MATHEMATICS

Paper 9709/11
Paper 11

Key Messages

Candidates who displayed good algebraic skills coped well with paper. The formulae provided were most useful and rarely misquoted. Efficient application of the trigonometrical identity and solution of trigonometrical equations was a feature of the better candidates' answers. In questions where no diagram is given it is often beneficial for candidates to produce their own sketch to refer to in their working.

General Comments

When answers were expressed clearly and logically Examiners were most able to award method and follow-through marks. Presentation of solutions was an issue for some candidates, this makes their logic path very difficult to follow. Where candidates list methods and formulae which may be useful they are strongly advised to indicate clearly which ones they eventually use and the order in which they have been used.

When an answer is given every effort should be made to ensure the justification of that answer is clear.

Comments on Specific Questions

Question 1

This type of question is testing the use of $\sin^2\theta + \cos^2\theta = 1$ in part **(i)**. Successful candidates realised this and recognised the implication of the statement, "θ is an obtuse angle."

The use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ with the given value of $\sin\theta$ and the candidate's result from part **(i)** generally led to full marks for part **(ii)**.

In part **(iii)** the correct answer was seen quoted directly and from the expansion of $\sin(\theta + \pi)$. Some also tested the effect on the sine of an obtuse angle by adding π to the angle.

Answer: **(i)** $-\sqrt{1-k^2}$ **(ii)** $\dfrac{-k}{\sqrt{1-k^2}}$ **(iii)** $-k$

Question 2

Those candidates who, in part **(i)**, realised *XP* was found by simple addition and that *PQ* could be found by simple substitution usually went on to find an expression for the area of triangle *XPQ*. Some overlooked the request for the area in terms of *p* and used *x*, gaining no credit.

In part **(ii)** it was expected that the chain rule, $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}p} \times \dfrac{\mathrm{d}p}{\mathrm{d}t}$ would be used. Those familiar with this area of calculus scored well.

Answer: **(i)** $A = 2p^2 + p^3$ **(ii)** 0.4

Question 3

The number of correct answers to part **(i)** of this question demonstrates how well this area of the syllabus is understood. Occasionally the first three terms in descending order were given and some candidates did not class the initial '1' as a term.

It was expected that the product of the two expressions found in part **(i)** would be used in part **(ii)** and those who recognised this were able to produce the required three terms in x^2.

Answers: **(i)(a)** $1-6x+15x^2$ **(b)** $1+12x+60x^2$ **(ii)** 3

Question 4

Most candidates elected to use the scalar product to find $\cos AOB$ in part **(i)**. The unnecessary step of finding angle AOB was often seen. Sometimes this was the only evidence that cos AOB had been found. $\overrightarrow{AB}$ was generally found correctly to start part **(ii)**. Finding the expression for $\left|\overrightarrow{OC}\right|$ proved challenging but those who realised the two moduli were equal usually went on to find two solutions. Some incorrectly assumed $\left|\overrightarrow{AB}\right|=\left|\overrightarrow{OC}\right|$ implied $\left|\overrightarrow{AB}\right|=\left|\overrightarrow{OC}\right|$ and then found three inconsistent values of k.

Answers: **(i)** $\frac{2}{7}$ **(ii)** $3, -\frac{5}{3}$

Question 5

In this type of questions the successful candidates are able to introduce and then eliminate a variable. Simple diagrams can prove to be most helpful. In part **(i)** those who introduced the angle at the centre, θ or the arc length, l, were able to deduce $24 = 2r + r\theta$ or $24 = 2r + l$ and eliminate their variable using the given sector area formula or $A = \frac{rl}{2}$ .Candidates' use of the given formula ensured arc length and sector area were rarely confused.

In part **(ii)** the absence of a constant term caused some confusion but those adept at dealing with negative terms were able to obtain a correct expression.

To find r in part **(iii)** the connection to part **(ii)** was needed but a correct version of the perimeter was required to find the equivalent sector area.

Answer: ***(ii)*** $36 - (r - 6)^2$ ***(iii)*** $r = 6$, $\theta = 2$

Question 6

In part **(i)** equating two separate expressions for the gradient of AB or forming the equation of AB were the favoured methods for finding the coordinates of A and B. Expressions for the area occasionally omitted the $\frac{1}{2}$ or the t^2.

