MATHEMATICS

Paper 9709/11
Paper 11

General comments

It is pleasing to record improvement in some of the areas mentioned in last year's report. For example, although there were still some candidates who divided each page into two columns, this practice was less prevalent than in previous years. If candidates need further reminding, they should work straight down the page rather than try to fit one question beside another question on the page.

General setting out was mostly satisfactory but there were a few questions where this was not the case. In **Question 3(i)**, for example, it was often the case that it was not at all clear which area was being considered. Answers to **Questions 4(i)**, **6** and **9(ii)** also suffered from poor or unclear setting out.

In previous reports comments were made that candidates were losing many marks on routine procedures. Although some improvement in this respect has been noted there is still room for further improvement.

Comments on specific questions

Question 1

Although most candidates attempted this question, often their attempts earned little or no credit. A large proportion of candidates did not recall that the standard method of showing that a given function is an increasing function is to consider the derivative of the function and to show that it is positive for all values of x. Large numbers of candidates did not differentiate but instead substituted a few particular values of x intending to show that f(x) increases as x increases. Unfortunately this is not a satisfactory method since it does not consider the general case. The candidates who attempted differentiation often made a mistake, either forgetting to multiply by the factor 2, or omitting completely the derivative of x, the second term. Finally, those candidates who obtained the correct derivative were often unsure what conclusion to draw. What was required was to state that 1 plus the square of any quantity is always positive and hence the function is increasing.

Answer: $\mathrm{f}'(x) = 6(2x-5)^2 + 1$. This is > 0 for all values of x and hence the function is increasing.

Question 2

Most candidates started well and applied the binomial theorem for the first three terms. A few candidates confused the words 'ascending' with 'descending' and unfortunately this was a costly mistake. Much more common errors, however, were sign errors and not raising p to the power 2 when simplifying $15(-px)^2$. This last error was particularly costly since it prevented a quadratic equation being formed in part **(ii)**.

Answers: **(i)** $1-6px+15p^2x^2$; **(ii)** $-2/5$.

Question 3

Many candidates thought that the radius of the semicircle and the radius of the sector were the same. A particular feature of candidates' answers in this question was working in which it was not clear exactly what was being considered at each stage. For example words such as 'Area of sector = ...', 'Area of semicircle = ...' would have been helpful both to candidates themselves and to Examiners. The result for candidates was sometimes confused and incorrect work.

Answers: **(i)** $\pi/8$; **(ii)** $8+5\pi$.

Question 4

This question was generally very well done and full marks was frequently the outcome. In part **(i)** Examiners expected to see two equations, $ar^2 = -108$ and $ar^5 = 32$, and for candidates to proceed from there by eliminating a to find r. In reality, although many candidates did employ this method, it was also the case that many candidates employed rather more 'ad hoc' methods to reach the answer.

Answers: **(i)** $-2/3$; **(ii)** – 243; **(iii)** –145.8.

Question 5

Part **(i)** was usually done well. In part **(ii)**, most candidates reached a correct expression for $\sin^2\theta$ or $\cos^2\theta$ or $\tan^2\theta$ but very few candidates remembered the ± sign on taking the square root. Most candidates, therefore, only found two of the four solutions.

Answers: **(ii)** 54.7°, 125.3°, 234.7°, 305.3°.

Question 6

This question tended to expose a general lack of understanding and confidence in dealing with a number of processes involving vectors. Part **(i)** was done reasonably well with candidates required to demonstrate that the scalar product of **OA** and **OC** is zero. Part **(ii)**, however, was not so well done. It was disappointing to see many errors, even in the first step (finding **CA**) of this part. Candidates should have found that the **j** and **k** components were both zero and with this straightforward case the magnitude of **CA** is simply the coefficient of the **i** component. However, the majority of candidates who reached **CA** correctly did not recognise this and proceeded to square, add and square root etc., often not reaching the correct answer. In part **(iii)** many candidates made errors, including sign errors, in reaching **BA**. However, it was pleasing to see that, compared to previous years, a greater proportion of candidates were employing the correct method for finding a unit vector.

Answers: **(i)** **OA.OC** = $-4p^2 - q^2 + 4p^2 + q^2 = 0$ hence perpendicular; **(ii)** |**CA**| = $1 + 4p^2 + q^2$;
(iii) 1/9 (**i** + 4**j** +8**k**).

Question 7

Part **(i)** was generally answered well, although a common incorrect answer was (1½, 1½), obtained by subtracting instead of adding the end-points. Although part **(ii)** requires standard procedures few correct answers were seen. There is more than one way of tackling this question but the expected method is to equate the equation of the curve with the equation of the line, rearrange to make zero on one side and then apply the condition for equal roots ($b^2 - 4ac = 0$). Candidates who attempted this method were usually successful but other methods were far less successful.

Answers: **(i)** (2½, 2½); **(ii)** $m = -8$, (–2, 16).

Question 8

In part **(i)**, completing the square is a topic which appears almost every year and candidates are advised to practise this process so that they are confident that they can perform it accurately. In this particular case there were many correct answers but there was still a significant proportion of candidates who did not achieve all 3 marks. Parts **(ii)** and **(iii)** were answered reasonably well. Part **(iv)** was answered very well. Candidates were perhaps fortunate that the correct answer required the <u>positive</u> square root. There will be occasions, of course, when the answer will require the negative square root so the correct procedure is to apply the '±' sign in the first instance and then to decide which sign is appropriate.

Answers: **(i)** $2(x-3)^2 - 5$; **(ii)** 3; **(iii)** $y \geq 27$; **(iv)** $3 + \sqrt{\tfrac{1}{2}(x+5)}$ for $x \geq 27$.

Question 9

Most candidates used the suggested substitution and were able to transform the resulting equation into a 3-term quadratic equation and solve it. Some candidates forgot to square the roots in order to find values of *x*. In part **(ii)** the second derivative was usually found correctly. The expected method for determining the nature of the stationary points was to substitute the values of *x* found in part **(i)** into the second derivative. Other valid methods are accepted but it is necessary to show the actual substitution of particular values of *x*. Part **(iii)** was usually done very well.

Answers: **(i)** $\frac{1}{9}$, 9; **(ii)** $f''(x) = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$, Maximum at $x = \frac{1}{9}$, Minimum at $x = 9$;

(iii) $f(x) = 2x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - 10x + 5$.

Question 10

Parts **(i)** and **(ii)** were done very well and many candidates achieved full marks. Part **(iii)**, however, was far more challenging. Most candidates made the mistake of applying the same limits to the curve and the line when integrating. Other candidates ignored the tangent and simply integrated the equation of the curve between limits 5/4 and 2. Few candidates were able to score more than 1 mark for this part.

Answers: **(i)** *B*(5/4,0), *C*(0,3/4); **(ii)** $\sqrt{17}/4$; **(iii)** 3/40.

MATHEMATICS

Paper 9709/12
Paper 12

Key Messages

- Candidates need to be advised of the need to read questions carefully, for on questions such as **Question 8 (ii)**, at least a third of all solutions had a value for *r* instead of *S* and in **Question 3** many candidates failed to give answers for both *m* and the coordinates of P.
- Candidates should also be aware that in questions demanding a proof, such as **Questions 5(i)** and **9(ii)**, testing a value of a variable is insufficient to show that a result is true for all values of the variable.

General Comments

The paper gave all candidates the chance to show what they had learnt. It was particularly pleasing that the concept of "unit vector", which in previous years had presented so many problems, was fully understand by the majority of candidates. Although the standard of presentation was generally good, there were many scripts which were difficult to mark because of poor presentation and lack of working. Again candidates (and Centres) need to realise that the splitting of a page into columns, whilst being acceptable in the classroom, is an unacceptable practice in an examination. It is a practice which makes marking extremely difficult.

Comments on Specific Questions

Question 1

This proved to be a good starting question and the majority of solutions were correct. Common errors however were to integrate $6x^{-2}$ as $6x^{-1}$ rather than $-6x^{-1}$ or to omit the constant of integration. A significant proportion of candidates still assumed that the equation of the curve is the same as the equation of the tangent and failed to perform any integration.

Answer: $y=-\dfrac{6}{x}+12$.

Question 2

(i) Most candidates realised that the term in x^2 came from the 3rd term of the progression, ${}_6C_2(2x)^4\times\left(\dfrac{-1}{2x}\right)^2$. Unfortunately errors were common, particularly with incorrect expansion of the brackets. It was very common to see $(2x)^4$ given as $2x^4$ and $\left(\dfrac{-1}{2x}\right)^{-2}$ expanded as either $\dfrac{1}{2x^2}$ or as $-\dfrac{1}{2x^2}$. Use of the binomial coefficient was accurate.

(ii) This caused considerable difficulty with a large proportion of candidates failing to realise the need to find the constant term in the expansion (−20). Dealing with the minus sign in the original expansion proved a further problem with +20 being given instead of – 20 as the constant term.

Answers: **(i)** 60. **(ii)** 40.

Question 3

Most candidates realised the need to equate the two expressions for y and that the resulting equation had equal roots. Use of the determinant $\left(b^2 - 4ac = 0\right)$ was generally well done, though the answer $m = +3$ was a common error. A large number of candidates preferred to equate the gradient of the line and the curve and obtained the equation $m = -\dfrac{12}{x^2}$. This was usually substituted into the equation formed by equating the two expressions for y, though a common error was to replace mx by $-\dfrac{12}{x^2}$ rather than to replace m. A small proportion of candidates failed to read the question carefully and failed to give answers for both m and the coordinates of P.

Answer: $m = -8$. $P(2, 8)$.

Question 4

(i) This was well answered, though with the answer being given, it was difficult to judge the accuracy of many answers. The majority of candidates realised that BOC was equal to either $2 \times \tan^{-1} COM$, where M is the midpoint of BC or to $(\pi - 2 \times \tan^{-1} COD)$. Alternatives were to find OB ($\sqrt{125}$) and to use sine or cosine in triangle OCD or to use the cosine rule. Use of radians was pleasing as was the way candidates using degrees were able to convert to radians.

(ii) This was usually correct with candidates finding the radius OB and using the formula $s=r\theta$ accurately. The only common error was to assume that the radius of the arc was 10 cm.

(iii) This proved to be an easy question, though accuracy presented many candidates with a problem. Many answers were given as "8 cm^2" instead of "7.96 cm^2". As in part **(ii)**, the only method error was to assume that the radius was 10 cm.

Answers: **(i)** Proof. **(ii)** 20.4 cm. **(iii)** 7.95 or 7.96 cm^2.

Question 5

(i) This presented most candidates with problems. The squaring of the brackets was poor with such errors as $(-3\cos\theta)^2 = -9\cos^2\theta$ or $3\cos^2\theta$ being very common. Consequently it was relatively rare for candidates to finish with an expression of the form $k\cos^2\theta + k\sin^2\theta$. Many candidates obtained the answer $10\sin^2\theta + 10\cos^2\theta$ but failed to realise that this had a constant value of 10.

(ii) This was well answered and the majority of candidates obtained the equation $\tan\theta = -7$ and proceeded to give correct values in the second and fourth quadrants. Even when poor algebra led to a value for $\tan\theta$ other than -7, the method mark and follow-through mark for the second angle were usually obtained.

Answers: **(i)** 10. **(ii)** 98.1°, 278.1°.

Question 6

This question was extremely well answered and it was particularly pleasing that the concept of unit vector was much better understood than in previous years.

(i) Of the three parts, this caused most difficulty with many candidates failing to realise that if two vectors are parallel, all of the three components must be in same ratio, leading directly to $p = -6$ and $q = 6$.

(ii) This was nearly always correct, though the solution of the equation $3 - 2p + 4p = 0$ was often given as $+1.5$ instead of -1.5. Only a few candidates failed to realise that the scalar product was equal to 0.

(iii) Most candidates realised that $\overrightarrow{AB}$ was obtained from **b** – **a** and obtained a correct expression. The vast majority of candidates also realised the need to divide $\overrightarrow{AB}$ by its modulus. Only a small proportion of candidates incorrectly used $\overrightarrow{AB}$ as **a** + **b**. It was of concern that several candidates left the answer as a number rather than a vector (usually $\frac{2+3+6}{7}$).

Answers: **(i)** $p = -6$, $q = 6$. **(ii)** -1.5. **(iii)** $\frac{1}{7}(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k})$.

Question 7

This presented the majority of candidates with problems. Many candidates automatically associated the word "reflection" with the inverse of a function and attempted to find the equation of a line which was the mirror image of the original line in the line $y = x$. Very little progress was subsequently made. Those giving the situation a little more thought realised that R lay on the perpendicular from $(-1, 3)$ to the original line. These candidates had little difficulty in finding the equation of this perpendicular which was then solved simultaneously with the original equation to obtain the point (3, 9). A common error was to assume that this was the point R, but most of these candidates proceeded to find the coordinates of R by vector steps or use of the midpoint.

Answer: (7, 15)

Question 8

(i) This was badly answered. Many candidates made no attempt whilst many others attempted to contrive the answer. Many used the formula "$V = \pi r^2 h$" to express h in terms of h, but errors over "π" were common and the surface area formula "$S=2\pi rh$" was often misquoted in order to make the answer fit the given answer.

(ii) This was well answered. The standard of differentiation was good and virtually all candidates recognised the need to set the differential to 0. Algebraic errors often led to $r = 125$, instead of $r^3 = 125$, and at least a quarter of all candidates failed to read the question, giving the final answer as $r = 5$, instead of finding the value of S.

(iii) This was also well answered, nearly always by finding the second differential. Again the differentiation was good and conclusions were generally correct. Of the few attempting to find the nature by examining values of either S or dS/dx, it was often difficult to follow working and in many cases, insufficient working was given.

Answers: **(ii)** $S = 150\pi$. **(iii)** Minimum.

Question 9

It was pleasing that the majority of candidates realised the difference between $f'(x)$ and $f^{-1}(x)$, though a significant proportion used $f^{-1}(x)$ in all three parts of the question. Knowledge of $f'(x)$ caused more misunderstanding than $f^{-1}(x)$.

(i) The standard of differentiation was good with only a small minority failing to recognise that the function was composite and needed "×3", the differential of the bracket, in the answer.

(ii) Candidates realised that for a function to be increasing, the gradient needed to be positive, but far too many candidates tested one value of x and assumed that the gradient was therefore positive for **all** values of x. This gained no credit. Only a small proportion realised that the denominator $(1 - 3x)^2$ was always positive.

(iii) The expression for $f^{-1}(x)$ was nearly always correct and most realised that the range of $f^{-1}(x)$ was the same as the given domain for $f(x)$. Finding the domain was more difficult and only about a quarter of all attempts realised that the lower limit was $x \geq -2.5$. Finding the upper limit of $x < 0$ proved too difficult for all but a handful of candidates.

