UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*:	State or imply non-modular inequality $x^2 < (5+2x)^2$, or corresponding equation, or pair of linear equations $x = \pm(5+2x)$	M1	
		Obtain critical values –5 and $-\frac{5}{3}$ only	A1	
		Obtain final answer $x < -5$, $x > -\frac{5}{3}$	A1	
	OR:	State one critical value e.g. –5, by solving a linear equation or inequality, or from a graphical method, or by inspection	B1	
		State the other critical value, e.g. $-\frac{5}{3}$, and no other	B1	
		Obtain final answer $x < -5$, $x > -\frac{5}{3}$	B1	[3]
		[Do not condone $\le$ or $\ge$.]		

2	**(i)**	Use law for the logarithm of a product or quotient	M1	
		Use $\log_2 32 = 5$ or $2^5 = 32$	M1	
		Obtain $x^2 + 5x - 32 = 0$, or horizontal equivalent	A1	[3]
	(ii)	Solve a 3-term quadratic equation	M1	
		Obtain answer $x = 3.68$ only, or exact equivalent, e.g. $\dfrac{\sqrt{153}-5}{2}$	A1	[2]

3	Use correct trig formula (or formulae) and obtain an equation in $\cos\theta$	M1	
	Obtain $8\cos^2\theta + \cos\theta - 7 = 0$, or equivalent	A1	
	Solve a 3-term quadratic in $\cos\theta$ and reach $\theta = \cos^{-1}(a)$	M1	
	Obtain answer 29.0°	A1	
	Obtain answer 180° and no others	A1	[5]
	[Ignore answers outside the given interval. Treat answers in radians (0.505 and 3.14 or π) as a misread.]		
	[SR: The answer 180° found by inspection can earn B1.]		

4	**(i)**	State or imply $CT = r\tan x$ or $OT = r\sec x$, or equivalent	B1	
		Using correct area formulae, form an equation in r and x	M1	
		Obtain the given answer correctly	A1	[3]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain the final answer 1.35	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p. , or show there is a sign change in the interval (1.345, 1.355)	A1	[3]

5 **(i)** *EITHER*: State $\frac{dx}{dt} = \sec^2 t / \tan t$, or equivalent B1

State $\frac{dy}{dt} = 2\sin t \cos t$, or equivalent B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Obtain correct answer in any form, e.g. $2\sin^2 t \cos^2 t$ A1

OR: Obtain $y = e^{2x} / (1 + e^{2x})$, or equivalent B1

Use correct quotient or product rule M1

Obtain correct derivative in any form, e.g. $2e^{2x} / (1 + e^{2x})^2$ A1

Obtain correct derivative in terms of t in any form, e.g. $(2\tan^2 t) / (1 + \tan^2 t)^2$ A1 [4]

(ii) State or imply $t = \frac{1}{4}\pi$ when $x = 0$ B1

Form the equation of the tangent at $x = 0$ M1

Obtain correct answer in any horizontal form, e.g. $y = \frac{1}{2}x + \frac{1}{2}$ A1 [3]

[SR: If the *OR* method is used in part **(i)**, give B1 for stating or implying $y = \frac{1}{2}$ or $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.]

6 **(i)** Show that the differential equation is $\frac{dy}{dx} = 2xy$ B1

Separate variables correctly and attempt integration of both sides M1

Obtain term $\ln y$, or equivalent A1

Obtain term x^2, or equivalent A1

Evaluate a constant, or use limits $x = 1$, $y = 2$, in a solution containing terms $a\ln y$ and bx^2 M1

Obtain correct solution in any form A1

Obtain the given answer correctly A1 [7]

(ii) State that the gradient at $(-1, 2)$ is -4 B1

Show the sketch of curve with correct concavity, positive y-intercept and axis of symmetry $x = 0$ B1 [2]

[SR: A solution with $k \neq 2$, or not evaluated, can earn B0M1A1A1M1A1A0 in part **(i)**.]

[SR: If given answer is assumed valid, give B1 if $\frac{dy}{dx}$ is shown correctly to be equal to $2xy$, is stated to be proportional to xy, and shown to be equal to 4 at $(1, 2)$.]

7 **(a)** **(i)** *EITHER*: Multiply numerator and denominator by $a - 2\text{i}$, or equivalent — M1

Obtain final answer $\dfrac{5a}{a^2+4} - \dfrac{10\text{i}}{a^2+4}$, or equivalent — A1

OR: Obtain two equations in x and y, solve for x or for y — M1

Obtain final answer $x = \dfrac{5a}{a^2+4}$ and $y = \dfrac{10}{a^2+4}$, or equivalent — A1 [2]

(ii) Either state $\arg(u) = -\dfrac{3}{4}\pi$, or express u^* in terms of a (f.t. on u) — B1√

Use correct method to form an equation in a, e.g. $5a = -10$ — M1

Obtain $a = -2$ correctly — A1 [3]

(b) Show a point representing $2 + 2\text{i}$ in relatively correct position in an Argand diagram — B1

Show the circle with centre at the origin and radius 2 — B1

Show the perpendicular bisector of the line segment from the origin to the point representing $2 + 2\text{i}$ — B1√

Shade the correct region — B1 [4]

[SR: Give the first B1 and the B1√ for obtaining $y = 2 - x$, or equivalent, and sketching the attempt.]

