UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 State or imply $4 - 2^x = -10$ and 10 — B1

Use correct method for solving equation of form $2^x = a$ — M1

Obtain 3.81 — A1 [3]

2 **(i)** Either: Obtain correct (unsimplified) version of x or x^2 term from $(1-4x)^{\frac{1}{2}}$ — M1

Obtain $1 + 2x$ — A1

Obtain $+ 6x^2$ — A1

Or: Differentiate and evaluate f(0) and f′(0) where f′$(x) = k(1-4x)^{-\frac{3}{2}}$ — M1

Obtain $1 + 2x$ — A1

Obtain $+ 6x^2$ — A1 [3]

(ii) Combine both x^2 terms from product of $1 + 2x$ and answer from part **(i)** — M1

Obtain 5 — A1 [2]

3 **(i)** Substitute $x = 2$ and equate to zero, or divide by $x - 2$ and equate constant remainder to zero, or equivalent — M1

Obtain $a = 4$ — A1 [2]

(ii) **(a)** Find further (quadratic or linear) factor by division, inspection or factor theorem or equivalent — M1

Obtain $x^2 + 2x - 8$ or $x + 4$ — A1

State $(x-2)^2(x+4)$ or equivalent — A1 [3]

(b) State any two of the four (or six) roots — B1√

State all roots ($\pm\sqrt{2}$, $\pm 2\mathrm{i}$), provided two are purely imaginary — B1√ [2]

4 **(i)** Either: Expand $(1 + 2\mathrm{i})^2$ to obtain $-3 + 4\mathrm{i}$ or unsimplified equivalent — B1

Multiply numerator and denominator by $2 - \mathrm{i}$ — M1

Obtain correct numerator $-2 + 11\mathrm{i}$ or correct denominator 5 — A1

Obtain $-\frac{2}{5} + \frac{11}{5}\mathrm{i}$ or equivalent — A1

Or: Expand $(1 + 2\mathrm{i})^2$ to obtain $-3 + 4\mathrm{i}$ or unsimplified equivalent — B1

Obtain two equations in x and y and solve for x or y — M1

Obtain final answer $x = -\frac{2}{5}$ — A1

Obtain final answer $y = \frac{11}{5}$ — A1 [4]

(ii) Draw a circle — M1

Show centre at relatively correct position, following their u — A1√

Draw circle passing through the origin — A1 [3]

5 **(i)** Differentiate to obtain $4\cos\frac{1}{2}x - \frac{1}{2}\sec^2\frac{1}{2}x$ B1

Equate to zero and find value of $\cos\frac{1}{2}x$ M1

Obtain $\cos\frac{1}{2}x = \frac{1}{2}$ and confirm $\alpha = \frac{2}{3}\pi$ A1 [3]

(ii) Integrate to obtain $-16\cos\frac{1}{2}x \ldots$ B1

$\ldots + 2\ln\cos\frac{1}{2}x$ or equivalent B1

Using limits 0 and $\frac{2}{3}\pi$ in $a\cos\frac{1}{2}x + b\ln\cos\frac{1}{2}x$ M1

Obtain $8 + 2\ln\frac{1}{2}$ or exact equivalent A1 [4]

6 **(i)** Obtain $2y\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^2 B1

Obtain $-4y - 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $-4xy$ B1

Substitute $x = 2$ and $y = -3$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$

(dependent on at least one B1 being earned and $\frac{\mathrm{d}(45)}{\mathrm{d}x} = 0$) M1

Obtain $\frac{12}{7}$ or equivalent A1 [4]

(ii) Substitute $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ in an expression involving $\frac{\mathrm{d}y}{\mathrm{d}x}$, x and y and obtain $ay = bx$ M1

Obtain $y = x$ or equivalent A1

Uses $y = x$ in original equation and demonstrate contradiction A1 [3]

7 Separate variables correctly and attempt integration on at least one side M1

Obtain $\frac{1}{3}y^3$ or equivalent on left-hand side A1

Use integration by parts on right-hand side (as far as $axe^{3x} + \int be^{3x}\,\mathrm{d}x$) M1

Obtain or imply $2xe^{3x} + \int 2e^{3x}\,\mathrm{d}x$ or equivalent A1

Obtain $2xe^{3x} - \frac{2}{3}e^{3x}$ A1

Substitute $x = 0$, $y = 2$ in an expression containing terms Ay^3, Bxe^{3x}, Ce^{3x}, where ABC ≠ 0, and find the value of c M1

Obtain $\frac{1}{3}y^3 = 2xe^{3x} - \frac{2}{3}e^{3x} + \frac{10}{3}$ or equivalent A1

Substitute $x = 0.5$ to obtain $y = 2.44$ A1 [8]

8 **(i)** Either Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line) or equivalent B1