Answers: **(i)** $\frac{15}{(1-3x)^2}$. **(ii)** Increasing. **(iii)** $f^{-1}(x) = \frac{x-5}{3x}$. Range $f^{-1}(x) \geq 1$. Domain $-2.5 \leq x \leq 0$.

Question 10

(a) This produced many correct answers with candidates confidently setting the sum of the first four terms to 57, finding the value of the common difference and using this to find the value of n.

(b) Although many candidates obtained one correct value of k, the second was usually missing when the solution of $r^2=4$ was given as $r = 2$, instead of $r = \pm 2$. Surprisingly many candidates failed to cancel a in the equation $ar^2 = 4a$ and never obtained a numerical value for r whilst others obtained $r = \pm 2$ and automatically gave the answers as ±63.

Answers: **(a)** $n = 25$. **(b)** $k = 63$ or -21.

Question 11

(i) This part of the question was very well answered. The differentiation was generally accurate, with only a small number of candidates omitting the "×4" from the differential. Finding the gradient of both the tangent and the normal and hence the equation of the normal presented few problems. The only common error was to give the equation of the tangent rather than the normal.

(ii) Most candidates realised the need to integrate a composite function and the standard of integration was good. Coming to a final answer presented more difficulty, and although the lower limit of $-\frac{1}{4}$ was used by most candidates, it was common to see an upper limit of 2, rather than 0. It was also rare to see a correct answer for the area of the triangle *BOC*. Some weaker candidates also assumed that the shaded region was a sector of a circle.

Answers: **(i)** $2y + x = 2$. **(ii)** $1\frac{1}{6}$.

MATHEMATICS

Paper 9709/13
Paper 13

General Comments

- Candidates would do well to read both the Instructions on the front of the question paper, and then each question, carefully. All too often marks are lost by not giving answers to the required degree of accuracy or in the form required and often by not actually answering the question asked. Additionally, the instructions in questions such as 'Show that...' or 'Hence...' were sometimes ignored.
- Many candidates displayed a lack of working. In simple calculations or algebraic solutions this is not an issue but in more complex work where there are both method and accuracy marks available, a wrong answer with no working will lose all the potential marks.
- There was some evidence of formulae used in wrong contexts – the most common being the misuse of the formula for segment area in **Question 2** and the use of the S_n formula in **9(b)**.
- The question paper instructions remind candidates of "the need for clear presentation" in their answers. Much work was far from this; presented in two columns; or with part answers completed later squeezed into small spaces.

Comments on specific questions

Question 1

The use of integration was widespread and fairly well done, although some of the candidates forgot to include a constant of integration, while others could not deal correctly with the contents of the bracket.

Answer: y = ⅓, $\sqrt{(2x+5)^3} - 4$.

Question 2

(i) Use of $\frac{1}{2}r^2\theta$ twice with subtraction gave the shaded area to then be compared with the area of the small circle. 'Show that...' meant the result was known but some candidates got wrong answers (often from taking the radius of the larger circle as 6 cm, not 9 cm) and carried on, even, in some cases, using their wrong answer in **(ii)**.

(ii) The common error here was to find the total perimeter for each sector and subtract but most candidates correctly found the two requisite arc lengths and added these to *QR* and *PS*.

Answers: **(ii)** 21.4 cm.

Question 3

(i) Two common methods were seen; writing tan in terms of sec or as sin/cos (and then using $\sin^2\theta + \cos^2\theta = 1$).

(ii) This question was well done by many, but some chose to ignore the upper limit for solutions of π or failed to give answers 'in terms of π' as required. The main problem centred on making their quartic in cosθ into a quadratic. Some put $\cos^2\theta = x$ and solved (a proportion of these forgetting to take the square root later) but far too many wrote "let $\cos^2\theta = \cos\theta$". Some of these recovered by saying "$\cos\theta = ½$ therefore $\cos^2\theta = ½$ etc..." (a grammatical error, effectively) but others, again, failed to appreciate the need to take the square root.

Answers: **(i)** $2\cos^4\theta + \cos^2\theta - 1 = 0$; **(ii)** $\frac{1}{4}\pi$, $\frac{3}{4}\pi$.

Question 4

(i) This question was handled well by many of the candidates, with the main errors arising from the extraction of the 2 from the bracket and having the third term as ax^2 not the correct $(ax)^2$ i.e. a^2x^2. Some candidates wasted their time by performing the complete expansion.

(ii) Two terms were needed, added and equated to 240. A quadratic equation in a was thus formed, and solved. Those who had ax^2 in **(i)** only had a linear equation in a and thus not 'values' as required by the question.

Answers: **(i)** $32 + 80ax + 80a^2x^2$; **(ii)** $a = -3$, $a = 1$.

Question 5

(i) Many good sketches were seen. Some candidates chose to plot accurate graphs (at the expense of their time) and some had 3, or more, curves and failed to label them, but the majority produced what was required in order for them to answer **(ii)**. The main error was the lack of points of inflection on the $\cos x - 1$ sketch with a parabola being offered.

(ii) (a) 'Hence...' required candidates to rewrite the equation as $\sin 2x = -½$ and to then see where the horizontal line at $-½$ crossed their $\sin 2x$ curve.

(b) Rearrangement of the given equation leads then to the idea that solutions will occur where their sketches cross each other. Some candidates had 'one solution' ignoring those at 0 and 2π.

Note: The solution of either **(a)** or **(b)** by algebraic methods was time consuming and worth no marks. The wording 'Hence…' requires their sketches to be used here. Similarly answers to **(ii)** if no sketch graphs were offered were also worthless.

Answer: **(i)** Sketch; **(ii)**: **(a)** 4 solutions, **(b)** 3 solutions.

Question 6

The most straightforward way to answer this question was to find an equation for 'u' in terms of just x or just y and to put the first differential equal to 0 and proceed as normal. Some candidates opted for more complicated methods however (including implicit differentiation). A large number failed to give the value of u required. 'Determining' maximum or minimum proved difficult for some but most candidates found the second differential and considered its sign. Of those who quoted the shape of the curve (a cubic) some failed to address the issue of the two turning points.

Some candidates seemed unsure of what to do, and ignored the linear equation, at first working purely on x^2y as if one or other of the x or y were a constant. Others who used correct methods were careless in their use of letters, e.g. the second differential of u with respect to x was often called d^2y/dx^2.

Answers: 12; maximum.

Question 7

(i) Finding the gradient of AB and of its perpendicular and forming, and solving simultaneously, two line equations gave the x–value at X. Some candidates forgot to go on and find the corresponding y–value.

(ii) Two methods were used, either vector steps (or its equivalent, similar triangles) or calculation of lengths AX and XB by Pythagoras's Theorem. Some candidates failed to answer the question asked and gave the ratio $AX:AB$.

Answers: **(i)** (11, 8); **(ii)** 3:1.

Question 8

(i) This question was well handled by the majority of the candidates. Some candidates were not sure which two vectors to use, while others had trouble finding an expression for vector $\overrightarrow{OC}$.

Candidates who used the cosine rule could pick up a maximum of 2 marks as the use of a scalar product was demanded by the question.

(ii) A straightforward question which some candidates got very confused about. The magnitude of vector $\overrightarrow{OC}$ was found in **(i)** and all that was required here was to see by what factor this needed to be increased to give **35.**

Answers: **(i)** 54.3° or 0.948 rads.; **(ii)** **either** $5(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})$ **or** $-5(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})$.

Question 9

(a) Many candidates immediately equated the given expression with the general expression for the sum of n terms of an arithmetic progression and solved to find a and d. A common error occurred with the ½ being lost when expanding the general expression.

Others realised that the sum of the first term IS the first term and that the sum of the first two terms is twice this plus the common difference, which was a much easier/quicker route to the solution.

(b) Some candidates tried to involve the arithmetic progression before processing the information on the geometric progression (i.e. finding r and the 3rd term). As a result, some got lost in their working. However, those who dealt fully with the geometric progression now had an arithmetic progression with 3 known terms and thus d and then n were obtained fairly easily.

Answers: **(a)** $a = 10$, $d = 4$; **(b)** $n = 15$.

Question 10

(i) This was well done by most candidates.

(ii) The equating of the two expressions for y resulted in a quadratic equation. The discriminant b^2-4ac was then used. Most candidates realised this must be negative and solved.

(iii) Two methods were adopted (treating $(8-y)$ as 'c' and using the formula for solving a quadratic; and completing the square) both of which resulted in a term involving $\sqrt{(1+x)}$. Some candidates had this in their answer with a ± sign, ignoring the fact that an inverse is single-valued.

Answers: **(i)** $x = 4$; **(ii)** $k < -8$; **(iii)** $3 + \sqrt{(1+x)}$.

Question 11

(i) Some candidates could not start this question, as they did not realise that the line *CB* being a tangent to the curve at *B* meant the gradients there were the same, and that they could then use the differential of the curve. Of those who did use this method, there was some confusion on dealing with a negative fractional power; some candidates confusing the issue further by combining the two terms and then differentiating as a quotient. Often the value of the gradient was found at $x = 1$ rather than at $x = 4$.

(ii) The actual integration of y with respect to x proved troublesome for several candidates but in general this was well done, with candidates subtracting the area under the line (found either by integration or, more simply, as the area of a triangle) from the area under the curve.

Answers: **(i)** $(1, 4\frac{1}{2})$; **(ii)** $1\frac{3}{4}u^2$.

MATHEMATICS

Paper 9709/21
Paper 21

Key Messages

Candidates should read the questions carefully, making sure that they answer them fully. The appropriate degree of accuracy should be used and candidates should also check that they have their calculators in the correct mode for the question they are doing.

General Comments

A basic lack of understanding of logarithms appeared to be quite common among many candidates. This clearly affected their performance on both **Questions 1** and **2**.

Comments on Specific Questions

Question 1

Most candidates attempted to solve either the equation $(2^x - 7)^2 = 1$ or the equations $2^x - 7 = \pm 1$. While most candidates were able to obtain the solution $x = 3$, many had difficulty with the solution which resulted from $2^x = 6$ not recognising that the use of logarithms was required..

Answer: $x = 3$ and $x = 2.58$

Question 2

This question was probably the least well done on the paper. The basic laws of logarithms were seldom applied correctly, with the result that many candidates were unable to gain any marks. Few candidates were able to recognise that $2\ln x = \ln x^2$ and many candidates erroneously thought that $\ln(3-2x) = \ln 3 - \ln 2x$. The syllabus demands of the topic of logarithms is clearly an area that needs to be concentrated upon more.

For those candidates that were able to obtain the correct quadratic equation, it was pleasing to see that many gave or indicated that the positive solution was the only valid solution.

Answer: $x = 0.6$

Question 3

(i) Few candidates were able to show the required trigonometric relationship given by not considering double angles to start with. Many tried to make use of $\sin^2 x + \cos^2 x = 1$ which was of no real help. For those candidates that did try to use the double angle formulae, sign errors often prevented them from obtaining full marks. A few candidates started with $\frac{3}{2}(1 - \cos 4x)$ and were much more successful in obtaining $12\sin^2 x\cos^2 x$ as a result.

(ii) This part of the question was done by most candidates with a good deal more success. The given answer did help some candidates identify errors in their work which they were then able to correct. It should be noted that if a candidate fails to obtain a given answer and cannot see where they went wrong, it is far better to leave their work unaltered than to try to contrive to obtain the given answer by incorrect means. There are very often method marks available which candidates are otherwise unable to obtain if they use an incorrect method.

Question 4

This question was done very well by most candidates.

(i) Apart from those candidates who made simple algebraic or arithmetic errors, most were able to obtain the required values.

(ii) The required factorisation was usually done correctly using a variety of methods. Those candidates who had incorrect values from their work in part **(i)** were usually able to gain a method mark. It needs to be pointed out that those candidates who use synthetic division often ended up with a quadratic factor of $2x^2 - 8x + 6$ rather than $x^2 - 4x + 3$, but then had an extra factor of 2 when giving their final linear factors. Candidates should be encouraged to check that their linear factors are appropriate in such cases.

Answer: **(i)** $a = 2,\ b = -6$ **(ii)** $(x-1)(x-3)(2x+3)$

Question 5

This question was done very well by many candidates.

(i) Provided candidates recognised that they had to differentiate y as a product, most were able to gain full marks for this part. Again, the given answer was a help to candidates and acted as a prompt for those who did not readily recognise that they had to use the product rule.

(ii) Most candidates were able to make use of the given answer to find the appropriate gradient, together with the relevant coordinates and produce the equation of the normal.

Answer: **(ii)** $y = \frac{1}{2} - \frac{1}{2}x$

Question 6

(i) It was expected that candidates would make sketches of the graphs of $y = 4x - 2$ and $y = \cot x$ and show that there was one point of intersection in the given range. Most were able to produce a good sketch of $y = 4x - 2$ although it appeared that many did not make use of a ruler to draw a straight line. Few candidates were able to draw the graph of $y = \cot x$. When asked to produce a sketch of a graph, it is not necessary to use graph paper, but better to do the sketch within the body of the rest of the question solution.

(ii) Most candidates chose to adopt a change of sign method with great success as long as $\mathrm{f}(x) = \cot x - 4x + 2$ or equivalent was used. For those candidates who chose to substitute the given values into $\cot x = 4x - 2$, credit was only given if a fully correct explanation was given.

(iii) This part of the question was misunderstood by many candidates who seemed to think that some sort of numerical substitution was needed rather than just a 're-writing of the equation $\cot x = 4x - 2$

(iv) The iteration process to obtain the root correct to 2 decimal places was usually done with success provided candidates had their calculator in the correct mode.

Answer: **(iv)** 0.76

Question 7

(i) A standard straightforward application of the syllabus which most candidates did really well apart from those who did not give the exact value of R but chose instead to give a value to 3 significant figures; candidates need to ensure that they understand what is meant by the phrase ‘exact value of’.

(ii) Most solutions seen were calculated correctly with candidates performing the correct order of operations. Very few candidates however were able to obtain all the solutions in the given range, with most just giving the first 2 possible solutions.

(iii) This part of the question was ‘the discriminator’ for the paper. Very few correct solutions were seen as it required insight and deduction, using the maximum value the expression that had obtained in part **(i)**, but very little work as indicated by the mark allocation.

Answer: **(i)** $R = \sqrt{29}$, $\theta = 21.80^\circ$, **(ii)** $13.3^\circ, 55.1^\circ, 193.2^\circ, 235.2^\circ$ **(iii)** $\frac{1}{116}$

MATHEMATICS

Paper 9709/22
Paper 22

Key Messages

Candidates should read the questions carefully, making sure that they answer them fully. The appropriate degree of accuracy should be used and candidates should also check that they have their calculators in the correct mode for the question they are doing. An understanding of what is required when answers are needed in an exact form is also essential.

General Comments

A basic lack of understanding of logarithms appeared to be quite common among many candidates. This clearly affected their performance on both **Questions 1** and **4**.

