1. Use logarithms to solve the equation $5^{2x-1} = 2(3^x)$, giving your answer correct to 3 significant figures. \[4\]

2. The curve $y = \frac{\ln x}{x^3}$ has one stationary point. Find the $x$-coordinate of this point. \[4\]

3. Show that $\int_0^1 (1-x)e^{-\frac{1}{x^2}} \, dx = 4e^{-\frac{1}{2}} - 2$. \[5\]

4. (i) Show that the equation
\[
\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k
\]
can be written in the form
\[
(2\sqrt{3})(1 + \tan^2 \theta) = k(1 - 3\tan^2 \theta).
\]
\[4\]

(ii) Hence solve the equation
\[
\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},
\]
giving all solutions in the interval $0^\circ \leq \theta \leq 180^\circ$. \[3\]

5. The polynomial $ax^3 + bx^2 + 5x - 2$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of $a$ and $b$. \[5\]

(ii) When $a$ and $b$ have these values, find the quadratic factor of $p(x)$. \[2\]

6. (i) By sketching a suitable pair of graphs, show that the equation
\[
\cot x = 1 + x^2,
\]
where $x$ is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. \[2\]

(ii) Verify by calculation that this root lies between 0.5 and 0.8. \[2\]

(iii) Use the iterative formula
\[
x_{n+1} = \tan^{-1}\left(\frac{1}{1 + x_n^2}\right)
\]
to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. \[3\]
7 (i) Find the roots of the equation
\[ z^2 + (2\sqrt{3})z + 4 = 0, \]
giving your answers in the form \( x + iy \), where \( x \) and \( y \) are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation
\[ z^6 = -64. \] [3]

8

\[ \begin{align*}
\text{The diagram shows the curve } & y = 5 \sin^3 x \cos^2 x \text{ for } 0 \leq x \leq \frac{\pi}{2}, \text{ and its maximum point } M. \\
\text{(i) Find the } x \text{-coordinate of } M. & \quad [5] \\
\text{(ii) Using the substitution } u = \cos x, \text{ find by integration the area of the shaded region bounded by the } \\
& \text{curve and the } x\text{-axis.} \quad [5]
\end{align*} \]

9 In a chemical reaction, a compound \( X \) is formed from two compounds \( Y \) and \( Z \). The masses in grams of \( X \), \( Y \) and \( Z \) present at time \( t \) seconds after the start of the reaction are \( x \), \( 10 - x \) and \( 20 - x \) respectively. At any time the rate of formation of \( X \) is proportional to the product of the masses of \( Y \) and \( Z \) present at the time. When \( t = 0 \), \( x = 0 \) and \( \frac{dx}{dt} = 2. \)

(i) Show that \( x \) and \( t \) satisfy the differential equation
\[ \frac{dx}{dt} = 0.01(10 - x)(20 - x). \] [1]

(ii) Solve this differential equation and obtain an expression for \( x \) in terms of \( t \). [9]

(iii) State what happens to the value of \( x \) when \( t \) becomes large. [1]

10 With respect to the origin \( O \), the lines \( l \) and \( m \) have vector equations \( \mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \) and \( \mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \) respectively.

(i) Prove that \( l \) and \( m \) do not intersect. [4]

(ii) Calculate the acute angle between the directions of \( l \) and \( m \). [3]

(iii) Find the equation of the plane which is parallel to \( l \) and contains \( m \), giving your answer in the form \( ax + by + cz = d \). [5]