UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a product, power or quotient	M1*
	Obtain a correct linear equation, e.g. $(2x-1)\ln 5 = \ln 2 + x\ln 3$	A1
	Solve a linear equation for x	M1(dep*)
	Obtain answer $x = 1.09$	A1 [4]
	[SR: Reduce equation to the form $a^x = b$ M1*, obtain $\left(\frac{25}{3}\right)^x = 10$ Al, use correct method to calculate value of x M1(dep*), obtain answer 1.09 A1.]	
2	Use correct quotient or product rule	M1
	Obtain correct derivative in any form, e.g. $-\frac{3\ln x}{x^4} + \frac{1}{x^4}$	A1
	Equate derivative to zero and solve for x an equation of the form $\ln x = a$, where $a > 0$	M1
	Obtain answer exp($\frac{1}{3}$), or 1.40, from correct work	A1 [4]
3	Attempt integration by parts and reach $k(1-x)e^{-\frac{1}{2}x} \pm k\int e^{-\frac{1}{2}x}\,dx$, or equivalent	M1
	Obtain $-2(1-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$, or equivalent	A1
	Integrate and obtain $-2(1-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$, or equivalent	A1
	Use limits $x = 0$ and $x = 1$, having integrated twice	M1
	Obtain the given answer correctly	A1 [5]
4	**(i)** Use tan($A \pm B$) formula correctly at least once and obtain an equation in tanθ	M1
	Obtain a correct horizontal equation in any form	A1
	Use tan60° = $\sqrt{3}$ throughout	M1
	Obtain the given equation correctly	A1 [4]
	(ii) Set $k = 3\sqrt{3}$ and obtain $\tan^2\theta = \frac{1}{11}$	B1
	Obtain answer 16.8°	B1√
	Obtain answer 163.2°	B1√ [3]
	[Ignore answers outside the given interval. Treat answers in radians (0.293 and 2.85) as a misread.]	

5	**(i)**	Substitute $x = \frac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g. $\frac{1}{8}a + \frac{1}{4}b + \frac{5}{2} - 2 = 0$	B1	
		Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12	M1	
		Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$	A1	
		Solve for a or for b	M1	
		Obtain $a = 2$ and $b = -3$	A1	[5]
	(ii)	Attempt division by $2x - 1$ reaching a partial quotient $\frac{1}{2}ax^2 + kx$	M1	
		Obtain quadratic factor $x^2 - x + 2$	A1	[2]
		[The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B, or an unknown factor of $\frac{1}{2}ax^2 + Bx + C$ and an equation in B and/or C.]		
6	**(i)**	Make recognisable sketch of a relevant graph over the given range	B1	
		Sketch the other relevant graph and justify the given statement	B1	[2]
	(ii)	Consider the sign of $\cot x - (1 + x^2)$ at $x = 0.5$ and $x = 0.8$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once with $0.5 \le x_n \le 0.8$	M1	
		Obtain final answer 0.62	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	A1	[3]
7	**(i)**	Use the quadratic formula, completing the square, or the substitution $z = x + \mathrm{i}y$ to find a root and use $\mathrm{i}^2 = -1$	M1	
		Obtain final answers $-\sqrt{3} \pm \mathrm{i}$, or equivalent	A1	[2]
	(ii)	State that the modulus of both roots is 2	B1√	
		State that the argument of $-\sqrt{3} + \mathrm{i}$ is 150° or $\frac{5}{6}\pi$ (2.62) radians	B1√	
		State that the argument of $-\sqrt{3} - \mathrm{i}$ is –150° (or 210°) or $-\frac{5}{6}\pi$ (–2.62) radians or $\frac{7}{6}\pi$ (3.67) radians	B1√	[3]
	(iii)	Carry out an attempt to find the sixth power of a root	M1	
		Verify that one of the roots satisfies $z^6 = -64$	A1	
		Verify that the other root satisfies the equation	A1	[3]