Comments on Specific Questions

Question 1

Many candidates failed to recognise that the integration involved a logarithmic function, but those that did were usually able to obtain most of the marks available with most errors involving the multiple of the logarithmic function. Candidates need to be able to recognise standard types of integrals readily.

Answer: $y = 2 - 2\ln(7 - 2x)$

Question 2

Most candidates attempted to find the critical values by considering either $(x-8)^2 = (2x-4)^2$ or $x - 8 = \pm(2x-4)$ or equivalent. For those that chose to use $(x-8)^2 = (2x-4)^2$, to obtain critical values, a significant number of them only stated one solution for the equation $x^2 = 16$, thus losing 3 marks.

Answer: $-4 < x < 4$

Question 3

This question was done exceptionally well by many candidates, most of whom gained full marks. The only real problems were minor slips in either the arithmetic or algebraic manipulation of the equations involved.

Answer: **(i)** $a = 7$ **(ii)** 18

Question 4

(i) Most candidates attempted to use logarithms to varying degrees of success. Most also attempted to re-arrange their resulting equation into a straight line form.

(ii) There were a surprising number of candidates who, having got a correct straight line form for part **(i)** were unable to correctly identify the gradient of their line and the intercept of their line. Of those that did correctly identify the gradient, many did not realise the significance of the word 'exact' and gave answers correct to 3 significant figures.

Answer: **(ii)** gradient = $\frac{3\ln 2}{\ln 5}$ or equivalent, intercept (0, –1)

Question 5

(i) The fact that the answer was given as part of the question enabled many candidates to obtain full marks as they were able to identify errors, usually sign errors and subsequently correct them. It should be noted that if a candidate fails to obtain a given answer and cannot see where they went wrong, it is far better to leave their work unaltered than to try to contrive to obtain the given answer by incorrect means. There are very often method marks available which candidates are otherwise unable to obtain if they use an incorrect method.

(ii) The given answer in part **(i)** also enabled those candidates who had not obtained a correct differentiation a chance to obtain full marks in a relatively straightforward process of finding a normal. Some candidates did mistakenly find the equation of the tangent, but this was fairly rare. Too many candidates however, did not read fully what was required and failed to give their answer in the required form.

Answer: **(ii)** $5x + 12y + 2 = 0$

Question 6

(i) It was expected that candidates sketch the graphs of $y = 3e^x$ and $y = 8 - 2x$, showing one point of intersection, which was what most candidates were able to do. It should be noted that it is not necessary to draw sketches on graph paper and that they may be drawn within the body of the question solution itself.

(ii) Most candidates chose to adopt a change of sign method with great success as long as $f(x) = 3e^x - 8 + 2x$ or equivalent was used. For those candidates who chose to substitute the given values into $3e^x = 8 - 2x$, credit was only given if a fully correct explanation was given.

(iii) This part of the question was misunderstood by many candidates who seemed to think that some sort of numerical substitution was needed rather than just a re-writing of the equation $3e^x = 8 - 2x$

(iv) The iteration process to obtain the root was usually done with great success by most candidates, however there were many who did not round their final answer correctly and the incorrect root of $x = 0.767$ was all too common.

Answer: **(iv)** 0.768

Question 7

(a) Most candidates realised that integration was involved and were able to obtain an answer in the form ke^{2x-2}. There was less success with the correct value of k itself. Again, many candidates failed to appreciate the implications of the word 'exact', giving their answer in decimal form and thus losing the final accuracy mark.

(b) Differentiation of a quotient was attempted by most with a good deal of success. Correct solutions to find the exact x-coordinate of the minimum point were quite common with most being given in the exact required form, although most calculators will now give the required exact form anyway.

Answer: **(a)** $\frac{1}{2}e^3 + 1$ **(b)** $\frac{\pi}{8}$

Question 8

(i) Correct solutions were commonplace. Occasionally candidates made sign errors and errors in the values of the trigonometric ratios.

(ii) Most candidates realised that the equation $2\operatorname{cosec}x = 3\cot^2 x - 2$ needed to be solved and either used a correct identity to obtain an equation in terms of $\operatorname{cosec}x$ or $\sin x$. Some candidates made errors when dealing with the 2, sometimes omitting it altogether, but these candidates were usually able to gain subsequent method marks. There were many completely correct solutions which was very pleasing to see.

Answer: **(ii)** 36.9°, 143.1°, 270°

MATHEMATICS

Paper 9709/23
Paper 23

Key Messages

Candidates should read the questions carefully, making sure that they answer them fully. The appropriate degree of accuracy should be used and candidates should also check that they have their calculators in the correct mode for the question they are doing.

General Comments

A basic lack of understanding of logarithms appeared to be quite common among many candidates. This clearly affected their performance on both **Questions 1** and **2**.

Comments on Specific Questions

Question 1

Most candidates attempted to solve either the equation $\left(2^x - 7\right)^2 = 1$ or the equations $2^x - 7 = \pm 1$. While most candidates were able to obtain the solution $x = 3$, many had difficulty with the solution which resulted from $2^x = 6$ not recognising that the use of logarithms was required.

Answer: $x = 3$ and $x = 2.58$

Question 2

This question was probably the least well done on the paper. The basic laws of logarithms were seldom applied correctly, with the result that many candidates were unable to gain any marks. Few candidates were able to recognise that $2\ln x = \ln x^2$ and many candidates erroneously thought that $\ln(3 - 2x) = \ln 3 - \ln 2x$. The syllabus demands of the topic of logarithms is clearly an area that needs to be concentrated upon more.

For those candidates that were able to obtain the correct quadratic equation, it was pleasing to see that many gave or indicated that the positive solution was the only valid solution.

Answer: $x = 0.6$

Question 3

(i) Few candidates were able to show the required trigonometric relationship given by not considering double angles to start with. Many tried to make use of $\sin^2 x + \cos^2 x = 1$ which was of no real help. For those candidates that did try to use the double angle formulae, sign errors often prevented them from obtaining full marks. A few candidates started with $\frac{3}{2}(1 - \cos 4x)$ and were much more successful in obtaining $12\sin^2 x \cos^2 x$ as a result.

(ii) This part of the question was done by most candidates with a good deal more success. The given answer did help some candidates identify errors in their work which they were then able to correct. It should be noted that if a candidate fails to obtain a given answer and cannot see where they went wrong, it is far better to leave their work unaltered than to try to contrive to obtain the given answer by incorrect means. There are very often method marks available which candidates are otherwise unable to obtain if they use an incorrect method.

Question 4

This question was done very well by most candidates.

(i) Apart from those candidates who made simple algebraic or arithmetic errors, most were able to obtain the required values.

(ii) The required factorisation was usually done correctly using a variety of methods. Those candidates who had incorrect values from their work in part **(i)** were usually able to gain a method mark. It needs to be pointed out that those candidates who use synthetic division often ended up with a quadratic factor of $2x^2 - 8x + 6$ rather than $x^2 - 4x + 3$, but then had an extra factor of 2 when giving their final linear factors. Candidates should be encouraged to check that their linear factors are appropriate in such cases.

Answer: **(i)** $a = 2,\ b = -6$ **(ii)** $(x-1)(x-3)(2x+3)$

Question 5

This question was done very well by many candidates.

(i) Provided candidates recognised that they had to differentiate y as a product, most were able to gain full marks for this part. Again, the given answer was a help to candidates and acted as a prompt for those who did not readily recognise that they had to use the product rule.

(ii) Most candidates were able to make use of the given answer to find the appropriate gradient, together with the relevant coordinates and produce the equation of the normal.

Answer: **(ii)** $y = \frac{1}{2} - \frac{1}{2}x$

Question 6

(i) It was expected that candidates would make sketches of the graphs of $y = 4x - 2$ and $y = \cot x$ and show that there was one point of intersection in the given range. Most were able to produce a good sketch of $y = 4x - 2$ although it appeared that many did not make use of a ruler to draw a straight line. Few candidates were able to draw the graph of $y = cot\, x$. When asked to produce a sketch of a graph, it is not necessary to use graph paper, but better to do the sketch within the body of the rest of the question solution.

(ii) Most candidates chose to adopt a change of sign method with great success as long as $\mathrm{f}(x) = \cot x - 4x + 2$ or equivalent was used. For those candidates who chose to substitute the given values into $\cot x = 4x - 2$, credit was only given if a fully correct explanation was given.

(iii) This part of the question was misunderstood by many candidates who seemed to think that some sort of numerical substitution was needed rather than just a re-writing of the equation $\cot x = 4x - 2$

(iv) The iteration process to obtain the root correct to 2 decimal places was usually done with success provided candidates had their calculator in the correct mode.

Answer: **(iv)** 0.76

Question 7

(i) A standard straightforward application of the syllabus which most candidates did really well apart from those who did not give the exact value of R but chose instead to give a value to 3 significant figures; candidates need to ensure that they understand what is meant by the phrase 'exact value of'.

(ii) Most solutions seen were calculated correctly with candidates performing the correct order of operations. Very few candidates however were able to obtain all the solutions in the given range, with most just giving the first 2 possible solutions.

(iii) This part of the question was 'the discriminator' for the paper. Very few correct solutions were seen as it required insight and deduction, using the maximum value the expression that had obtained in part **(i)**, but very little work, as indicated by the mark allocation.

Answer: **(i)** $R = \sqrt{29}$, $\theta = 21.80^{\circ}$, **(ii)** $13.3^{\circ}, 55.1^{\circ}, 193.2^{\circ}, 235.2^{\circ}$ **(iii)** $\frac{1}{116}$

MATHEMATICS

Paper 9709/31
Paper 31

General comments

No question or part of a question on this paper seemed to be too difficult for the more able candidates, and most questions discriminated well within this group. The questions that candidates found relatively easy were **Question 1** (polynomial division), **Question 2** (binomial expansion), **Question 3** (partial fractions), **Question 4 (i)** (moduli equation), **Question 5 (i)** (differentiation via the quotient rule), **Question 8 (a)** (integration by parts), **Question 9 (i)** (expressing a trigonometric relation in terms of R and ϑ), and **Question 10 (ii)** and **(iii)** (iterative formula). Those that they found difficult were **Question 4 (ii)** (a moduli equation requiring substitution), **Question 6** (vector equation of a plane), **Question 9 (ii) (b)** (integration using substitution of an earlier derived relation) and **Question 10 (i)** (establishing a differential equation and solving it).

In general, the presentation of the work often fell below that expected from candidates attempting this paper. When attempting a question, candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrived at their answer, whether they are working towards a given answer, for example as in **Question 8 (a)**, or an answer that is not given, as in **Question 5 (a)** and **(b)**. Candidates should also be aware that the questions set do not require pages of heavy algebra, as many candidates produced in **Question 4 (ii)**, and if this is occurring they would be well advised to look again at what the question is requiring them to do and whether there is a linkage with an earlier part of the question. This is something that most candidates fail to utilise, namely that where questions are displayed in sections **(i)**, **(ii)**, etc., then there is often a direct link between the individual sections, for example **Question 4 (i)** and **(ii)**, **Question 6 (i)** and **(ii)**, **Question 9 (i)**, **(b) (ii)** and **(iii)**. However, this is not the case when questions are divided into parts **(a)** and **(b)**.

Candidates should be using scientific calculators to evaluate their answers. Where they use a calculator capable of giving exact answers to questions such as 7**(a)** and 8 **(b)**, candidates should be aware that answers given devoid of working will receive no credit.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on individual questions

Question 1

Most candidates dealt easily with this question, however, too many stopped once they had a quotient of $2x$. The fact that there was no linear or constant term in the numerator appeared to imply that they could now cease the division process. Another common error was to express the remainder as $\frac{8}{x+2}$.

Answer: $2x-4,\ 8$

Question 2

(i) Usually candidates produced a correct solution. However several candidates produced an expansion for $(1-x)^{1/2}$ instead of $(1-x)^{-1/2}$.

Answer: $1+2x-\frac{3}{2}x^2$

Question 3

(i) This question was answered well by most candidates who obtained a correct form for the partial fractions. However, too many had $\frac{B}{x+1}$ and $\frac{C}{x-1}$ instead of $\frac{Bx+C}{x^2+1}$.

Answer: $\frac{2}{x}+\frac{5x-3}{x^2+1}$

Question 4

(i) This part was answered well, with most candidates opting to solve the quadratic equation rather than the linear equations.

Answer: $-\frac{2}{3}$ and $\frac{4}{5}$

(ii) This was the point in the paper when candidates started to experience problems. Only a few realised the linkage with **(i)** and as a result tried to solve the actual moduli equation, with the appropriate consequence of pages of incorrect algebra.

Answer: –0.161

Question 5

(i) Differentiation by the quotient rule was virtually always correct, however many failed to substitute for $x = 0$.

Answer: $-\frac{1}{2}$

(ii) Implicit differentiation of y^3 was mostly successful, however applying the product rule to $5xy$ was usually incorrect, with $5\frac{dy}{dx}+5x$ common. Several candidates who differentiated correctly failed to realise that they then needed to use the value of $y(0)$.

Answer: $-\frac{5}{6}$

Question 6

(i) This question proved very difficult for nearly all the candidates, with **OA** believed usually to be $\frac{1}{2}$**PQ**, and the normal to the plane being either the actual line vector or the cross product of **OP** and **OQ**.

Answer: $12x+6y-6z=48$

(ii) Since few candidates could produce the equation of the straight line through P parallel to the x-axis, and they had the equation of the plane incorrect, only the method mark for finding the distance AB was possible.

Answer: $3\sqrt{3}$ or 5.20

Question 7

(a) This question proved difficult as few realised that they needed to express $w = u + v\,\mathrm{i}$, together with its conjugate w^*, and take real and imaginary parts. Just manipulating w and w^* failed to produce any progress towards the form required.

Answer: $7 - 2\,\mathrm{i}$

(b) Only a minority of candidates knew what was meant by arg($z - 2\,\mathrm{i}$), however most could handle the other equation to establish $y = x$. In fact a considerable number of candidates realised that arg(z) was $\frac{\pi}{4}$ yet they were unable to make any progress with the modulus.

Answer: $6.69\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$

Question 8

(i) Many candidates scored nearly full marks on this integration by parts, however with a **given answer** candidates must show their working in going from 32 ln4 – 8 ln2 to 56 ln2, otherwise the final answer mark will be withheld.

(ii) Many candidates either failed to substitute correctly for $\cos^2(4x)$ in terms of u or muddled their substitution of dx, the latter usually resulting in another $\cos^2(4x)$ appearing in the numerator instead of a $\cos(4x)$ cancelling from the numerator and the denominator.

Answer: $\frac{11}{96}$

Question 9

(i) Many candidates produced correct values for R and α, although a few candidates used the expansion of $\cos(\vartheta + \alpha)$.

Answer: 0.6435

(ii) Again most candidates obtained the answer 1.80, however the other answer proved more difficult, since instead of $2\pi - \cos^{-1}(2/5) + \alpha$ candidates assumed the answer was $2\pi - (\cos^{-1}(2/5) + \alpha)$.

