MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/32  Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.
Mark Scheme Notes

Marks are of the following three types:

M  Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A  Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B  Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol \( \triangledown \) implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

• Note:  B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking \( g \) equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

- **AEF** Any Equivalent Form (of answer is equally acceptable)
- **AG** Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- **BOD** Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- **CAO** Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
- **CWO** Correct Working Only – often written by a “fortuitous” answer
- **ISW** Ignore Subsequent Working
- **MR** Misread
- **PA** Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- **SOS** See Other Solution (the candidate makes a better attempt at the same question)
- **SR** Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### Penalties

- **MR –1** A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

- **PA –1** This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.
1  EITHER: State or imply non-modular equation \((x - 2)^2 = \left(\frac{1}{3}x\right)^2\),

\[2 \quad (x - 2) = \pm \frac{1}{3} \sqrt{x}\]  \(M1\)

Obtain answer \(x = 3\)  \(A1\)

Obtain answer \(x = \frac{3}{2}\), or equivalent  \(A1\)

OR: Obtain answer \(x = 3\) by solving an equation or by inspection  \(B1\)

State or imply the equation \(x - 2 = -\frac{1}{3}\), or equivalent  \(M1\)

Obtain answer \(x = \frac{3}{2}\), or equivalent  \(A1\) \([3]\)

2  (i) Use the iterative formula correctly at least once  \(M1\)

Obtain final answer 3.6840  \(A1\)

Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405)  \(A1\) \([3]\)

(ii) State a suitable equation, e.g. \(x = \frac{x(3 + 100)}{2(x^3 + 25)}\)  \(B1\)

State that the value of \(\alpha\) is \(3\sqrt{50}\), or exact equivalent  \(B1\) \([2]\)

3  EITHER: State or imply \(\ln y = \ln A - kx^2\)  \(B1\)

Substitute values of \(\ln y\) and \(x^2\), and solve for \(k\) or \(\ln A\)  \(M1\)

Obtain \(k = 0.42\) or \(A = 2.80\)  \(A1\)

Solve for \(\ln A\) or \(k\)  \(M1\)

Obtain \(A = 2.80\) or \(k = 0.42\)  \(A1\)

OR1: State or imply \(\ln y = \ln A - kx^2\)  \(B1\)

Using values of \(\ln y\) and \(x^2\), equate gradient of line to \(-k\) and solve for \(k\)  \(M1\)

Obtain \(k = 0.42\)  \(A1\)

Solve for \(\ln A\)  \(M1\)

Obtain \(A = 2.80\)  \(A1\)

OR2: Obtain two correct equations in \(k\) and \(A\) and substituting \(y-\) and \(x^2\) – values in

\[y = Ae^{kx^2}\]  \(B1\)

Solve for \(k\)  \(M1\)

Obtain \(k = 0.42\)  \(A1\)

Solve for \(A\)  \(M1\)

Obtain \(A = 2.80\)  \(A1\) \([5]\)

[SR: If unsound substitutions are made, e.g. using \(x = 0.64\) and \(y = 0.76\), give B1M0A0M1A0 in the EITHER and OR1 schemes, and B0M1A0M1A0 in the OR2 scheme.]
4 (i) Substitute \( x = -\frac{1}{3} \), or divide by \( 3x + 1 \), and obtain a correct equation,

\[ e.g. \frac{1}{27} a - \frac{20}{9} - \frac{1}{3} + 3 = 0 \]

Solve for \( a \) an equation obtained by a valid method

\[ \text{Obtain } a = 12 \] A1

(ii) Commence division by \( 3x + 1 \) reaching a partial quotient \( \frac{1}{3}ax^2 + kx \)

Obtain quadratic factor \( 4x^2 - 8x + 3 \) A1

Obtain factorisation \( (3x+1)(2x-1)(2x-3) \) A1

[The M1 is earned if inspection reaches an unknown factor \( \frac{1}{3}ax^2 + Bx + C \) and an equation in \( B \) and/or \( C \), or an unknown factor \( Ax^2 + Bx + 3 \) and an equation in \( A \) and/or \( B \), or if two coefficients with the correct moduli are stated without working.]

[If linear factors are found by the factor theorem, give B1B1 for \( (2x-1) \) and \( (2x-3) \), and B1 for the complete factorisation.]

[Synthetic division giving \( 12x^2 - 24x + 9 \) as quadratic factor earns M1A1, but the final factorisation needs \( (x + \frac{1}{3}) \), or equivalent, in order to earn the second A1.]

[SR: If \( x = \frac{1}{3} \) is used in substitution or synthetic division, give the M1 in part (i) but give M0 in part (ii).]

5 EITHER: State \( 2ay \frac{dy}{dx} \) as derivative of \( ay^2 \)

State \( y^2 + 2xy \frac{dy}{dx} \) as derivative of \( xy^2 \)

Equate derivative of LHS to zero and set \( \frac{dy}{dx} \) equal to zero M1

Obtain \( 3x^2 + y^2 - 6ax = 0 \), or horizontal equivalent A1

Eliminate \( y \) and obtain an equation in \( x \) M1

Solve for \( x \) and obtain answer \( x = \sqrt{3}a \) A1

OR1: Rearrange equation in the form \( y^2 = \frac{3ax^2 - x^3}{x + a} \) and attempt differentiation of one side