Use scalar product to find cosine of angle between PA and line M1

Obtain $\frac{42}{\sqrt{14\times230}}$ or equivalent A1

Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent A1

Or 1 Obtain $\pm\begin{pmatrix}2n+2\\n-1\\3n-15\end{pmatrix}$ for PN (where N is foot of perpendicular) B1

Equate scalar product of PN and line direction to zero
Or equate derivative of PN^2 to zero
Or use Pythagoras' theorem in triangle PNA to form equation in n M1
Solve equation and obtain $n = 3$ A1

Obtain $\sqrt{104}$ or 10.2 or equivalent A1

Or 2 Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line) B1

Evaluate vector product of PA and line direction M1

Obtain $\pm\begin{pmatrix}12\\-36\\-4\end{pmatrix}$ A1

Divide modulus of this by modulus of line direction and obtain $\sqrt{104}$ or 10.2 or equivalent A1

Or 3 Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line) B1

Evaluate scalar product of PA and line direction to obtain distance AN M1

Obtain $3\sqrt{14}$ or equivalent A1

Use Pythagoras' theorem in triangle PNA and obtain $\sqrt{104}$ or 10.2 or equivalent A1

Or 4 Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line) B1

Use a second point B on line and use cosine rule in triangle ABP to find angle A or angle B or use vector product to find area of triangle M1

Obtain correct answer (angle $A = 42.25\ldots$) A1

Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent A1 [4]

(ii) Either: Use scalar product to obtain a relevant equation in a, b, c, e.g. $2a + b + 3c = 0$ or $2a - b - 15c = 0$ — M1
State two correct equations in a, b and c — A1√
Solve simultaneous equations to obtain one ratio — M1
Obtain $a : b : c = -3 : 9 : -1$ or equivalent — A1
Obtain equation $-3x + 9y - z = 28$ or equivalent — A1

Or 1: Calculate vector product of two of $\begin{pmatrix}2\\1\\3\end{pmatrix}$, $\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ and $\begin{pmatrix}8\\2\\-6\end{pmatrix}$ or equiv — M1
Obtain two correct components of the product — A1√
Obtain correct $\begin{pmatrix}-3\\9\\-1\end{pmatrix}$ or equivalent — A1
Substitute in $-3x + 9y - z = d$ to find d or equivalent — M1
Obtain equation $-3x + 9y - z = 28$ or equivalent — A1

Or 2: Form a two-parameter equation of the plane — M1
Obtain $\mathbf{r} = \begin{pmatrix}1\\3\\-4\end{pmatrix} + s\begin{pmatrix}2\\1\\3\end{pmatrix} + t\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ or equivalent — A1√
State three equations in x, y, z, s, t — A1
Eliminate s and t — M1
Obtain equation $3x - 9y + z = -28$ or equivalent — A1 [5]

9 State or imply form $A + \frac{B}{2x+1} + \frac{C}{x+2}$ — B1
State or obtain $A = 2$ — B1
Use correct method for finding B or C — M1
Obtain $B = 1$ — A1
Obtain $C = -3$ — A1
Obtain $2x + \frac{1}{2}\ln(2x+1) - 3\ln(x+2)$ [Deduct B1√ for each error or omission] — B3√
Substitute limits in expression containing $a\ln(2x + 1) + b\ln(x + 2)$ — M1
Show full and exact working to confirm that $8 + \frac{1}{2}\ln 9 - 3\ln 6 + 3\ln 2$, or an equivalent expression, simplifies to given result $8 - \ln 9$ — A1 [10]

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B0√B1√B1√M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{M}{2x+1} + \frac{Nx}{x+2}$ or $\frac{Px}{2x+1} + \frac{Q}{x+2}$, give B0B0M1A0A0 in (**i**); B1√B1√B1√, if recover correct form, M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{B}{2x+1} + \frac{Dx+E}{x+2}$, give M1A1 for one of $B = 1$, $D = 2$, $E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -3$.]

[SR: For a solution starting with $\frac{Fx+G}{2x+1} + \frac{C}{x+2}$, give M1A1 for one of $C = -3$, $F = 4$, $G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = 1$.]

10 **(i)** Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero — M1
Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$ — A1
Obtain $kt(t^3 + et + f) = 0$ or equivalent — M1
Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$ — A1 [4]

(ii) Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent — M1
Justify the given statement with correct calculations (–0.06 and 0.02) — A1 [2]

(iii) Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$ — M1
Obtain final answer 1.276 — A1
Show sufficient iterations to justify answer or show there is a change of sign in interval (1.2755, 1.2765) — A1 [3]

(iv) Evaluate $\tan^{-1}$ (answer from part **(iii)**) to obtain at least one value — M1
Obtain –2.24 and 0.906 — A1
State $-\pi$, 0 and π — B1 [3]
[SR If A0, B0, allow B1 for any 3 roots]