Answer: 1.80 and 5.77

(iii) Only a minority of candidates realised that if they used the result from **(i)** then this integral could be converted into the integral of $2\sec^2(\vartheta - \alpha)$.

Answer: $2\tan(\vartheta - 0.6435)$

Question 10

(i) It was rare to see any candidate write the correct differential equation. Without the correct differential equation little or no progress was possible. Unfortunately, in the odd case when the correct differential equation was produced candidates failed to introduce an arbitrary constant in their integration and use the condition that at $t = 0$, $V = 0$.

Answer: $\frac{dV}{dt} = 80 - kV$

(ii) The iterative formula usually produced the correct answer and many candidates gained full marks for this section.

Answer: 0.14

(iii) Whilst candidates should have been using their most accurate value of k and not k =0.14, on this occasion values between 530 and 540 were allowed for the liquid in the tank after 20 minutes and between 567 and 571 for the liquid in the tank after a long time.

Answer: 530 to 540, 567 to 571

MATHEMATICS

Paper 9709/32
Paper 32

General comments

No question on the paper seemed to be of undue difficulty. The questions or parts of questions that were generally done well were **Question 1** (modular equation), **Question 2 (i)** (iteration), **Question 4** (algebra), **Question 8 (i)** (partial fractions) and **Question 10 (i)** (vector geometry). Those that were done least well were **Question 3** (logarithmic transform to linear form), **Question 5** (implicit differentiation), **Question 6** (calculus), **Question 9 (a)** (complex numbers), and **Question 10 (ii)** (vector geometry).

In general the presentation of work was good, though there were some very untidy and unnecessarily lengthy scripts, and most candidates appeared to have sufficient time to attempt all the questions. Candidates need to be advised that where a question is presented in parts **(i)**, **(ii)**, **(iii)** etc. it is possible that one of the later parts may be most conveniently answered by using a result obtained in an earlier part. In this paper such possibilities occurred in **Question 6**, **Question 7** and **Question 8**, but some candidates seemed unaware of the situation.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

This was generally very well answered. Most candidates squared the given equation and then solved a quadratic equation. A minority considered two linear equations.

Answer: $\frac{3}{2}$, 3

Question 2

Part **(i)** was well answered. The iteration converges rapidly and candidates seemed quite ready to give iterates to 6 decimal places and their final answer to 4 decimal places. In part **(ii)** most candidates could state a correct equation satisfied by a but errors in the subsequent simplification were surprisingly frequent. The final answers indicated that the distinction between an exact answer and an approximate answer is not clearly understood by some candidates.

Answers: **(i)** 3.6840; **(ii)** $\sqrt[3]{50}$

Question 3

For many candidates this proved a difficult question. The given function expresses y in terms of x, but the logarithmic transform to a linear form means that points on the graph have coordinates of the form (x^2, ln y). Candidates who obtained the linear form ln y = ln A – kx^2 and understood the nature of the given coordinates, or worked with an equation such as $Y = mX + c$, where $Y \equiv \ln y$ and $X \equiv x^2$ was understood or stated, had no difficulty in using the given points and completing the problem quickly. The only common error was that of equating the gradient of the line to k rather than to $-k$. A minority calculated the y values at the given points and completed the problem by substituting in the given equation and solving for A and k.

However the majority of candidates took one or both coordinates to be x– and y-coordinates and, whichever route they took, lacked a sound basis for solving the problem. Erroneous forms such as ln $y = -kx^2$ ln A and

In $y = -kx^2 \ln(Ae)$ were seen and some made futile attempts to use $\frac{dy}{dx} = -2kxAe^{-kx^2}$. Overall the impression given is that candidates need more experience of the use of logarithms to transform a relationship to linear form.

Answer: $A = 2.80$, $k = 0.42$

Question 4

This was generally well answered. In part **(i)** the most common approach was to equate $p(-\frac{1}{3})$ to zero and solve for *a.* Another less popular and more demanding method was to divide $p(x)$ by $(3x + 1)$ using long division or Synthetic Division, to equate the remainder to zero and solve for *a.* Marks were usually lost on account of sign errors.

In part **(ii)** those with $a = 12$ usually went on to find the quadratic factor $4x^2 - 8x + 3$ and to then present a complete factorisation. The distinction between factors and roots (or zeros) seems to be better known than in the past. Synthetic Division gave the factor $12x^2 - 24x + 9$, but almost all who used this method took out the factor of 3 before giving their factorisation of $p(x)$.

Answer: **(i)** 12; **(ii)** $(3x + 1)(2x-1)(2x - 3)$

Question 5

This question discriminated well. Most candidates scored some part marks for correct implicit differentiation of xy^2 and ay^2 though a few failed to take into account the fact that *a* was constant. Many then went on to reach the equation $3x^2 + y^2 - 6ax = 0$, or some equivalent, correctly. To obtain the value of *x* at *M,* a further step was needed. Only a few realised that by using the equation of the curve to eliminate y^2 they could reach an equation in *x* and thus solve the problem. The alternative approach equating the derivative of an expression for *y* (or y^2) in terms of *x* to zero was also seen occasionally and some candidates kept sufficient control of the algebra to obtain the correct answer this way.

Answer: $\sqrt{3a}$

Question 6

Part **(i)** was quite well answered. Applying the chain or quotient rule, most attempts reached $\frac{\sin x}{\cos^2 x}$, but some did not show the consequent working that leads to the given answer. In part **(ii)** some used the chain rule, incorporated the derivative of sec *x* given in part **(i)** and obtained the given answer in a couple of lines. Those who did not think to do this usually converted the problem to finding the derivative of an expression such as $\ln(1 + \sin x) - \ln \cos x$, but only a few maintained sufficient accuracy of manipulation to reach the given answer this way. The error of taking $\ln(\sec x + \tan x)$ to be $\ln \sec x + \ln \tan x$ was frequently seen. The attempts at the substitution in part **(iii)** were mostly very poor. Among the errors noted were the following: **(a)** persistent omission of $d\theta$, **(b)** taking $\sqrt{(3+\tan^2\theta)}$ to be $\sqrt{3}+\sqrt{\tan\theta}$, and **(c)** using *x* instead of θ. It was hoped that the result given in part **(ii)** would assist candidates with the conclusion to part **(iii)**. However those who obtained $\int \sec\theta \; d\theta$ often failed to follow it with $\ln(\sec\theta + \tan\theta)$. Instead answers such as $\ln(\cos\theta)$ or $-\ln(\cos\theta)/\sin\theta$ were given.

Answer: $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$

Question 7

Most candidates had a sound approach to part **(i)** and obtained a correct value for *R,* but some lacked the precision needed to obtain α to the required accuracy. In part **(ii)** the majority found one angle in the given interval by a correct method, but the second angle proved more elusive.

Answers: **(i)** $R = 2.236$, $\alpha = 71.57°$; **(ii)** 261.9°, 315°

Question 8

Part **(i)** was very well answered. In part **(ii)** nearly all candidates separated correctly and integrated the partial fractions found in part **(i)**, the integral of $\frac{1}{x^2}$ proving the most prone to error. However some ignored their previous work and produced incorrect attempts at integrating $\frac{1}{x^2(2x+1)}$ such as $\frac{\ln(2x^3+x^2)}{6x^2+2x}$. At the end some candidates spoiled otherwise correct work by giving a decimal approximation instead of an exact answer.

Answers: $\frac{1}{x^2} - \frac{2}{x} + \frac{4}{2x+1}$; **(ii)** $\ln y = 1 - \frac{1}{x} + 2\ln\left(\frac{2x+1}{3x}\right)$, $\frac{25}{36}e^{\frac{1}{2}}$

Question 9

(i) At the start most candidates failed to substitute correctly and clearly did not know how to continue and find x and y. Errors in simplifying terms, especially those involving i were common and only a few attempted to equate real and imaginary parts.

(ii) There were some excellent diagrams but also some very poor ones. A correct calculation of the maximum value of $|z|$ was rarely seen. Approximate answers obtained by estimating coordinates from the diagram scored nothing.

Answers: **(i)** $2\sqrt{2} - 2\,\mathrm{i}$; **(ii)** 3.70

Question 10

(i) Most candidates had the right approach and were successful. Arithmetic slips in calculating the components of the line vector for *AB* were costly and could have been avoided by careful checking. At the conclusion, some candidates with otherwise correct solutions lost the final mark because they multiplied the position vector by 2 and gave $13\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ as their final answer.

(ii) This proved a challenging test. A substantial number could make no progress of value. However some made good attempts at finding one or two of the key equations in b and c, and there were some fully correct solutions. Slips in forming the modulus of the vector $\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ were quite common.

Answers: $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$; **(ii)** $x - 4y - z = 12$

MATHEMATICS

Paper 9709/33

Paper 33

General comments

In general the presentation of the work was good and most candidates attempted all questions. When attempting a question, candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrived at their answer, whether they are working towards a given answer, for example as in **Question 4 (ii)** and **Question 6 (i)**, or an answer that is not given, as **Question 8(i)** and **Question 9(ii)**. Candidates are expected to use a scientific calculator and if they have a calculator that will produce the solution immediately, such as the answer of $k = 9$ for **Question 8(i)** from 2 equations involving logarithms and an exact answer of $\frac{8}{15}$ for **Question 9 (ii)** with no working, then no marks will be awarded for those sections. In addition, candidates should realise that where they have made an error, as often happened in **Question 10(iii)**, then it is even more necessary to show the details of the solution of their quadratic equation and not just 2 incorrect answers from their calculator. The latter will result in the method mark for the solution to this equation being withheld.

The questions that candidates found relatively easy were **Question 1** (inequalities), **Question 2** (laws of logarithms), **Question 3** (solution of trigonometric equation), **Question 4 (i)** (expressing trigonometric expression in a given form), **Question 5 (i)** (remainder theorem) and **Question 6 (i)**, **(ii)**, **(iii)** (numerical iteration). Those that they found difficult were **Question 4 (ii)** (integration using result from **(i)**), **Question 5 (ii)** (long division), **Question 8** (stationary points and integration) and **Question 10** (vectors). **Question 7 (ii)** proved to be difficult and few candidates made much progress despite the information given in **(i)**.

Whilst answers in radians involving fractions of π, such as $\frac{\pi}{6}$ or $\frac{\pi}{3}$, are perfectly acceptable answers, decimal multiples of π, such as 0.173π, are not.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

Most candidates solved the quadratic equation successfully. Unfortunately, they then often either produced the incorrect inequalities with $x > -1$ and $x < -\frac{3}{5}$ or they mixed their notation with an answer of $-1 > x > -\frac{3}{5}$.

Answer: $x < -1$, $x > -\frac{3}{5}$

Question 2

Again many correct answers were seen, however whilst most candidates were able to simplify the left hand side of the equation, the right hand side proved more difficult. Many either could not introduce lne =1 or they had e + x^3 despite having the correct left hand side of $\frac{y+1}{y}$.

Answer: $y = (ex^3 - 1)^{-1}$

Question 3

Most candidates were successful in obtaining the answer 40.2° and errors in solving the equation were few, although some candidates had a problem in going from $\sqrt{\frac{5}{7}}$ to the correct angle. However, many failed to find the other solution arising from tan x = $-\sqrt{\frac{5}{7}}$.

Answer: 40.2° and 139.8°

Question 4

(i) Both R and α were usually obtained correctly, however 30° instead of $\frac{\pi}{6}$ was common.

Answer: 2 and $\frac{\pi}{6}$

(ii) This section was one where candidates wasted a lot of time and became involved in heavy algebra involving difficult trigonometric expressions since they failed to use the information given in **(i)**. Often the integral of $\frac{1}{\cos(x-\alpha)}$, instead of $\frac{1}{\cos^2(x-\alpha)}$, was attempted and even when $\frac{1}{\cos^2(x-\alpha)}$ was written it was common to see an answer of $\frac{\cos^{-1}(x-\alpha)}{\sin(x-\alpha)}$ or other combinations of $\sin(x-\alpha)$ and $\cos(x-\alpha)$. The few candidates that did reach $\tan(x-\frac{\pi}{6})$ often failed to show the limits actually substituted; instead they just quoted the given answer of $\frac{\sqrt{3}}{4}$. On the mark scheme a method mark was available for substituting the limits into an expression of the correct form and an accuracy mark for obtaining the answer given. Candidates who just jumped from $\frac{1}{4}\left[\tan(x-\frac{\pi}{6})\right]_{\pi/6}^{\pi/2}$ to $\frac{\sqrt{3}}{4}$ lost both these marks.

Question 5

(i) A correct equation was usually obtained from the candidate's knowledge that (2x + 1) was a factor, but the knowledge that (2x –1) resulted in a remainder of 1 when dividing p(x) was muddled. Candidates who tried to obtain the remainder by long division often experienced problems due to the presence of non-numerical values as coefficients in p(x). Many who used the easier approach of substituting $x = \frac{1}{2}$ then equated their remainder to 0 or –1 instead of 1

Answer: a = – 10 b = – 1

(ii) Those who tried long division or synthetic division were usually successful, although some candidates muddled the two approaches, subtracting when they should have been adding and vice-versa. Some candidates attempted to substitute $x=\sqrt{\frac{1}{2}}$ but just obtained a constant value for the remainder instead of a linear term. Whilst this approach will work if one sets the remainder as $cx + d$, and then compares coefficients, it is not standard and requires some careful thought before being applied.

Answer: $3x - 2$

Question 6

(i) Many incorrectly equated the 2 values of y instead of the 2 values of $\frac{dy}{dx}$. In the differentiation it was common to see factors of 2 omitted. Again with a given answer it was necessary to see the actual logarithm rules being applied without huge jumps, such as $\ln(1/a)$ to $-\ln a$.

(ii) Some candidates incorrectly chose the function as just the right hand side of the equation in **(i)**, believing that this should show a change in sign between 1 and 2. Most who chose the correct function to investigate were successful, but candidates should realise that to acquire full marks requires more than just stating the values of the function at the 2 points, a reference to a change of sign taking place is necessary.

(iii) Very well done but for odd candidates either not working to 4 decimal places or presenting their final answer to 2 decimal places.

Answer: 1.35

Question 7

(i) Clear evidence was required that $i^2 = -1$ was being used, since just writing $(a + bi)(a - bi) = a^2 + b^2$ is what candidates were being asked to show.

(ii) It was rare to see anything approaching a correct solution as most candidates failed to use the information given in **(i)**. The most common error was squaring leading to $(z - 10i)^2 = 4(z - 4i)^2 = z^2 - 20iz - 100 =$ ....etc. However, candidates who set $z = a + bi$ and $z^* = a - bi$ were often successful.

(iii) A few successful diagrams or statements were seen.