Use correct quotient or product rule to differentiate RHS M1

Obtain correct derivative of RHS in any form A1

Set \( \frac{dy}{dx} \) equal to zero and obtain an equation in \( x \) M1

Obtain a correct horizontal equation free of surds A1

Solve for \( x \) and obtain answer \( x = \sqrt{3}a \) A1

OR2: Rearrange equation in the form \( y = \left( \frac{3ax^2 - x^3}{x + a} \right)^{\frac{1}{2}} \) and differentiation of RHS B1

Use correct quotient or product rule and chain rule M1

Obtain correct derivative in any form
Equate derivative to zero and obtain an equation in $x$ M1
Obtain a correct horizontal equation free of surds A1
Solve for $x$ and obtain answer $x = \sqrt{3}a$ A1 [6]

6 (i) Use correct quotient or chain rule to differentiate $\sec x$ M1
Obtain given derivative, $\sec x \tan x$, correctly A1
Use chain rule to differentiate $y$ M1
Obtain the given answer A1 [4]

(ii) Using $\frac{d\sqrt{3}\sec^2 \theta d\theta}{d\theta}$, or equivalent, express integral in terms of $\theta$ and $d\theta$ M1
Obtain $\int \sec \theta d\theta$ A1
Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k \ln (\sec \theta + \tan \theta)$ M1
Obtain a correct exact final answer in the given form, e.g. $\int \frac{2 + \sqrt{3}}{\sqrt{3}}$ A1 [4]

7 (i) Use $\cos (A + B)$ formula to express the given expression in terms of $\cos x$ and $\sin x$ M1
Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}} \sin x$, or equivalent A1
Obtain $R = 2.236$ A1
Use trig formula to find $\alpha$ M1
Obtain $\alpha = 71.57^\circ$ with no errors seen A1 [5]

(ii) Evaluate $\cos^{-1} (2/2.236)$ to at least 1 d.p. (26.56 to 2 d.p., use of $R = \sqrt{5}$ gives 26.57$^\circ$) B1
Carry out an appropriate method to find a value of $x$ in the interval $0^\circ < x < 360^\circ$ M1
Obtain answer, e.g. $x = 315^\circ$ (315.0$^\circ$) A1
Obtain second answer, e.g. 261.9$^\circ$ and no others in the given interval A1 [4]
[Ignore answers outside the given range.]
[SR: Conversion of the equation to a correct quadratic in $\sin x$, $\cos x$, or $\tan x$ earns B1, then M1 for solving a 3-term quadratic and obtaining a value of $x$ in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]

8 (i) Use any relevant method to determine a constant M1
Obtain one of the values $A = 1$, $B = -2$, $C = 4$ A1
Obtain a second value A1
Obtain the third value A1 [4]
[If $A$ and $C$ are found by the cover up rule, give B1 + B1 then M1A1 for finding $B$. If only one is found by the rule, give B1M1A1A1.]

(ii) Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction M1
Obtain $\ln y = -\frac{1}{2} - 2 \ln (2x + 1) + c$, or equivalent A3
Evaluate a constant, or use limits \( x = 1, \ y = 1, \) in a solution containing at least three terms of the form \( k \ln y, \ l/x, \ m \ln x \) and \( n \ln (2x + 1), \) or equivalent

Obtain solution \( \ln y = -\frac{1}{2} - 2 \ln x + 2 \ln(2x + 1) + c, \) or equivalent

Substitute \( x = 2 \) and obtain \( y = \frac{25}{36} e^{\frac{1}{2}}, \) or exact equivalent free of logarithms

(The f.t. is on \( A, B, C. \) Give A2 if there is only one error or omission in the integration; A1 if two.)

9 (a) Substitute \( w = x + iy \) and state a correct equation in \( x \) and \( y \)  
Use \( i^2 = -1 \) and equate real parts
Obtain \( y = -2 \)
Equate imaginary parts and solve for \( x \)
Obtain \( x = 2 \sqrt{2}, \) or equivalent, only

(b) Show a circle with centre \( 2i \)
Show a circle with radius 2
Show half line from \(-2 \) at \( \frac{\pi}{4} \) to real axis
Shade the correct region
Carry out a complete method for calculating the greatest value of \( |z| \)
Obtain answer 3.70

10 (i) Carry out a correct method for finding a vector equation for \( AB \)
Obtain \( r = 2i - 3j + 2k + \lambda (3i + j - k) \) or
\( r = \mu (2i + 3j + 2k) + (1 - \mu)(5i - 2j + k), \) or equivalent
Substitute components in equation of \( p \) and solve for \( \lambda \) or for \( \mu \)
Obtain \( \lambda = \frac{3}{2} \) or \( \mu = -\frac{1}{2} \) and final answer \( \frac{13}{2} i - \frac{3}{2} j + \frac{1}{2} k, \) or equivalent

(ii) Either equate scalar product of direction vector of \( AB \) and normal to \( q \) to zero or substitute for \( A \) and \( B \) in the equation of \( q \) and subtract expressions
Obtain \( 3 + b - c = 0, \) or equivalent
Using the correct method for the moduli, divide the scalar product of the normals to \( p \) and \( q \) by the product of their moduli and equate to \( \pm \frac{1}{2}, \) or form horizontal equivalent
Obtain correct equation in any form, e.g. \( 1 + b \sqrt{(1 + b^2 + c^2)(1 + 1)} = \pm \frac{1}{2} \)
Solve simultaneous equations for \( b \) or for \( c \)
Obtain \( b = -4 \) and \( c = -1 \)
Use a relevant point and obtain final answer \( x - 4y - z = 12, \) or equivalent
(The f.t. is on \( b \) and \( c. \)