UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

# MARK SCHEME for the May/June 2012 question paper

# for the guidance of teachers

## 9709 MATHEMATICS

**9709/32** Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

| | | | | |
|---|---|---|---|---|
| **1** | *EITHER*: | Use law of the logarithm of a power or quotient and remove logarithms | M1 | |
| | | Obtain a 3-term quadratic equation $x^2 - x - 3 = 0$, or equivalent | A1 | |
| | | Solve 3-term quadratic obtaining 1 or 2 roots | M1 | |
| | | Obtain answer 2.30 only | A1 | |
| | *OR*1: | Use an appropriate iterative formula, e.g. $x_{n+1} = \exp\left(\frac{1}{2}\ln(3x_n + 4)\right) - 1$ correctly at least once | M1 | |
| | | Obtain answer 2.30 | A1 | |
| | | Show sufficient iterations to at least 3 d.p. to justify 2.30 to 2 d.p., or show there is a sign change in the interval (2.295, 2.305) | A1 | |
| | | Show there is no other root | A1 | |
| | *OR*2: | Use calculated values to obtain at least one interval containing the root | M1 | |
| | | Obtain answer 2.30 | A1 | |
| | | Show sufficient calculations to justify 2.30 to 3 s.f., e.g. show it lies in (2.295, 2.305) | A1 | |
| | | Show there is no other root | A1 | [4] |

| | | | | |
|---|---|---|---|---|
| **2** | **(i)** | Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}bh$, form an equation an $a$ and $\theta$ | M1 | |
| | | Obtain given answer | A1 | [2] |
| | **(ii)** | Use the iterative formula correctly at least once | M1 | |
| | | Obtain answer $\theta = 1.32$ | A1 | |
| | | Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change in the interval (1.315, 1.325) | A1 | [3] |

**3** *EITHER*: State a correct unsimplified term in $x$ or $x^2$ of $(1-x)^{\frac{1}{2}}$ or $(1+x)^{-\frac{1}{2}}$ B1

State correct unsimplified expansion of $(1-x)^{\frac{1}{2}}$ up to the term in $x^2$ B1

State correct unsimplified expansion of $(1+x)^{-\frac{1}{2}}$ up to the term in $x^2$ B1

Obtain sufficient terms of the product of the expansions of $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$ M1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1

*OR*1: State that the given expression equals $(1-x)(1-x^2)^{-\frac{1}{2}}$ and state that the first term of the expansion of $(1-x^2)^{-\frac{1}{2}}$ is 1 B1

State correct unsimplified term in $x^2$ of $(1-x^2)^{-\frac{1}{2}}$ B1

State correct unsimplified expansion of $(1-x^2)^{-\frac{1}{2}}$ up to the term in $x^2$ B1

Obtain sufficient terms of the product of $(1-x)$ and the expansion M1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1

*OR*2: State correct unsimplified expansion of $(1+x)^{\frac{1}{2}}$ up to the term in $x^2$ B1

Multiply expansion by $(1-x)$ and obtain $1-2x+2x^2$ B1

Carry out correct method to obtain one non-constant term of the expansion of $\left(1-2x+2x^2\right)^{\frac{1}{2}}$ M1

Obtain a correct unsimplified expansion with sufficient terms A1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1 [5]

[Treat $(1+x)^{-1}(1-x^2)^{\frac{1}{2}}$ by the *EITHER* scheme.]

[Symbolic coefficients, e.g. $\binom{\frac{1}{2}}{2}$, are not sufficient for the B marks.]

**4** Use trig formulae to express equation in terms of $\cos\theta$ and $\sin\theta$ M1

Use Pythagoras to obtain an equation in $\sin\theta$ M1

Obtain 3-term quadratic $2\sin^2\theta-2\sin\theta-1=0$, or equivalent A1

Solve a 3-term quadratic and obtain a value of $\theta$ M1

Obtain answer, e.g. 201.5° A1

Obtain second answer, e.g. 338.5°, and no others in the given interval A1 [6]

[Ignore answers outside the given interval. Treat answers in radians (3.52, 5.91) as a misread and deduct A1 from the marks for the angles.]

**5** Separate variables correctly and attempt integration of both sides B1

Obtain term $-e^{-y}$, or equivalent B1

Obtain term $\frac{1}{2}e^{2x}$, or equivalent B1

Evaluate a constant, or use limits $x = 0$, $y = 0$ in a solution containing terms $ae^{-y}$ and $be^{2x}$ M1

Obtain correct solution in any form, e.g. $-e^{-y}=\frac{1}{2}e^{2x}-\frac{3}{2}$ A1

Rearrange and obtain $y=\ln(2/(3-e^{2x}))$, or equivalent A1 [6]