Answer: Circle, centre (0,2), radius 4

Question 8

(i) Most candidates separated correctly and integrated to acquire $\ln t$. Unfortunately the integration with respect to x, proved much more difficult. Whilst $\ln(k - x^3)$ was common a correct coefficient was not. The boundary conditions were usually applied correctly however the laws of logarithms were often very muddled as candidates attempted to determine k and an expression for x in terms of t. Candidates would be well advised to combine all their logarithm terms into a single logarithm term, such as $\ln\left(\frac{k-1}{8(k-9)}\right) = 0$ prior to removing logarithms.

Answer: $x = (9 - 8t^{-3/2})^{\frac{1}{3}}$

(ii) Success here much depended on acquiring something approaching the correct form being established in **(i)**.

Answer: $9^{\frac{1}{3}}$

Question 9

(i) Candidates should be able to apply the product rule to $\sin^2 2x$ and $\cos x$ without the need to convert to a form in just $\cos x$. However, whilst there is nothing wrong with this latter approach if candidates feel happier using it, it often leads to candidates having errors present even before they start to differentiate. Probably half the candidates failed to differentiate $\sin^2 2x$ correctly, with the most common error being a coefficient of 2 instead of 4. Most candidates knew how to complete the question and it was only their incorrect initial differentiation that was causing them to fail to score many of the marks. Although where the differentiation was very poor it was difficult to convert to an equation involving only a single trigonometric function.

Answer: 0.685

(ii) This question appeared straight forward but candidates often experienced problems early on since they either failed to establish $\frac{dx}{du}$ correctly or substituted it incorrectly, hence instead of the $\cos x$ terms cancelling they reinforced in the numerator. However the major problem was that instead of having $\sin^2 2x$ this slipped to become just $\sin^2 x$, hence making the integral very much easier as there was now no need to know how to acquire $\cos^2 x$ in terms of u.

Answer: $\frac{8}{15}$

Question 10

(i) Some good work was seen from many candidates, although some used the equation of the line with the normal vector in the scalar product. Whilst it was possible to recover, few did so. For some reason many candidates believed the scalar product should equal 6 (right hand side of equation of plane) instead of 0.

Answer: − 6

(ii) Most candidates knew what to do and only arithmetic errors prevented them acquiring full marks. A few candidates seemed not to realise that the parameter λ was already being used in the given equation and so they used λ again in their new line equation.

Answer: 4

(iii) Attempts at this section showed many errors, starting with the use of $\cos(\tan^{-1}2)$ instead of $\cos(90^\circ - \tan^{-1}2)$. Some candidates just set their scalar product equal to $\tan^{-1}2$, and were unable to evaluate $\tan^{-1}2$ as an angle or to find the actual angle itself or to go straight to the cosine or the sine of the angle. It was common to see $(a + 6)^2$ expanded without the $12a$ term and $\sqrt{(a^2 + 5)}$ as $a + \sqrt{5}$.

Answer: 0 and $\frac{60}{31}$

MATHEMATICS

Paper 9709/41
Paper 41

General Comments

In the question paper each of the following is essential to the question,

- the motion of a car in **Question 2**,
- the motion of a distress signal in **Question 3**,
- the motion of a train in **Question 4**

and
- the motion of a car in **Question 7**.

Many candidates made sketches of pleasing images of these moving items, with waves on the sea below in the case of the distress signal. However sketches of this type do not help the candidate and can be a distraction.

The essence of the subject is that a real moving body is treated as a particle and the study of the particle's motion provides realistic information about the motion of the real body. Thus a diagram of worth to answering the question should consist of a tiny blob to represent the moving body, (or stationary body if relevant) with a straight line representing the surface of the sea or a road or a railway line and useful annotations such as a length, a weight, the magnitude of a force or the size of an angle.

In a considerable number of cases, the presentation of candidates' work was untidy. In some such cases candidates answered one question on the left hand side of a page and another question on the right hand side of the same page. In this circumstance the work of the two questions sometimes became intertwined. This procedure should be discouraged.

Candidates should read the questions very carefully. Unfortunately very many candidates did not do so in the case of **Question 6**.

Comments on Specific Questions

Question 1

It is unusual for a question to ask candidates for a statement, but the requirement of such was fairly well attempted in this question.

(i) Most candidates correctly wrote that the minimum vertical force required to move the block is less than the minimum horizontal force required to move the block, giving a coherent reason why this applied in the given circumstances.

(ii) In applying Newton's second law a fairly large minority of candidates disappointingly omitted the force of friction.

Question 2

In this question a car moves from the bottom to the top of a straight hill. In order to construct an equation allowing candidates to calculate the speed of the car when it reaches the top of the hill, candidates form a linear combination in which the work done by the driving force plus the decrease in the kinetic energy is equal to the increase in potential energy plus the work done against the resistance.

Candidates were aware of this strategy, but many omitted the increase in the potential energy. Some candidates had a problem with units, with some terms in joules and others with kilojoules. Another problem is to ensure correct signs, plus or minus, for all four terms.

Question 3

The first two parts of the question are very straightforward and candidates attempted them very well.

(i) All candidates recognised the scenario as that in which the speed had to be found of a particle being projected vertically upwards and reaching a height of 45 m above the point of projection. Candidates had no difficulty in finding the required speed.

(ii) The most common of several approaches to this question was for candidates to consider a particle released from rest and falling for 5m, and finding the time taken from $5 = ½\ 10t^2$. The time is 1s and candidates allowed another 1s to accommodate the same time for the upward part as for the downward part.

Another method that many candidates used with complete satisfaction was to use the answer found in part **(i)**. The times at which the signal is at a height of 40 m are given by $40 = 30t – ½\ 10t^2$. The two answers are 2s and 4s and correspond to passing the cliff top on the way up and on the way down. The interval from 2s to 4s is the time when the signal is above the cliff top.

(iii) This part was more testing than the first two, however most high scoring candidates scored all three of the marks available from this part of this question.

Question 4

Candidates recognised that the acceleration to be found in part **(i)** is instantaneous, whereas the speed to be found in **(ii)** relates to a period while the acceleration is zero and the speed is therefore constant. These contrasting scenarios caused no problem for the majority of candidates.

(i) Almost all candidates seemed to be aware of finding the driving force by dividing the power by the speed and using this in the application of Newton's second law to find the required acceleration.

(ii) Almost all candidates interpreted 'steady' correctly, as constant speed. Again candidates were aware of finding the driving force by dividing the power by the speed, with again using it in the application of Newton's second law but this time to find the constant (steady) speed as required.

Question 5

Some candidates may have found it daunting that the first requirement, of a connected particles question, be to find the normal and frictional components of a contact force. However the question was fairly well attempted.

(i) As usual with connected particles questions candidates applied Newton's second law to each of the particles, facilitating simultaneous equations in tension and acceleration. Candidates coped with this very well.

(ii) Candidates found no difficulty to find the required distance.

Question 6

The question says that the particle lies on a horizontal plane and is subject to horizontal forces. This must be clearly understood by candidates, but nevertheless a considerable number of candidates had the weight of the particle acting along the negative *y*-axis, which is clearly in the horizontal plane.

(i) Almost all candidates found $F\cos\theta$ and $F\sin\theta$ by equating them to the sum of the *x*-components and the sum of the *y*-components, respectively, of the other horizontal forces. Unfortunately a large minority included the weight with the *y*-components.

(ii) Most candidates realised that the resultant of the three (not four) remaining forces has magnitude equal to that of the removed force, and direction opposite to that of the direction of the removed force. Most candidates appropriately used $F = 0.5a$ to obtain the required acceleration.

Question 7

Candidates seemed very familiar with the chain s(t) ➔ v(t) ➔ a(t) by differentiation, and the reverse by integration.

(i) Candidates seemed hesitant to find v(t) because the first requirement is to find a distance, for which the s(t) function of t is available. However most candidates realised the need to find v(t) and to solve v(t) = 0 to find the time at which the car is at B. Candidates who achieved this had no difficulty with substituting the value of t into s(t) to obtain the required answer.

(ii) This part requires the maximum speed to be found. Candidates were clearly aware that solving dv/dt = 0 was the starting point, to obtain the value of t at which the maximum speed occurs. Candidates who reached this far were aware that substitution into v(t) is all that is required.

(iii) Most candidates realised the need to evaluate a(0) and a(100) to obtain the required answers.

(iv) The attempts at a sketch of the velocity-time graph were generally disappointing. Most candidates showed the starting point and ending point at (0, 0) and (100, 0) as expected. Most candidates identified (66.7, 20.8) as the maximum point, but the sketch of many candidates consisted of straight line segments from (0, 0) to (66.7, 20.8) and from (66.7, 20.8) to (100, 0).

Very few candidates demonstrated the zero acceleration at the origin with the curve having the t-axis as a tangent. Furthermore few had the slope of the curve continuously increasing from zero at the origin to a point where the slope starts continuously decreasing until it become zero at the maximum velocity point.

MATHEMATICS

Paper 9709/42

Paper 42

General Comments

The paper was generally well done by many candidates and the presentation of the work was good in most cases.

Candidates should not divide their written answer page vertically as this often makes it much more difficult to follow the candidate's work and their arguments.

Many candidates lost marks due to not giving answers to 3sf as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3sf then their working should be performed to at least 4sf.

Candidates should be reminded to read the question paper very carefully, as many candidates misread some of the values which were stated in the examination paper.

On several questions, in this paper **Questions 2**, **3**, **4** and **7**, sines and cosines of angles were given in the question. This should indicate to candidates that there is no necessity to use a calculator in order to determine the angle itself. However, many candidates often then proceeded to find the angle and immediately lost accuracy and marks.

One of the rubrics on this paper is to take g = 10 and it has been noted that almost all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used. This was the case in this paper in **Question 2 (ii)**.

Comments on Specific Questions

Question 1

Generally part **(i)** of this question was well done. In part **(ii)** many candidates continued to use R = 30 rather than resolving vertically and involving the 25N force. In addition some candidates misread the question as being one involving an inclined plane.

Answers: **(i)** Coefficient of friction μ = 0.8 **(ii)** a = 2.55 ms^{-2}

Question 2

Candidates found this question quite difficult. Any method should include work done by the driving force, work done against resistance, kinetic energy and potential energy. Many candidates omitted to include one or more of these elements in their solution. In part **(ii)** many assumed either that the block began moving with a speed of 10 ms^{-1} or started from rest, neither of which was specified in the question. Some candidates assumed that the problem involved constant acceleration when in fact this was not stated in the question.

Answer: **(i)** 750J

Question 3

The majority of candidates performed well on this question. Several different methods were seen. Most resolved horizontally and vertically. Other methods used included the triangle of forces or Lami's theorem. Candidates who used the exact values of the sine and cosine of the angles produced excellent solutions and did not suffer from premature approximation errors which cost some candidates marks. Some assumed the answer of 20N for one of the tensions rather than proving it which again lost them some of the marks. A few wrongly assumed that the tensions were the same in both strings.

Answer: Tension in *BP* = 13N

Question 4

This question was generally well attempted by candidates either by finding the acceleration down the plane and using the constant acceleration equations or by considering energy conservation. An error which occurred most often was to assume that the acceleration is g = 10. Another very common error was to assume that because sin α was given in the form 16/65 that the length of the plane was 65 m. A surprisingly common misread was 16/25 for 16/65.

Answers: **(i)** *S* = 13, speed = 5.66 ms^{-1} **(ii)** distance = 3.25 m

Question 5

This was probably the question which most candidates answered correctly. The majority of candidates realised the implication of the term "steady speed". The most common loss of marks here was due to misreading the question and quoting the answer as *P* = 20000 instead of the correct answer of *P* = 20.

Answers: **(i)** *P* = 20 **(ii)** Steady speed = 33.3 ms^{-1}

Question 6

The first two marks were almost always achieved by most candidates. However, the mark for shading the region representing *s* was almost always incorrect with candidates generally shading only that part of the graph between *t* = 20 and *t* = 26. In part **(iii)** candidates either chose to integrate their value of *a* which most found correctly as *a* = 0.75 or they started by differentiating the given result. It was possible to achieve full marks by a complete method in either of these cases.

Question 7

The first part of this question was well done by many candidates. It was slightly more complicated than the usual pulley type question as candidates had to be careful when considering particle *A* and its weight component along the plane. A significant number of candidates wrongly used the equation of motion for particle *A* as $T - 0.26g = 0.26a$. However a good number of candidates achieved full marks on this part. Again a significant number misread 16/25 for 16/65.

In part **(ii)** candidates correctly used their value of *a* to determine the speed at which particle *B* reached the floor. However once the string became slack, many still used their value of *a* from part **(i)** rather than realising that the particle *A* was now moving under gravity. In the final stage many lost a mark because they did not keep enough accuracy in their working leading up to the final answer.

Answers: **(i)** Acceleration = 5.85 ms^{-2}, Tension in string = 2.16N
(ii) Speed of *B* as it reaches the floor = 2.65 ms^{-1}, Distance of *A* from *P* = 0.475m

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

- Candidates need to be aware that when using a previously calculated value corrected to 3 significant figures, this will not always lead to 3 significant figure accuracy in the final answer (e.g. **Question 6**).
- 'Hence' questions require the use of the solution from the previous part of the question rather than another method as in **Question 2(ii)**.
- Candidates should check that if they write down a work / energy equation then all the terms represent either work or energy rather than force (e.g. **Question 1**). They also need to be careful to consider 'work done' rather than assuming 'kinetic energy gained = potential energy lost'.

General comments

As usual the full range of marks was seen. Many candidates presented their work as expected with solutions supported appropriately by diagrams and clear working. There was, however, a significant number of candidates whose work was written in unclear handwriting, sometimes using a minimum of space and showing a minimum of working, which made it very difficult to follow.

On this paper **Question 3** and **Question 4** were the best answered questions. **Question 5(ii)** was found to be particularly difficult while **Question 1** and **Question 2(ii)** were also less well answered.

Comments on specific questions

Question 1

Candidates found this to be a challenging question at the start of the paper. The two approaches using Newton's Second Law and a constant acceleration formula or considering work and energy were both used successfully but often a missing or incorrect term in the equations formed led to an incorrect speed. Those using constant acceleration sometimes believed a=gsinθ despite the rough plane and found the speed to be 17.5 ms^{-1}. Those forming a work / energy equation sometimes included the frictional force rather than 'work done by the frictional force', thus obtaining a speed of 17.5 ms^{-1}. Others equated 'gain in kinetic energy' to 'loss in potential energy' omitting the work done against friction.

Answer: 16.9 ms^{-1}

Question 2

This was a difficult question for some candidates.

(i) The best solutions calculated the difference between the potential energy of particle B at the start and particle A when B reached the level of the bottom of the plane. Candidates attempted a variety of alternative solutions involving acceleration, work done or kinetic energy. Some candidates appeared to look for a calculation which gave the answer 23.328 (provided in the question) without making it clear that this was equivalent to the loss in potential energy.