Similar methods to part **(i)** were used in part **(ii)** to find the x-coordinate of C. Those able to recall and use the mid-point formula correctly were able to demonstrate the point lay on $y = x$. Some found the point as (t,t) without ever stating this was on $y = x$.

Answer: ***(i)*** $16t^2$ ***(ii)*** (t,t) *on* $y = x$

Question 7

A variety of methods were used to find the common ratio in part **(a)**. Some were clear and well set out but many were difficult to interpret. Again candidates showed themselves to be familiar with the contents of the formulae booklet and there was little evidence of the use of incorrect formulae.

When the common ratio was found correctly the first term and the sum to infinity were usually also found correctly. It should be again noted that no credit is given for using the formula for the sum to infinity with a value or *r* outside the range $|r| < 1$.

In part **(b)** those candidates who chose to use $S_n = \frac{n}{2}(a + l)$ found the largest term very quickly whilst those who persevered with simultaneous equations in *a* and *d* were less successful. Answers were acceptable in degrees or radians.

Answers: **(a)** $2\frac{1}{4}$ **(b)** 115.2° or 2.01

Question 8

In part **(i)** the successful candidates realised $f(x) = 7$ gave an equation in *x* and used the correct order of operations in their solutions. Others found f(7) or treated $\cos x$ as a variable rather than a function. When the range is given in radians all candidates should realise the answer is required to be given in radians. Also, when the question asks for "your **answer** to 2 decimal places," there will only be one answer in the given range.

The sketch in part **(ii)** was well drawn by some with others showing some understanding of the transformation of $\cos x$ to $f(x)$. To display a good understanding it is advisable to give some indication of scale on each axis

Many candidates in part **(iii)** correctly stated the inverse exists because the function is a one to one mapping (in the given range) or implied this with reference to their sketch. It was not sufficient to quote the 'vertical line rule' without stating that this indicated the function was a one to one mapping.

As in part **(i)**, in part **(iv)** the successful candidates were able to apply the correct order of operations, but this time to change the subject of the formula. It should be noted the final answer must be a function of *x*.

Answers: **(i)** 1.68 **(iii)** 1:1 mapping **(iv)** $2\cos^{-1}\left(\frac{x-5}{2}\right)$

Question 9

The techniques required to find the turning points in part **(i)** were well understood and often seen applied correctly. Although the origin was given as a turning point it was expected that candidates would confirm that $x = 0$ at $\frac{dy}{dx} = 0$ and $y = 0$. It was not unusual for candidates to find the two *x* values at the turning points without the corresponding y values even though the coordinates of the turning points were required.

In part **(ii)** the route to identifying the turning points using the second derivative was the most successful. Those who chose to examine the sign of the gradient at either side of the turning points rarely produced a convincing argument. Few candidates chose to quote the shape of the " $+ x^2$" curve but for those who did it produced the quickest route to the correct conclusions.

In part **(iii)** many candidates appreciated the need to differentiate *y* but only a few were able to apply the discriminant correctly after this.

Answers: **(i)** and **(ii)** (0,0) minimum, $\left(\frac{-2p}{2}, \frac{4p^3}{27}\right)$ maximum **(iii)** $0 < p < 3$

Question10

In part **(i)** the need to differentiate to find the gradient was generally appreciated. Candidates are increasingly confident in their differentiation of this type of function and many good answers were seen although the multiplication by 3 was missed by some. Finding the gradient of the normal using $m_1 m_2 = -1$ and the equation of a line passing through a point appeared to be familiar to many.

The integration to obtain the area under the curve (*P*) was often completed successfully although the substitution of the limit $x = 0$ was sometimes ignored or assumed to give a zero result. Various methods were used to find the area of the trapezium under the line *AB*. Those who found it successfully were able to reach a correct conclusion. Answers for the area under the curve without any supporting working out were not considered. When the question requires all necessary working to be shown answers obtained from functions within a calculator will not gain any credit.