8	**(i)**	Use product and chain rule	M1	
		Obtain correct derivative in any form, e.g. $15\sin^2 x\cos^3 x - 10\sin^4 x\cos x$	A1	
		Equate derivative to zero and obtain a relevant equation in one trigonometric function	M1	
		Obtain $2\tan^2 x = 3$, $5\cos^2 x = 2$, or $5\sin^2 x = 3$	A1	
		Obtain answer $x = 0.886$ radians	A1	[5]
	(ii)	State or imply $\mathrm{d}u = -\sin x\,\mathrm{d}x$, or $\dfrac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$, or equivalent	B1	
		Express integral in terms of u and $\mathrm{d}u$	M1	
		Obtain $\pm\int 5(u^2 - u^4)\,\mathrm{d}u$, or equivalent	A1	
		Integrate and use limits $u = 1$ and $u = 0$ (or $x = 0$ and $x = \frac{1}{2}\pi$)	M1	
		Obtain answer $\frac{2}{3}$, or equivalent, with no errors seen	A1	[5]
9	**(i)**	State or imply $\dfrac{\mathrm{d}x}{\mathrm{d}t} = k(10-x)(20-x)$ and show $k = 0.01$	B1	[1]
	(ii)	Separate variables correctly and attempt integration of at least one side	M1	
		Carry out an attempt to find A and B such that $\dfrac{1}{(10-x)(20-x)} \equiv \dfrac{A}{10-x} + \dfrac{B}{20-x}$, or equivalent	M1	
		Obtain $A = \frac{1}{10}$ and $B = -\frac{1}{10}$, or equivalent	A1	
		Integrate and obtain $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x)$, or equivalent	A1√	
		Integrate and obtain term $0.01t$, or equivalent	A1	
		Evaluate a constant, or use limits $t = 0$, $x = 0$, in a solution containing terms of the form $a\ln(10-x)$, $b\ln(20-x)$ and ct	M1	
		Obtain answer in any form, e.g. $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x) = 0.01t + \frac{1}{10}\ln 2$	A1√	
		Use laws of logarithms to correctly remove logarithms	M1	
		Rearrange and obtain $x = 20(\exp(0.1t) - 1)/(2\exp(0.1t) - 1)$, or equivalent	A1	[9]
	(iii)	State that x approaches 10	B1	[1]

10	**(i)**	*EITHER*:	Express general point of l or m in component form, e.g. $(2+\lambda, -\lambda, 1+2\lambda)$ or $(\mu, 2+2\mu, 6-2\mu)$	B1	
			Equate at least two pairs of components and solve for λ or for μ	M1	
			Obtain correct answer for λ or μ (possible answers for λ are -2, $\frac{1}{4}$, 7 and for μ are 0, $2\frac{1}{4}$, $-4\frac{1}{2}$)	A1	
			Verify that all three component equations are not satisfied	A1	
		OR:	State a relevant scalar triple product, e.g. $(2\mathbf{i}-2\mathbf{j}-5\mathbf{k}) \,.\, ((\mathbf{i}-\mathbf{j}+2\mathbf{k}) \times (\mathbf{i}+2\mathbf{j}-2\mathbf{k}))$	B1	
			Attempt to use the correct method of evaluation	M1	
			Obtain at least two correct simplified terms of the three terms of the expansion of the triple product or of the corresponding determinant, e.g. $-4, -8, -15$	A1	
			Obtain correct non-zero value, e.g. -27, and state that the lines do not intersect	A1	[4]
	(ii)		Carry out the correct process for evaluating scalar product of direction vectors for l and m	M1	
			Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	
			Obtain answer $47.1°$ or 0.822 radians	A1	[3]
	(iii)	*EITHER*:	Use scalar product to obtain $a - b + 2c = 0$	B1	
			Obtain $a + 2b - 2c = 0$, or equivalent, from a scalar product, or by subtracting two point equations obtained from points on m, and solve for one ratio, e.g. $a : b$	M1*	
			Obtain $a : b : c = -2 : 4 : 3$, or equivalent	A1	
			Substitute coordinates of a point on m and values for a, b and c in general equation and evaluate d	M1(dep*)	
			Obtain answer $-2x + 4y + 3z = 26$, or equivalent	A1	
		OR1:	Attempt to calculate vector product of direction vectors of l and m	M1*	
			Obtain two correct components	A1	
			Obtain $-2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or equivalent	A1	
			Form a plane equation and use coordinates of a relevant point to evaluate d	M1(dep*)	
			Obtain answer $-2x + 4y + 3z = 26$, or equivalent	A1	
		OR2:	Form a two-parameter plane equation using relevant vectors	M1*	
			State a correct equation e.g. $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mathbf{s}(\mathbf{i}-\mathbf{j}+2\mathbf{k}) + \mathbf{t}(\mathbf{i}+2\mathbf{j}-2\mathbf{k})$	A1	
			State three correct equations in x, y, z, s and t	A1	
			Eliminate s and t	M1(dep*)	
			Obtain answer $-2x + 4y + 3z = 26$, or equivalent	A1	[5]