8 **(i)** State or imply partial fractions are of the form $\dfrac{A}{1+x} + \dfrac{Bx+C}{2+x^2}$ — B1

Use a relevant method to determine a constant — M1

Obtain one of the values $A = -2$, $B = 1$, $C = 4$ — A1

Obtain a second value — A1

Obtain the third value — A1 [5]

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$, $\left(1+\dfrac{1}{2}x^2\right)^{-1}$ or $(2+x^2)^{-1}$ in ascending powers of x — M1

Obtain correct unsimplified expansion up to the term in x^3 of each partial fraction — A1√ + A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ — M1

Obtain final answer $\dfrac{5}{2}x - 3x^2 + \dfrac{7}{4}x^3$, or equivalent — A1 [5]

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the first M1. The f.t. is on A, B, C.]

[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]

[In the case of an attempt to expand $(5x - x^2)(1 + x)^{-1}(2 + x^2)^{-1}$, give M1A1A1 for the expansions, M1 for the multiplying out fully, and A1 for the final answer.]

[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining f(0) = 0 and f '(0) = $\dfrac{5}{2}$, A1√ for f "(0) = –6, and A1 for f "'(0) = $\dfrac{21}{2}$ and the final answer (the f.t. is on A, B, C if used).]

[For the identity $5x - x^2 \equiv (2 + 2x + x^2 + x^3)(a + bx + cx^2 + dx^3)$ give M1A1; then M1A1 for using a relevant method to obtain two of $a = 0$, $b = \dfrac{5}{2}$, $c = -3$ and $d = \dfrac{7}{4}$; then A1 for the final answer in series form.]

9	**(i)**	State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$	B1		
		Carry out correct process for evaluating the scalar product of the two normals	M1		
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1		
		Obtain the final answer 79.7° (or 1.39 radians)	A1	[4]	

(ii) *EITHER*: Carry out a method for finding a point on the line — M1

Obtain such a point, e.g. (1, 3, 0) — A1

EITHER: State two correct equations for the direction vector (a, b, c) of the line, e.g. $a + 2b - 2c = 0$ and $2a + b + 3c = 0$ — B1

Solve for one ratio, e.g. $a : b$ — M1

Obtain $a : b : c = 8 : -7 : -3$, or equivalent — A1

State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ — A1√

OR1: Obtain a second point on the line, e.g. $\left(0, \frac{31}{8}, \frac{3}{8}\right)$ — A1

Subtract position vectors to find a direction vector — M1

Obtain $\mathbf{i} - \frac{7}{8}\mathbf{j} - \frac{3}{8}\mathbf{k}$, or equivalent — A1

State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} - \frac{7}{8}\mathbf{j} - \frac{3}{8}\mathbf{k})$ — A1√

OR2: Attempt to calculate the vector product of two normals — M1

Obtain two correct components — A1

Obtain $8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$, or equivalent — A1

State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ — A1√

OR3: Express one variable in terms of a second — M1

Obtain a correct simplified expression, e.g. $x = (31 - 8y) / 7$ — A1

Express the first variable in terms of a third — M1

Obtain a correct simplified expression, e.g. $x = (3 - 8z) / 3$ — A1

Form a vector equation of the line — M1

State a correct final answer, e.g. $\mathbf{r} = \frac{31}{8}\mathbf{j} + \frac{3}{8}\mathbf{k} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ — A1√

OR4: Express one variable in terms of a second — M1

Obtain a correct simplified expression, e.g. $y = (31 - 7x) / 7$ — A1

Express the third variable in terms of the second — M1

Obtain a correct simplified expression, e.g. $z = (3 - 3x) / 8$ — A1

Form a vector equation of the line — M1

State a correct final answer, e.g. $\mathbf{r} = \frac{31}{8}\mathbf{j} + \frac{3}{8}\mathbf{k} + \lambda(-8\mathbf{i} + 7\mathbf{j} + 3\mathbf{k})$ — A1√ [6]

[The f.t. is dependent on all M marks having been earned.]

10	**(i)**	Attempt integration by parts and reach $\pm x^2e^{-x} \pm \int 2xe^{-x}dx$	M1*	
		Obtain $-x^2e^{-x} + \int 2xe^{-x}dx$, or equivalent	A1	
		Integrate and obtain $-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$, or equivalent	A1	
		Use limits $x = 0$ and $x = 3$, having integrated by parts twice	M1(dep*)	
		Obtain the given answer correctly	A1	[5]
	(ii)	Use correct product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for non-zero x	M1	
		Obtain $x = 2$ with no errors send	A1	[4]
	(iii)	Carry out a complete method for finding the x-coordinate of P	M1	
		Obtain answer $x = 1$	A1	[2]