| | | | | |
|---|---|---|---|---|
| 6 | (i) | State derivative in any correct form, e.g. $3\cos x - 12\cos^2 x \sin x$ | B1 + B1 | |
| | | Equate derivative to zero and solve for sin 2*x*, or sin *x* or cos *x* | M1 | |
| | | Obtain answer $x = \frac{1}{12}\pi$ | A1 | |
| | | Obtain answer $x = \frac{5}{12}\pi$ | A1 | |
| | | Obtain answer $x = \frac{1}{2}\pi$ and no others in the given interval | A1√ | [6] |
| | (ii) | Carry out a method for determining the nature of the relevant stationary point | M1 | |
| | | Obtain a maximum at $\frac{1}{12}\pi$ correctly | A1 | [2] |
| | | [Treat answers in degrees as a misread and deduct A1 from the marks for the angles.] | | |
| 7 | (i) | *EITHER*: Multiply numerator and denominator by 1 + 3i, or equivalent | M1 | |
| | | Simplify numerator to –5 + 5i, or denominator to 10, or equivalent | A1 | |
| | | Obtain final answer $-\frac{1}{2} + \frac{1}{2}\text{i}$, or equivalent | A1 | |
| | | *OR*: Obtain two equations in *x* and *y*, and solve for *x* or for *y* | M1 | |
| | | Obtain $x = -\frac{1}{2}$ or $y = \frac{1}{2}$, or equivalent | A1 | |
| | | Obtain final answer $-\frac{1}{2} + \frac{1}{2}\text{i}$, or equivalent | A1 | [3] |
| | (ii) | Show *B* and *C* in relatively correct positions in an Argand diagram | B1 | |
| | | Show *u* in a relatively correct position | B1√ | [2] |
| | (iii) | Substitute exact arguments in the LHS arg(1 + 2i) – arg(1 – 3i) = arg *u*, or equivalent | M1 | |
| | | Obtain and use $\arg u = \frac{3}{4}\pi$ | A1 | |
| | | Obtain the given result correctly | A1 | [3] |

**8** **(i)** State or imply $2u\,\mathrm{d}u = -\mathrm{d}x$, or equivalent — B1

Substitute for $x$ and d$x$ throughout — M1

Obtain integrand $\dfrac{-10u}{6-u^2+u}$, or equivalent — A1

Show correct working to justify the change in limits and obtain the given answer correctly — A1 [4]

**(ii)** State or imply the form of fractions $\dfrac{A}{3-u}+\dfrac{B}{2+u}$ and use a relevant method to find $A$ or $B$ — M1

Obtain $A = 6$ and $B = -4$ — A1

Integrate and obtain $-6\ln(3-u)-4\ln(2+u)$, or equivalent — A1√ + A1√

Substitute limits correctly in an integral of the form $a\ln(3-u)+b\ln(2+u)$ — M1

Obtain the given answer correctly having shown sufficient working — A1 [6]

[The f.t. is on $A$ and $B$.]

**9** **(i)** Use correct product rule — M1

Obtain derivative in any correct form, e.g. $\dfrac{\ln x}{2\sqrt{x}}+\dfrac{\sqrt{x}}{x}$ — A1

Carry out a complete method to form an equation of the tangent at $x = 1$ — M1

Obtain answer $y = x - 1$ — A1 [4]

**(ii)** State or imply that the indefinite integral for the volume is $\pi\int x(\ln x)^2\,\mathrm{d}x$ — B1

Integrate by parts and reach $ax^2(\ln x)^2 + b\int x^2.\dfrac{\ln x}{x}\,\mathrm{d}x$ — M1*

Obtain $\dfrac{1}{2}x^2(\ln x)^2 - \int x\ln x\,\mathrm{d}x$, or unsimplified equivalent — A1

Attempt second integration by parts reaching $cx^2\ln x + d\int x^2.\dfrac{1}{x}\,\mathrm{d}x$ — M1(dep*)

Complete the integration correctly, obtaining $\dfrac{1}{2}x^2(\ln x)^2 - \dfrac{1}{2}x^2\ln x + \dfrac{1}{4}x^2$ — A1

Substitute limits $x = 1$ and $x = \mathrm{e}$, having integrated twice — M1(dep*)

Obtain answer $\dfrac{1}{4}\pi(\mathrm{e}^2-1)$, or exact equivalent — A1 [7]

[If π omitted, or 2π or π/2 used, give B0 and then follow through.]

[Integration using parts $x\ln x$ and $\ln x$ is also viable.]

**10** **(i)** *EITHER*: Substitute coordinates of a general point of $l$ in given equation of plane $m$ M1
Obtain equation in $\lambda$ in any correct form A1
Verify that the equation is not satisfied for any value of $\lambda$ A1

*OR*1: Substitute for **r** in the vector equation of plane $m$ and expand scalar product M1
Obtain equation in $\lambda$ in any correct form A1
Verify that the equation is not satisfied for any value of $\lambda$ A1

*OR*2: Expand scalar product of a normal to $m$ and a direction vector of $l$ M1
Verify scalar product is zero A1
Verify that one point of $l$ does not lie in the plane A1

*OR*3: Use correct method to find perpendicular distance of a general point of $l$ from $m$ M1
Obtain a correct unsimplified expression in terms of $\lambda$ A1
Show that the perpendicular distance is 4/3, or equivalent, for all $\lambda$ A1

*OR*4: Use correct method to find the perpendicular distance of a particular point of $l$ from $m$ M1
Obtain answer 4/3, or equivalent A1
Show that the perpendicular distance of a second point is also 4/3, or equivalent A1 [3]

**(ii)** *EITHER*: Express general point of $l$ in component form, e.g. $(1 + 2\lambda, 1 + \lambda, -1 + 2\lambda)$ B1
Substitute in given equation of $n$ and solve for $\lambda$ M1
Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ from $\lambda = 2$ A1

*OR*: State or imply plane $n$ has vector equation $\mathbf{r}.(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 7$, or equivalent B1
Substitute for **r**, expand scalar product and solve for $\lambda$ M1
Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ from $\lambda = 2$ A1 [3]

**(iii)** Form an equation in $\lambda$ by equating perpendicular distances of a general point of $l$ from $m$ and $n$ M1*
Obtain a correct modular or non-modular equation in $\lambda$ in any form A1√
Solve for $\lambda$ and obtain a point, e.g. $7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ from $\lambda = 3$ A1
Obtain a second point, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ from $\lambda = 1$ A1
Use a correct method to find the distance between the two points M1(dep*)
Obtain answer 6 A1 [6]
[The f.t. is on the components of $l$.]