(ii) This question asked for a solution 'hence', so candidates were expected to use the PE loss of 23.328J given in part **(i)**. Many equated this to ½ mv^2 but they were not always clear which value to use for m. m=2 was frequently seen leading to 4.83 ms^{-1}. Another commonly seen incorrect answer was 8.05 ms^{-1} obtained from equating PE loss for B to KE gain for B but omitting the work done by the tension. An alternative method (not 'hence') leading to the correct speed was also seen regularly. Candidates formed two equations using Newton's Second Law for each particle to find the acceleration of the system and then using $v^2=u^2+2$ as (or equivalent) to calculate v.

Answer: 23.328 J; 3.6 ms^{-2}

Question 3

The majority of candidates knew how to find the values of P and R by forming two equations in P and R and then solving them simultaneously. There were many fully correct solutions. However, candidates often completed their solution working with 'P watts' instead of 'P kilowatts' as stated in the question. Whilst candidates generally knew 'P=Fv' they were sometimes unsure about the meaning of 'F', stating e.g. P=mav rather than considering the driving force.

Answer: 27.2; 825

Question 4

This was one of the best answered questions on the paper with many candidates gaining full marks. The majority recognised that they needed to use integration to find the velocity of the aeroplane in part **(i)** and then use integration a second time to find the distance travelled as required in part **(ii)**. If error occurred in part **(i)** it was usually either in the solution of the equation v(t)=90, or in attempting to solve for constant instead of variable acceleration. Some candidates used integration in part **(i)** successfully to find t but then attempted to use constant acceleration e.g. 's= ½ (u+v)t' in part **(ii)**. A few integrated without completing their solution by using substitution to find the distance travelled.

Answer: 50; 2125 m

Question 5

Candidates found this question very challenging and it was not common to see fully correct solutions. Some candidates did not attempt part **(ii)**.

(i) Candidates were expected to find the difference between the times taken by each particle to reach the ground. A significant number of candidates actually found the difference between the times taken by each particle to reach their greatest height, thus giving a value of 1 instead of 2 for T.

(ii) In part **(ii)** the problem involved a difference in height as well as a difference in time and whilst many candidates recognised $s_P-s_Q=5$, they often formed an equation suggesting that $t_P=t_Q$ rather than $t_P=t_Q+2$. The most common solution involved the use of $s=ut+ ½ at^2$ for both particles and then substitution into $s_P=s_Q+5$ or equivalent which frequently led erroneously to t=0.5s ($t_P=t_Q$). Various other approaches were attempted to overcome the problem of different starting times. Some found distances travelled starting from the positions of P and Q when $t_P=2$. Others attempted to use $v^2=u^2+2$ as forming equations in v_P and v_Q but were not always able to progress further.

Answer: 2; 12 ms^{-1} and 2 ms^{-1} downwards

Question 6

(i) Many candidates were able to gain some marks for resolving forces in the *x* and *y* directions to form two equations and then solving them simultaneously. Common errors were to omit 136 when resolving in the *x*- direction ($100\cos30^\circ+120\cos60^\circ-F\cos\alpha=0$) or to include 400 when resolving in the *y*-direction ($100\sin30^\circ+F\sin\alpha-120\sin60^\circ-400=0$). Some final answers were incorrect to 3 significant figures having used approximate values from earlier calculations e.g. F= 55.1 or 54.9.

(ii) Candidates were expected to recognise that 'constant speed' allowed them to 'state' the magnitude of the frictional force as 136 N. Whilst candidates knew that F=μR, they were often unsure what values to substitute for F and R, sometimes using e.g. F=55.0 from part **(i)** or a combination of *x*-components. The normal reaction R was sometimes treated as if it was acting in the plane of the horizontal floor rather than normal to the floor.

Answer: F=55.0; α=78.9; 0.34

Question 7

This question was generally well attempted with candidates often managing the first two parts successfully.

(i) Candidates usually attempted to form an equation of motion for each particle and then solved these simultaneously to find the acceleration. Since 2.5 ms^{-2} was given in the question, some candidates used this (without showing it first) together with **one** equation of motion to find the tension T. Other candidates calculated the acceleration correctly but forgot to find T.

(ii) This was usually well answered using $v^2=0^2+2as$ and $a=2.5$ as given in the question even if not found in part **(i)**. Those who attempted to use energy to find the speed invariably omitted to consider the work done due to the tension erroneously assuming $\frac{1}{2}mv^2=mgh$.

(iii) This part of the question discriminated between those who believed $a=2.5$ as previously ($v^2=1.5^2+2\text{x}2.5\text{x}0.03 \rightarrow v=1.55$) and those who realised that the particle A decelerates after B reaches the ground. Some calculated $a=20/7$ instead of $a=-20/7$ leading to $v=1.56$ ms^{-1} but a good proportion of candidates concluded successfully $v=1.44$ ms^{-1}. A distance of 0.3 m rather than 0.03 m was sometimes seen.

Answer: 2.5 ms^{-2}; 6.75 N; 1.5 ms^{-1}; 1.44 ms^{-1}

MATHEMATICS

Paper 9709/51
Paper 51

General Comments

The presentation of the work was good in most cases and usually easy to follow.

Generally $g = 10$ was used with only a few candidates still using $g = 9.8$ or 9.81.

Comments on Specific Questions

Question 1

This question was generally well done and source of good marks.

(i) Many candidates found the correct expressions for x and y in terms of t.

(ii) Some candidates, instead of eliminating t from the 2 expressions found, used the trajectory equation. This was perfectly acceptable.

(iii) Some lengthy methods were seen. All that the candidate needed to do was to put $y = 0$ in the given equation and then solve it.

Answers: **(i)** $x = (20\cos 45)t$, $y = (20\sin 45)t - gt^2/2$
(ii) $y = x - x^2/40$
(iii) Distance = 40m

Question 2

(i) Candidates often used Newton's Second Law. Unfortunately, sign errors occurred or the weight component was omitted.

(ii) A 3 term energy equation was often seen. Again, as above, sign errors appeared.

Answers: **(i)** Initial acceleration = 70 ms^{-2}
(ii) Speed at A = 12 ms^{-1}

Question 3

Candidates found this question difficult.

(i) Candidates tried to set up a moment equation. In doing so errors were made.

(ii) Most candidates attempted to take moments about A.

(iii) $F = \mu R$ was attempted. Errors in finding F and R were seen.

Answers: **(i)** Distance of centre of mass from AB = 0.04m
(ii) Weight = 30N
(iii) Least value of $\mu = 0.133$

Question 4

(i) a = 10 – 0.45v was usually found correctly.

(ii) $\int \frac{dv}{10 - 0.45v} = \int dt$ was often seen. On a number of occasions ln(10–0.45v) = t + c was seen instead of $-\frac{1}{0.45}$ ln(10–0.45v) = t + c. The value of c was too often incorrectly calculated. Some candidates tried to use the rectilinear equations of motion.

Answers: **(i)** a = 10 – 0.45v
(ii) v = 12.9

Question 5

This question proved to be a good source of marks.

(i) Some candidates found the vertical component of the speed as 50sin40 + 2.5g instead of 50sin40 – 2.5g. The horizontal and vertical components of the speed were usually found. The resultant was then found by using Pythagoras's theorem.

(ii) On occasions tan $\theta = v_y/v_x$ was used instead of tan $\theta = s_y/s_x$, where θ was the required angle.

Answers: **(i)** Speed of P = 39.0 ms^{-1}
(ii) Angle = 27.1°

Question 6

The first part of the question was quite well done. The second part proved to be rather difficult.

(i) a = 1.2^2/(0.2cos30) was used quite often. Unfortunately at times 0.3x1.2^2/(0.2cos30) was seen instead of Tcos30 = 0.3x1.2^2/(0.2cos30). r = 0.2 was used on occasions instead of r = 0.2cos30.

(ii) (a) The correct solution should be.
Resolve vertically: Tcos30 = 0.3g, T = 6
Resolve horizontally: 6cos30 = 0.3ω^2x(0.2cos30), ω = 10. (Newton's Second Law)

(b) KE = 0.3x$(10\text{x}0.2\cos30)^2$/2 = 0.45J

Answers: **(i)** Tension = 2.88N
(ii)(a) ω cannot exceed 10rads^{-1}
(ii)(b) Greatest KE = 0.45J

Question 7

This question proved to be a very difficult one. Many candidates did not score high marks.

Answers: $\theta \geq 0.224$

MATHEMATICS

Paper 9709/52
Paper 52

General Comments

Generally the work submitted was neat and easy to follow.

Only a few candidates used g = 9.8 or 9.81 instead of g = 10.

Comments on Specific Questions

Question 1

This question was generally well done. Many candidates scored all 4 marks. A number of candidates gave an answer of 5 which came from incorrect working.

Answers: **(i)** 3N
(ii) 5N

Question 2

(i) Some candidates used the incorrect formula to find the centre of mass. A formula booklet is provided and should always be referred to. Too often the answer was left as $1/3\pi$. Answers should be given to 3 significant figures.

(ii) The use of $\tan\theta = \dfrac{theirOG}{0.25}$ was often seen. From time to time $\tan\theta = \dfrac{0.25}{theirOG}$ appeared. This gives the angle with the horizontal. The question requires the angle with the vertical.

(iii) A moment equation is now required. On occasions W = 6sin45 was seen. No moment terms were present.

Answers: **(i)** Centre of mass from AB = 0.106m
(ii) Angle with the vertical = 23.0°
(iii) Weight = 20.0N

Question 3

This question was generally well done.

(i) Some candidates set up an energy equation to show that the KE when OP = 1.8 was zero. Having done that they failed to make the conclusion that v = 0.

(ii) Quite often all marks were scored. Some candidates lost the final mark as no conclusion was made.

Answers: **(i)** v = 0 (i.e. P is at instantaneous rest)
(ii) P is on the point of losing contact with the surface.

Question 4

This question proved to be the easiest one on the paper. The majority of candidates scored all 6 marks.

(i) Speed of projection = 14ms^{-1}

(ii) Greatest height = 4.05m

(iii) Distance from O when B hits the ground = 19.3m

Question 5

(i) The extension of 0.3m was often seen. Quite a number of candidates formed an energy equation with too many terms present.

(ii) This part of the question was well done by many candidates. The use of $F = \mu R$ was regularly applied.

Answers: **(i)** Speed of P when B is first in limiting equilibrium = 1 ms^{-1}
(ii) $\mu = 0.5$

Question 6

(i) Too many candidates had incorrect distances in their moment equation.

(ii) Many candidates attempted to resolve horizontally and vertically. Sign errors were seen or a wrong angle was used. In trying to solve 2 simultaneous equations simple mistakes were made.

(iii) The use of $v = r\,\omega$ was frequently seen to give the correct value for the velocity.

Answers: **(i)** Height of centre of mass above the horizontal plane = 0.221m
(ii) Tension in the string = 1.96N. Magnitude of the force exerted by P on the cone = 1.18N
(iii) Speed of P = 1.39 ms^{-1}

Question 7

(i) $a = 0.32e^{x}$ was often found. On occasions $a = -0.32e^{-x}$ was seen.

Using $a = v\,dv/dx$ allowed the setting up of a differential equation. When solved this often resulted in the correct answer.

(ii) From $v = dx/dt = 0.8e^{x/2}$ another differential equation could be solved to find the required time. Some candidates used the rectilinear equations of motion. This was not permissible.

Answers: **(i)** $v = 0.8e^{x/2}$
(ii) Time taken to travel 1.4m from O = 1.26s

MATHEMATICS

Paper 9709/53
Paper 53

General Comments

The work was generally well presented and usually easy to follow.

Most candidates now use g = 10 and not g = 9.8 or 9.81.

Comments on Specific Questions

Question 1

Many candidates scored full marks on this question.

Direction of motion is 28.0° above the horizontal

Question 2

This question was well done. An energy equation was usually set up which resulted in a quadratic equation.

Extension at the lowest point is 0.263m.

Question 3

(i) Candidates often managed to find the horizontal and vertical components of the velocity at the point when the ball hit the ground. The speed was then calculated by using Pythagoras's theorem. The angle of direction was then found by using trigonometry.

(ii) Many candidates found the required distance.

Answers: **(i)** Speed = 5 ms^{-1}, Angle above the horizontal = 78.5°
(ii) Distance = 12.2m

Question 4

(i) By resolving vertically the tension was found. Some candidates made errors in calculating the angle needed to find the vertical component of the tension.

(ii) Newton's Second Law was attempted horizontally in order to find the least angular speed. Again an incorrect angle caused candidates to get the incorrect answer.

(ii) By applying Newton's Second Law with v = 1.8, the force on P could be calculated.

Answers: **(i)** Tension = 2.5N
(ii) Least value of angular speed = 5$rads^{-1}$
(iii) Magnitude of force exerted on P = 0.66N

Question 5

This question proved to be quite difficult except for part **(i)**.

(i) Both marks were usually scored.

(ii) Some candidates had not realised that one string was elastic and the other was inextensible. This caused various problems for candidates.

(iii) This part of the question required the candidate to set up a 4 term energy equation. This proved to be too difficult for many of the candidates. The equation should be $0.4v^2/2 = 0.4g(0.6–0.54) – (20(0.1)^2/(2x0.5)–20(0.4)^2/(2x0.5))$

Answers: **(i)** OP = 0.6m
(ii)(a) 4N **(b)** 0N **(c)** 2.4N
(iii) Greatest speed = 0.6 ms^{-1}

Question 6

This proved to be a difficult question.

(i) A moment equation about the base of F was needed.

(ii) (a) $F = \mu R$ must be applied.

(b) A moment equation involving toppling has to be set up. This equation is $4(1.2 – 0.4) \prec 16(1.2 – 0.4)\tan\theta$

(iii) A diagram with F sitting on its curved surface would help to find the correct distances needed. Trigonometry is then used to find the required angle.

Answers: **(i)** Distance of the centre of mass of F from its larger plane face = 0.174m
(ii)(a) $\mu = 0.25$, **(b)** $\theta \succ 14.0$
(iii) $\theta = 28.0$

Question 7

This was a difficult question for many candidates.

(i) In order to start this part of the question it was necessary to set up an equation using Newton's Second Law. This equation should be $0.2a = 0.2v\,dv/dx = –k/(1 – x)$. The variables must now be separated and then an integration applied. Using the correct limits will give the value of k.

(ii) Another equation is formed by using Newton's Second Law. The variables can be be separated and integration applied. From this result the value of v can be found.

Answers: **(i)** k = 0.1803
(ii) Speed of B = 1.35 ms^{-1}

MATHEMATICS

Paper 9709/61
Paper 61

General comments

The majority of candidates found the paper very challenging. It was evident that candidates from many centres were unfamiliar with both the Binomial and Normal Statistical Distributions. There is still a reluctance to set out work in a neat and precise manner. Inattention to accuracy can prove costly – if answers are expected to 3 significant figures, then all working prior to this should be in 4 or more significant figures. Most candidates attempted questions in numerical order with no evidence of more time being required. There was a pleasing increase in the number of candidates using a sketch/diagram in questions involving the Normal Distribution.

