8}{3}}$ and hence $-\frac{40}{3}x^3$ or equivalent	A1	[4]
•		$k\cos 2x$	N (1	

2	(i)	Obtain $\frac{k \cos 2x}{1 + \sin 2x}$ for any non-zero co	onstant k	M1	
		Obtain $\frac{2\cos 2x}{1+\sin 2x}$		A1	[2]

(ii) Use correct quotient or product rule M1 Obtain $\frac{x \sec^2 x - \tan x}{x^2}$ or equivalent A1 [2]

3 (i) Obtain
$$\pm \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$$
 as normal to plane B1

Form equation of p as 3x - 4y + 6z = k or -3x + 4y - 6z = k and use relevant point to find k M1 Obtain 3x - 4y + 6z = 80 or -3x + 4y - 6z = -80 A1 [3]

(ii) State the direction vector
$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$
 or equivalent B1
Carry out correct process for finding scalar product of two relevant vectors M1

Use correct complete process with moduli and scalar product and evaluate sin⁻¹ or cos⁻¹ of result M1 Obtain 30.8° or 0.538 radians A1 [4]



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4	(i)	Verify that	at -96 + 100 + 8 - 12 = 0		B1	
		Attempt to $12x^2$ Obtain 12	+ kx , inspection or use of an identity $x^2 + x - 6$	tial quotient	M1 A1	
		State $(x +$	2)(4x+3)(3x-2)		A1	[4]
		[The M1 of A and	can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ d/or <i>B</i> or equation $12x^2 + Bx + C$ and an equation in <i>B</i> and/o	and an equation in r C.]		
	(ii)	State $3^{y} =$	$\frac{2}{3}$ and no other value		B1	
		Use corre	ct method for finding y from equation of form $3^y = k$, where	k > 0	M1	
		Obtain –0	0.369 and no other value		A1	[3]
5	(i)	Use at lea	st one of $e^{2x} = 9$, $e^{y} = 2$ and $e^{2y} = 4$		B1	
		Obtain giv	ven result $58 + 2k = c$ AG		B1	[2]
	(ii)	Differenti	hate left-hand side term by term, reaching $ae^{2x} + be^{y}\frac{dy}{dx} + ce^{y}\frac{dy}{dx}$	$\frac{2y}{dx}\frac{dy}{dx}$	M1	
		Obtain 12	$e^{2x} + ke^{y}\frac{dy}{dx} + 2e^{2y}\frac{dy}{dx}$		A1	
		Substitute RHS	e (ln 3, ln 2) in an attempt involving implicit differentiation $a = 0$	at least once, where	M1	
		Obtain 10	98 - 12k - 48 = 0 or equivalent		A1	
		Obtain <i>k</i> =	= 5 and c = 68		A1	[5]
6	(i)	State or ir	nply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$		B1	
		Attempt t	o form equation from area of segment = $\frac{1}{5}$ of area of circle,	or equivalent	M1	
		Confirm §	given result $\theta = \frac{2}{5}\pi + \sin\theta$		A1	[3]
	(ii)	Use iterat	ive formula correctly at least once		M1	
		Obtain va	lue for θ of 2.11		A1	
		Show suf	ficient iterations to justify value of θ or show sign change in $(5, 2, 115)$	interval	A 1	
		Use corre	ct trigonometry to find an expression for the length of AB		M1	
		e.g. 20 sin Hence 17	1.055 or $\sqrt{200} - 200 \cos 2.11$		A 1	[5]
		$[2.1 \rightarrow 2.$	$1198 \rightarrow 2.1097 \rightarrow 2.1149 \rightarrow 2.1122]$			[~]