Some centres had issued booklets of up to 80 pages for a 75 minute examination of 7 questions, most of which can be answered in just one page per question. Whilst some candidates ignored the instruction in **Question 4** to use graph paper, at least one centre presented all their scripts on graph paper.

There is a disturbing trend of candidates submitting two or more attempts for an individual question. e.g. by giving solutions involving probabilities with one solution with adding, and an alternative with multiplying the component factors.

Comments on specific questions

Question 1

Few candidates understood the concept of a working mean and there were very few correct solutions to either part. Clearly, candidates prefer to insert large numbers into their calculators than to use a coded system.

In **(i)**, the erroneous (1957.5)/30 was invariably used.
In **(ii),** $\sum(x-c)$ was wrongly expanded to $\sum x - c$.

Answers: **(a)** 2.1
(b) 78.2

Question 2

The candidates who were aware of the Normal Approximation to the Binomial Distribution scored well on this Question. The continuity correction was often overlooked but there were few instances of confusion over variance/standard deviation. Marks were all too often lost by premature approximations of $\sqrt{(300/7)}$ and the resultant 'z' value.

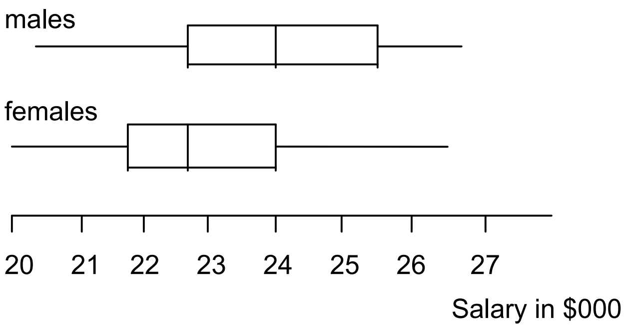
Answer: 0.703 or 0.704

Question 3

(i) Many candidates were unable to correctly determine the three quartiles. Care is required to ascertain the quartiles in ascending order. Paradoxically, weaker candidates who wrote down the 39 values in ascending order were invariably successful.

(ii) Many slipshod attempts were presented. Use of a ruler would assist in making accurate diagrams, choice of scale would eliminate inaccurate plots, (e.g. 2 cm ≡ 10 units, not 2 cm ≡ 15 units), use graph paper, all diagrams should be fully labelled (male/female, salary and $000), and the 'box' should not have a line through it.
Many candidates ignored the instruction of ' …a pair of plots in a single diagram on ...', often just presenting an unlabelled diagram

Answers: **(i)** Median $22 700 LQ $21 700 UQ $24 000
(ii)



Question 4

Very few attempts were made in either part.

(a) (0- μ)/½μ was rarely seen but, on the rare occasions when this was simplified to (-2), it was pleasing to find the correct tail of the distribution being obtained.

(b) Many realised the need to obtain a probability but then most attempts became confused with a probability being used as a 'z' value. Good solutions were then marred by incorrectly rounding to 3 decimal points, instead of to 3 significant figures.

Answers: **(a)** 0.0228 **(b)** 0.0323

Question 5

It was clear that many candidates were not familiar with the Binomial Distribution

(a) Very few candidates recognised that this situation was a Binomial Distribution and parameter is not a term in general use.

(b) Candidates made a better attempt here than in **(a)**, usually losing marks through the incorrect probability.

(c) Few recognised the need to associate 0.8^n with 0.01 or 0.99.

Answers: **(a)** B (or Bin(omial)), 12 and 0.2. **(b)** 0.422 **(c)** 21

Question 6

By far the most attempted question with different components contributing irregularly to the majority of candidates' total number of marks.

(i) Marks were lost in the omission of (!) in the constituent family sizes or for the 4 families. There was a significant number of additions instead of multiplications

(ii) 8! was often seen multiplied but the $_9C_6$ was not often obtained.

(iii) By far the most successful part; the sources of error were in adding $_8C_5$ and $_5C_3$, using $_8C_3$ on 2 or 3 successive occasions and then multiplying by 3 or 3! to obtain their final answer.

Answers: **(i)** 829440 **(ii)** 2438553600 (2.44×10^9) **(iii)** 560

Question 7

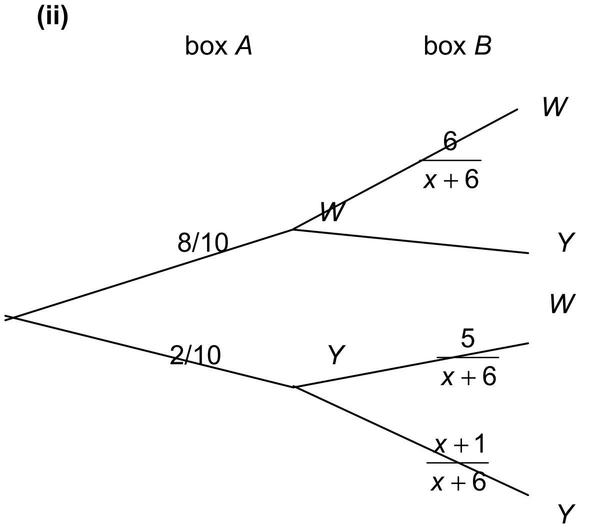
(i) Most candidates gave a satisfactory justification for P(Y) =x/(x+6).

(ii) The tree diagram was usually completed with only minor errors. Some candidates evaluated ‘x’ and substituted in the diagram’s probabilities.

(iii) The verification of x=12 took on many varied, but correct forms. The popular mistake was to precede their probability with (0.8) or (0.20) in a product.

(iv) The conditional probability was generally attempted successfully or not at all. Those candidates who did not substitute x=12 throughout their correct algebraic response to this part were not penalised.

Answers: **(i)** Sensible reason. E.g. P(Y) = x/(x+5+1) would have been the minimum response to be given credit.
(ii)



(iii) Solving x/(x+6) = 2/3 or “ 6/(6+x)” = 1/3 leading to the **AG** of x=12
(iv) 0.213 or 13/61

MATHEMATICS

Paper 9709/62
Paper 62

Key Messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks. Candidates should label graphs and axes including units, show dotted lines for finding the median or quartiles from a graph, and choose sensible scales.

General comments

This paper was well attempted by the majority of candidates who had worked and prepared for it. Most candidates appeared to have covered all the topics and were able to make a start on the questions. There were some discriminating questions, in particular **Questions 3** and **4**, in which part **(i)** was routine work and part **(ii)** required understanding of the underlying principles. Good candidates managed to answer both parts successfully.

Comments on specific questions

Question 1

This question was well attempted by the majority of candidates, although there were some who were confused by the words 'The mean is five times the standard deviation' and used it the wrong way round. It was pleasing to see many candidates using the tables backwards to find the correct z-value.

Answer: 15.5

Question 2

Candidates who knew that the variance formula is the same for coded data as uncoded data found this question straightforward. A few candidates expanded the brackets, involving a lot of extra work, and managed to score full marks.

Answers: 16.81, 63811

Question 3

(i) This was a straightforward question on the normal distribution which involved standardising and finding a probability. A surprising number of candidates used a continuity correction and also found the wrong area under the normal curve. A diagram of the normal curve with the area required shaded should show all candidates whether the required probability is greater than 0.5 or less than 0.5. Some candidates did not appear to understand the difference between 3 significant figures and 3 decimal places. The front page of the question paper states that all answers be given to 3 significant figures.

Answer: 0.0824

(ii) The second part of this question was easily solved using a diagram of the normal curve. Many candidates who did not draw a diagram thought that the area on the left of the upper limit was 0.94 instead of 0.97, thus losing the final accuracy mark.

Answer: 6.77

Question 4

(i) Many candidates thought that there were 10 random numbers between 1 and 9 inclusive, instead of 9. It was pleasing that many candidates did recognise the binomial distribution with n = 5 and so were awarded a method mark. But overall, a disappointingly small proportion of candidates managed to get the correct p (4/9) and the correct binomial distribution of 1 – P(0, 1), and evaluate it correctly.

Answer: 0.735

(ii) This was the most difficult part question on the paper as it involved candidates reading and absorbing the information in the question and applying it to a new situation. Many candidates wrote down the correct equations for the mean and variance of the binomial distribution, i.e. np = 96, npq = 32, but were unable to solve them and thus could not score anything at this stage. Those who did solve the equations then had to recognise that p = k/9, which was similar to part **(i)** where p = 4/9.

Answers: 144, 6

Question 5

This question should have been the easiest on the paper but it did not score the highest marks. Candidates struggled to cope with 3-figure information even though every number ended in a zero, and did not appreciate that 'leaves' can only be single digits. The stem of 0, 1, 2, 3, 4, 5 and leaves of 1, 4, 6, 8 (top row) and so on with a key of 1 | 3 represents $130 were seldom seen. Only about 10% of candidates remembered to put the dollar sign ($) in their key. The median and inter quartile range were well done with only a few candidates not using the (n+1)/4 th number for the lower quartile. The last part involved finding outliers and was well attempted by many candidates. Many did not read the question carefully and subtracted the wrong way round. Many multiplied the LQ and UQ by 1.5 instead of the IQR.

Answers: IQ R 70, outliers 10, 450, 570

Question 6

It was most pleasing to see that many candidates scored full marks on this question. Some candidates did not read the question carefully enough, where it says that the trees are all different and so treated the question as if the trees were all identical, thus they were only able to earn Method marks. Most candidates recognised that 'selections' meant using Combinations in part **(i)** and 'arrangements' meant using Permutations in part **(ii)** or plain factorials. In part **(iii)** many candidates appreciated that if no hibiscus tree is next to another, that left 8 trees to be arranged among themselves and the 4 hibiscus could then be interspersed in 9×8×7×6 or 9P4 ways.

Answers: 282, 207360, 121,927,680

Question 7

A few candidates mis-read the question and thought that 2 sweets were taken from Susan's bag initially before transferring one to Ahmad's bag, but were awarded marks for the correct method. Generally, this question was answered well, with a large number of candidates multiplying probabilities and adding options correctly. Some who could not answer **(i)** or **(ii)** managed to score full marks on part **(iii)**.

Answers: 1/12, 28/43, P(0) = 7/24, P(1) = 19/40, P(2) = 7/30.

MATHEMATICS

Paper 9709/63
Paper 63

Key Messages

- Candidates should be encouraged to show all necessary workings. A significant number of candidates did not show sufficient working to make their approach clear.
- To do well in this paper, candidates must work to 4 significant figures or more to achieve the accuracy required.
- Candidates need to make full use of the formula sheet provided.

General comments

Answers to questions **4, 5** and **6** were generally stronger than answers to other questions.

The majority of candidates used the answer booklets provided effectively, however a number failed to utilise the available space appropriately by answering the entire paper on a single page.

A number of candidates made more than a single attempt at a question and then did not indicate which was their submitted solution.

Comments on Specific Questions

Question 1

A large number of candidates answered this question well. However, many candidates did not show sufficient supporting evidence to provide a justification of their conclusion using an appropriate probability condition such as $P(Q \cap S) = P(Q) \times P(S)$. Most candidates considered the order of Event Q, many then failed to be consistent and consider the order for Event S. The use of an outcome table was often beneficial in achieving full credit.

Answer: Independent

Question 2

Good candidates recognised that the events were dependent, and clearly identified the required probabilities, including the impact of ordering the houses (e.g. ${}_3C_2$)

The majority of candidates applied the Binomial Distribution with constant probabilities, although a few candidates incorrectly attempted to apply the Normal Distribution. Some candidates interpreted ‘greater than’ as ‘greater or equal to’.

An alternative approach of considering the ‘number of combinations’ to calculate the probability frequently produced a fully correct solution. The most common errors were caused by failing to identify the houses that satisfied the various conditions.

Answer: 4/11

Question 3

Most candidates recognised that the Normal Distribution was required. Some candidates did not achieve the required degree of accuracy by not working to 4 significant figures throughout.

(i) Almost all candidates who attempted the question undertook the standardisation accurately. The most common error was to fail to calculate the correct tail.

(ii) Good solutions clearly identified that the short buildings were 1/3 of the building that were not tall, used the Normal Distribution table accurately to evaluate the z-value and then used clear algebra to solve their final standardised equation. However, many candidates found 1/3 of either their tall buildings value or of another spuriously calculated item. A number used the ratio in the reversed order. Candidates who were less successful used the table to evaluate $\Phi(z)$

Answers: **(i)** 0·106 **(ii)** 41·5m

Question 4

Many candidates gained full credit for this question. Good solutions used log identities appropriately in **(i)** and interpreted the information correctly for the normal distribution in **(ii)**.

(i) Although the majority of candidates evaluated the accurate solution, an unexpected proportion incorrectly interpreted their result and concluded that $n = 30$, $n > 31$ or $n \approx 31$. Some candidates used 0.01 as the required probability. Weaker candidates often used Trial and Improvement to reach the correct conclusion.

(ii) Most candidates evaluated μ and σ^2 accurately. Almost all attempted to use the Normal Distribution, but a common error was not to use a correct continuity correction.

Answers: **(i)** 31 **(ii)** 0·974

Question 5

Many fully correct solutions were seen from candidates of all levels of ability.

(a) Few candidates used a Tree Diagram to clarify the information in this question. A small number of candidates did not use the appropriate probability condition. A few candidates only provided a 2 significant figure answer. It was good to see many candidates having a fraction as a completely accurate solution. Candidates should be aware that it was not necessary to convert this into a decimal.

(b) The majority of candidates equated the size of the 2 packs initially, and then solved the equation formed by comparing the probability. Weaker candidates found the sum rather than the product of the probabilities. However a significant number used an alternative approach of comparing the probabilities initially. Many of these then failed to consider the number of cards that would be needed.

Several candidates used the combination approach to probabilities, but this was less successful due to the more complex algebra involved.

Answers: **(a)** 12/19 or 0.632 **(b)** $x = 8$

Question 6

(i) There were many good graphs, but overall the standard was disappointing. Scales were nearly always linear, but poor labelling was common. The vertical scales were frequently inappropriate, making it very complex to plot the points accurately (e.g. 2 cm = 12 or 15 people). A large number of candidates plotted on an adjusted upper boundary, such as 39·5 or 40·5.

(ii) Few candidates used the correct cumulative frequency, and used their graph accurately to read off for 64 people weighing less than c kg. Many candidates clearly indicated on their graph their method.

(iii) Fully correct solutions were not very common. Good candidates provided clear calculations and workings, and gained credit for their method when numerical errors occurred.

Almost all candidates that attempted the calculation did use a correct formula for the mean, but cumulative frequency instead of frequency and class width or upper boundary instead of mid-points were common errors.