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7	(i)	State or in Express the	mply $dx = 2t dt$ or equivalent he integral in terms of x and dx		B1 M1	
		Obtain gi	ven answer $\int_{1}^{5} (2x-2) \ln x dx$, including change of limits A	G	A1	[3]
	(ii)	Attempt i	ntegration by parts obtaining $(ax^2 + bx)\ln x \pm \int (ax^2 + bx) \frac{1}{x}$	dx or equivalent	M1	
		Obtain (x	$(x^2 - 2x)\ln x - \int (x^2 - 2x) \frac{1}{x} dx$ or equivalent		A1	
		Obtain (x	$(x^2 - 2x)\ln x - \frac{1}{2}x^2 + 2x$		A1	
		Use limits	s correctly having integrated twice		M1	
		Obtain 15	$5 \ln 5 - 4$ or exact equivalent		A1	[5]
		[Equivale	ent for M1 is $(2x - 2)(ax \ln x + bx) - \int (ax \ln x + bx) 2dx$]			
8	(i)	Fither	Multiply numerator and denominator by $(1 - 2i)$ or equival	ent	M1	
0	(1)	<u>Littlei</u> .	Obtain –3i		A1	
			State modulus is 3		A1	
			Refer to <i>u</i> being on negative imaginary axis or equivalent as as $-\frac{1}{2}\pi$	nd confirm argume	nt A1	
		<u>Or</u> :	Using correct processes, divide moduli of numerator and de	nominator	M1	
			Subtract argument of denominator from argument of numer	ator	M1	
			Obtain $-\tan^{-1}\frac{1}{2} - \tan^{-1}2$ or $-0.464 - 1.107$ and hence $-\frac{1}{2}\pi$	or –1.57	A1	[4]
	(ii)	Show cor	rect half-line from u at angle $\frac{1}{4}\pi$ to real direction		B1	
		Use corre	et trigonometry to find required value		M1	
		Obtain $\frac{3}{2}$	$\sqrt{2}$ or equivalent		A1	[3]
	(iii)	Show, or	imply, locus is a circle with centre $(1 + i)u$ and radius 1		M1	
		Use corre	et method to find distance from origin to furthest point of ci	rcle	M1	
		Obtain 3-	$\sqrt{2}$ +1 or equivalent		A1	[3]



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9	(i)	Express c Express c Obtain 8 Use cos2	$\cos 4\theta \text{ as } 2\cos^2 2\theta - 1 \text{ or } \cos^2 2\theta - \sin^2 2\theta \text{ or } 1 - 2\sin^2 2\theta$ $\cos 4\theta \text{ in terms of } \cos \theta$ $\cos^4 \theta - 8\cos^2 \theta + 1$ $\theta = 2\cos^2 \theta - 1 \text{ to obtain given answer } 8\cos^4 \theta - 3$	9 AG	B1 M1 A1 A1	[4]
	(ii)	 (a) State Obta (b) Integ Obta 	e or imply $\cos^4 \theta = \frac{1}{2}$ $\sin 0.572$ $\sin -0.572$ grate and obtain form $k_1\theta + k_2 \sin 4\theta + k_3 \sin 2\theta$ $\sin \frac{3}{8}\theta + \frac{1}{32} \sin 4\theta + \frac{1}{4} \sin 2\theta$		B1 B1 B1 M1 A1	[3]
		Obta	$ \lim \frac{3}{32}\pi + \frac{1}{4} $ following completely correct work		A1	[3]
10	(i)	Separate Carry out Obtain $\frac{2}{\Lambda}$	variables correctly and integrate of at least one side an attempt to find A and B such that $\frac{1}{N(1800 - N)} \equiv \frac{A}{N} + \frac{2}{1800 - N}$ or equivalent	$\frac{B}{0-N}$, or equivalent	M1 M1 A1	
		Integrates Obtain 2 Evaluate and a Obtain 2 Use laws Obtain <i>N</i>	is to produce two terms involving natural logarithms $\ln N - 2 \ln (1800 - N) = t \text{ or equivalent}$ a constant, or use $N = 300$ and $t = 0$ in a solution involving a ct $\ln N - 2 \ln (1800 - N) = t - 2 \ln 5 \text{ or equivalent}$ of logarithms to remove logarithms $V = \frac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}} \text{ or equivalent}$	a ln N, b ln(1800)	M1 A1 M1 A1 M1 A1	[9]

(ii) State or imply that N approaches 1800

B1 [1]