Where a value was obtained for the mean, the majority of candidates were able to use this within the standard deviation formula correctly.

Answers: **(i)** cumulative frequency graph **(ii)** 67.2 **(iii)** $\mu = 67.2$ kg, $\sigma = 11.3$

Question 7

Candidates often penalised themselves by not recognising that dogs within a breed are not all identical, and therefore order had to be considered throughout. Good solutions had clear explanations of each of the conditions being evaluated, and thus ensuring all possibilities were identified.

(i) Some candidates did not identify more than one selection of dogs which could fulfil the criteria, using the numbers provided and then calculating the possible combinations of choosing the remaining dog from the 24 dogs available regardless of breed. Inaccurate evaluation of expressions was more frequent than might be expected at this level.

(ii) The use of combinations was a common error in this part. 5! was a common final answer obtained by assuming that the pairs of dogs were single units. A surprising number of candidates were inconsistent in their approach within this part, using both combinations and permutations.

(iii) Good solutions considered how the spaniels and retrievers could be arranged, and then identified how to insert the poodles in the ‘gaps’ so that they were separated. A number of candidates attempted the complimentary approach, but although considered 3 poodles together, did not continue to eliminate where groups of 2 poodles were together.

Answers: **(i)** 168000 (168350) **(ii)** 480 **(iii)** 1440

MATHEMATICS

Paper 9709/71
Paper 71

General comments

Many candidates showed a reasonable understanding of the distributions and statistical techniques required for the syllabus. However, there were also some candidates whose responses indicated that they were not fully prepared for the demands of this paper. In general, candidates scored well on **Questions 2 and 4(i)** whilst **Questions 5 and 7** proved particularly demanding.

Overall work was reasonably well presented with methods usually clearly shown. Many candidates gave the required 3 significant figure accuracy, but some rounded to fewer figures, losing accuracy.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Candidates did not always find this question straightforward, and in many cases did not appreciate what was required. Some candidates correctly suggested that the probability of choosing Anthea, Bill or Charlie was not equally likely but then did not justify this with the actual values for these probabilities. Various suggestions were made for a fair method to use, but again few candidates scored well.

Answer: One of each is more likely; P(one of each)=0.5, P(HH) = P(TT) = 0.25
Choose Charlie only if H then T; throw again if T then H

Question 2

This was a reasonably well attempted question, with most candidates appreciating that a two-tail test was to be carried out. Calculation of the z value was found correctly by a good number of candidates; a valid comparison was then required. It is important that if z values are to be compared then a clear inequality statement or a clearly labelled diagram is shown. Alternatively areas (or X values) could be compared, but again the inequality must be clearly stated. The conclusion drawn should then be in context, and preferably not definite.

Answer: Claim cannot be accepted

Question 3

Many candidates realised what was required on this question and followed a valid method. However, some candidates omitted to include the box in their calculation, and errors were made in calculating the variance. The standardisation process was reasonably well attempted, as was using tables with their values to calculate the probability required.

Answer: 0.0147

Question 4

Part **(i)** of this question was well attempted, though some candidates confused the two alternative formulae for the unbiased estimate of the variance. Part **(ii)** was reasonably well attempted, but part **(iii)** was not. Very few candidates showed a good understanding of the Central Limit Theorem and its application.

Answer: 10.025 0.376
0.387
Yes; distribution of X unknown

Question 5

This was not a well attempted question. It was important that candidates clearly identified their answers to the distribution of X and to their approximating distribution; many candidates did not give clear answers so that it was difficult to tell which distribution they thought was X and which they thought was the approximate one. In cases where only one answer was offered it should have been clearly identified. It was also important that the distributions were fully described with the parameters clearly given. Very few candidates were able to justify their choice of approximating distribution. Those candidates who correctly identified Poisson as the approximating distribution usually went on to successfully find the probability required in part **(ii)**, however, the calculation to find the value of n was not always as well attempted. As the value of n was small, some candidates were able to find this value by direct calculation rather than to use the method expected, which was to solve an inequality in n.

Answer: X~B(520,0.008) →approx Po(4.16) because n(=500) large and np(=4.16)<5
0.597
Smallest n is 4

Question 6

This question was well attempted by some candidates. Weaker candidates, whilst understanding the methods to use, often struggled with the required integration. Common errors included the use of incorrect limits, and in part **(ii)** failure to subtract $(E(T))^2$ when calculating Var(T).

Answer: 8.41
19/3 2.09

Question 7

Questions involving Type I and Type II errors often cause problems for candidates, and this question was no exception. When describing what is meant by a Type I error in a given situation, it is important that candidates give an answer which relates to the question. Merely giving a text book definition will not score the marks. Calculation of the probability of a Type I error was not well attempted, nor was the calculation of a Type II error. However, candidates were more successful in calculating the confidence interval for p. Most candidates used the correct z value and there were good attempts at applying the formula for the confidence interval. It is important that this is written as an interval and not merely as two separate values.

Answer: 0.0355
0.210
0.514 to 0.736

MATHEMATICS

Paper 9709/72
Paper 72

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. In general, candidates scored well on **Questions 2**, **4(i)** and **(ii)** and **5** whilst **Question 6(i)** and **(ii)** proved particularly demanding.

Most candidates kept to the required level of accuracy; there were few cases where candidates lost marks for giving final answers to less than three significant figure accuracy. There were, however, a large number of candidates who lost marks through premature approximation. This was particularly noted on question **2(ii)**. It is important for candidates to realise that if a previous answer is used in a later part of the question, in order to maintain the required level of accuracy, more than three significant figures should be used in the subsequent calculations (see comments on **Question 2** below).

Timing did not appear to be a problem for candidates, though there were cases when candidates used lengthy methods when quicker ones could have been used.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Many candidates had difficulty in describing the distributions. For part **(i)** candidates needed to recognise that the distribution was Binomial, and also to give the parameters. Many candidates stated ‘Binomial’ but failed to give the values of n and p. The approximate distribution required in part **(ii)**, was not always correctly identified as Poisson, and even the candidates who did recognise this, were not always able to correctly identify the required reasons. Candidates who stated an incorrect approximate distribution for X could still gain some marks for method in part **(iii)**.

Answer: Binomial; $n=400$ $p=0.012$
Poisson n large and mean $4.8<5$
0.857

Question 2

This was a reasonably straightforward question with many candidates scoring well. Part **(ii)** required the answer to part **(i)**, and a common loss of marks was to use 1.56 rather than 14/9 causing their final answers to **(ii)** to not be accurate to the required 3.s.f. In part **(iii)** it was expected that candidates would compare their answer to part **(ii)** with 0.5 to show that the mean was less than the median. A more common (though more time consuming) approach used by candidates was to calculate the actual value of the median itself and compare this with their answer to part **(i)**. Whilst this was a valid method, it took longer and further errors were often made.

Answer: 14/9
0.473
Mean<median

Question 3

Part **(i)** was well attempted with many candidates correctly finding the value of n. However, some candidates correctly reached $\sqrt{n}= 4.242$ but then incorrectly concluded that $n=\sqrt{4.242}$. Very few candidates stated the necessary assumption that the standard deviation was still the same for the region. In part **(ii)** many candidates did unnecessary calculations. The null and alternative hypotheses needed to be stated, then a comparison (–1.563 with –1.555) needed to be shown before a conclusion was drawn. This comparison of z values was the easiest to make, however comparison of areas or x values were equally valid but required extra work to be done. It was important that the comparison was a valid one (z value with z value etc. and not a comparison of z with an area for example). A conclusion, in context and preferably not definite, with no contradictions was then required.

Answer: n=18 assume s.d. for the region is 5.7
Evidence that plants are shorter

Question 4

Parts **(i)** and **(ii)** of this question were well attempted, but part **(iii)** was not. Many candidates successfully calculated unbiased estimates of μ and σ^2 and went on to calculate the confidence interval as required. Common errors included calculating the biased estimate of σ^2 and use of an incorrect z value for the confidence interval. In part **(iii)** the calculation required was 0.02^2. Many candidates attempted lengthy and unnecessary calculations to find '0.02', and many thought 0.02 x 2 was the required probability.

Answer: 65 92.3
63.2 to 66.8
0.0004

Question 5

This was another well attempted question. The usual errors in finding the variance were seen, but the general method was usually recognised and applied. It was pleasing to note that many candidates were able to use a suitable method to find the correct area for the required probability for both part **(i)** and **(ii)**.

Answer: 0.746
0.0717

Question 6

Part **(i)** and **(ii)** of this question caused problems for many candidates. It is important that candidates realise that when finding the critical region all necessary working must be shown and full justification must be given. In this question it was necessary to calculate $P(X\le1)$ and $P(X\le2)$ in order to justify that the critical region was 0 or 1 cases. Just to state the correct critical region, without this justification did not score. It was also important here that candidates kept to the required level of accuracy with all their calculations. Having calculated the critical region, the test could quickly be carried out by stating whether 2 cases fell into the critical region or not. Many candidates did further unnecessary, though often correct, working in order to carry out the test. In part **(ii)**, as is often the case, some candidates merely quoted text book definitions rather than give an explanation using the context of the question. Many successfully stated the probability of a type I error (though 0.05 was a frequent incorrect answer). Part **(iii)** was particularly well attempted and was a good source of marks for many candidates.

Answer: CR is 0 or 1 cases. No evidence that the mean has decreased
Concluding that the mean has decreased when, in fact, it has not. 0.0314
0.312

MATHEMATICS

Paper 9709/73
Paper 73

Key Messages

Candidates need to read questions carefully and ensure that their responses are complete

General Comments

The paper enabled candidates to demonstrate their knowledge of the key topics on this component. Solutions were well presented with clear final answers. **Questions 3(i), 5** and **6** were particularly well done. Numerical calculations were done with care and to at least the required accuracy. A number of marks were available for assumptions and explanations of statistical principles and many candidates did not state these with the precision required.

Comments on Specific Questions

Question 1

This question was well answered by most candidates, who were able to accurately apply the rules regarding combinations of random variables. Some candidates made errors in part **(i)** by calculating the variance of the product rather than the sum of the random variable and / or in part **(ii)** by adding the constant into the calculation. Part **(iii)** was done correctly by most candidates finding the combined mean and variance. A few candidates applied rules for a mean to the variance calculation.

Answers: **(i)** 9.3 **(ii)** 27.9 **(iii)** mean = -2; variance =37.2

Question 2

Many candidates performed the significance test using the Binomial distribution to a high standard, and showed clear comparisons and conclusions. Few candidates stated the assumption that needed to be made which was related to the application of the Binomial distribution in the context of the question. A few candidates tried to apply the Normal approximation to the Binomial. This was not valid as the minimum requirements were not met. Many of these candidates made further errors and so did not score the available mark. Most candidates correctly stated the Hypotheses including the use of the correct parameter.

Answer: Insufficient evidence to show that the player had improved.

Question 3

The confidence interval for the mean revision time was accurately calculated by most candidates. A few candidates did not use the standard deviation that was given in the question, calculating an estimator using the given data, which led to the incorrect interval. The interpretation of the meaning of the interval proved the most challenging part of the question, with many candidates not stating that 92% of such intervals capture the true value of μ. In part **(iii)** many candidates correctly identified that a random sample means that all possible samples are equally likely. A significant number of candidates did not interpret “random” in an acceptable way.

Answers: **(i)** (56.6, 67.4) **(ii)** 92% of intervals capture μ **(iii)** Each possible sample is equally likely.

Question 4

Part **(i)** proved to be the most difficult part question on the paper for the majority of candidates. Many candidates did not appreciate that conditional probability was being tested. Most candidates calculated P(X=1|λ=2) and P(Y=4|λ=3) and multiplied these. They did not then meet the requirement that this was then expressed as a fraction of P(X+Y=5). Part **(ii)** was done well by most candidates who showed that in the conditions given that $3 \times 2^{r-1} = r!$ and verified that r=4 satisfied the equation. A few candidates failed to provide a convincing demonstration showing all the key stages in the manipulation.

Answers: **(i)** 0.259. **(ii)** $3 \times 2^{r-1} = 3!$, Valid for r=4

Question 5

Questions on continuous random variables have been well answered by candidates in past examination sessions, and this question continued that pattern. Part **(i)** required the verification that k=2. Most candidates used the correct limits (1, ∞) and showed the working required to complete a convincing proof. A few candidates used incorrect or no limits. The most common incorrect limits were (0, 1) which led to an evaluation involving a division by 0. Part **(ii)** was exceptionally well done with the correct probability nearly always found. Part **(iii)** required the calculation of E(X). Nearly all candidates knew how to start this, but a few candidates did not simplify the function before integrating. Most candidates correctly found the expectation of X using the same limits as part **(i)**. Those who used incorrect limits in part **(i)** also did so in this part. A few candidates who had solved part **(i)** correctly then used the limits from part **(ii)** (1, 2).

Answers: **(i)** k=2 **(ii)** 0.75 **(iii)** 2

Question 6

This question was very well answered, with many candidates producing fully correct solutions for both parts.

Part **(i)** required the correct calculation of the mean calls per 2.5 hours and the calculation of more than 3 calls by calculating 1 –P (3 or fewer calls |Po(3.5)). A few candidates calculated 1 –P (2 or fewer calls).

Part **(ii)** involved the approximation of a Poisson distribution by the Normal distribution. Almost all candidates recognised this. Most used the appropriate continuity correction during standardisation and found the correct area.

Answers: **(i)** 0.463 **(ii)** 0.972

Question 7

In part **(i)** the necessary assumption that the standard deviation was unchanged was identified by a minority of candidates. The majority of candidates performed the significance test in a clear and concise way. Most compared the test value of z with the critical value as an inequality and reached the correct conclusion that there was evidence that the mean profit had increased. Some candidates correctly compared probabilities. Fewer candidates than in previous sessions did not show a clear comparison of the test value and the critical value. Part **(ii)** was not done as well as part **(i)**. Few candidates correctly identified that as the distribution of the weekly store profit was unknown it was necessary to use the Central Limit theorem. A significant number of candidates indicated that as n was large the theorem was not required and normality of the sample mean could be assumed. Candidates often indicated that they were familiar with the application of the theorem, but not the conditions in which its application was required. In part **(iii)** many candidates correctly identified the significance level of the test as the Type I error. A few candidates used values from part **(i)**; either the probability of the weekly profit being more than £35400 or the test value of z. The final part **(iv)** of the question required the calculation of the critical value for the test in part **(i)** and then the calculation that, with a mean profit of 36 500, the probability of the profit being below this value. The need for the calculation of this critical value was identified by some candidates. Those candidates who did calculate the critical value then usually scored full marks for this part. Those who did not usually standardised with one of the two other values in the question and scored the method marks available for standardising.

Answers: **(i)** Evidence of an increase in profit **(ii)** As the distribution of weekly mean profit was unknown the CLT was required. **(iii)** 5% **(iv)** 0.0091