Answers: **(i)** $\left(4, \frac{20}{3}\right)$ **(ii)** $\frac{32}{3}$ (both areas)

MATHEMATICS

Paper 9709/12
Paper 12

Key Message

Many candidates showed their working out which meant that wrong final answers could still receive credit for correct working. All candidates would benefit from following the advice to show all necessary working.

General Comments

Many good and excellent scripts were seen but **Questions 2, 6** and **10(ii)** did prove to be a difficult challenge for many. The standard of presentation was generally good with candidates setting their work out in a clear readable fashion with very few candidates dividing the page into two vertically. Candidates found a number of questions to be reasonably straightforward but some questions proved more of a challenge. Most candidates appeared to have sufficient time to complete the paper although some good candidates failed to complete **Question 11** and were clearly working on it when time was called.

Candidates should be encouraged to look at the number of marks available for different parts of questions. A number of candidates spent a great deal of time unsuccessfully trying to obtain the one mark available in **Questions 5(i)** or **9(iii)**. It is also worth pointing out that centres and candidates should be aware that generally a question with parts labelled **(i),(ii)** and **(iii)** implies that there is a link between the parts as opposed to questions with parts labelled **(a)**, **(b)** and **(c)** where the link may not exist. In **Question 5** a number of candidates failed to use the result shown in **5(i)** in **5(ii)**.

Comments on Specific Questions

Question 1

This question was generally well answered by most candidates. Some candidates omitted the constant of integration or substituted for x only. A minority of the candidates misunderstood the question and found the equation of a straight line through (3, 5), others misunderstood the notation and tried to find the inverse function by rearranging $y = 5 – 2x^2$ to make x the subject. There were a number of minor errors even from good candidates. Candidates could perhaps be encouraged to go back and check question **1** again when they have finished the rest of the paper.

Answer: $f(x) = 5x - \frac{2x^3}{3} + 8$

Question 2

This question proved challenging for many candidates with a relatively small number of gaining the full 4 marks. Many demonstrated that they did not have a clear understanding of the structure of the diagram and what calculations were needed to find the required area. A reasonable number found the areas of the triangle and sector correctly but only stronger candidates were able to find the area of the semicircle. Recognising that the radius was $r\sin\theta$ proved problematic. A significant number used the cosine rule and a smaller number the sine rule to correctly find the length of AB but then struggled to follow through square rooting, halving and substituting correctly to find the area. Some candidates incorrectly simplified $\frac{1}{2}r^2\sin2\theta$ as $r^2\sin\theta$, or $\frac{1}{2}r^2 2\theta – \frac{1}{2}r^2\sin2\theta$ as $2\theta – \sin2\theta$. A significant number obviously spent a lot of time on the question with several attempts or replacement solutions.

Answer: $\frac{1}{2}\pi r^2\sin^2\theta - r^2\theta + \frac{1}{2}r^2\sin^2 2\theta$

Question 3

The majority of candidates gained full marks on this question. There were occasional errors with signs in part **(i)**, however many candidates were still able to get 2 marks in part **(ii)** due to the follow through available. In part **(ii)** it took some of the candidates a long time to reach the answer as they worked out the full expansion.

Answer: **(i)** 240, –160 ; **(ii)** 560.

Question 4

There was a mixed response to this question. Some candidates did not read the question carefully enough and tried to obtain u as a function of y instead of x. For those with good algebraic technique, it was straight-forward, but a good number failed to deal correctly with the denominator when substituting y with $(12–x)/3$, resulting in an incorrect function of x. Some candidates got to $x = \mathrm{f}(u)$ but did not then differentiate but put $\mathrm{f}(u) = 0$. Many correct answers for the stationary value of x were seen but fewer for the stationary value of u as requested in the question.

Answer: 6

Question 5

Candidates were generally more successful with part **(ii)** than with part **(i)**. Candidates often made the question unnecessarily complicated by multiplying the numerator and denominator by either $\sin\theta+\cos\theta$ or by $\sin\theta-\cos\theta$ but then failed to cope with the resulting trigonometric algebra. More successful approaches were dividing the numerator and denominator on the left hand side by $\cos\theta$ or changing $\sin\theta$ to $\tan\theta\cos\theta$ and then factorising. Part **(ii)** was well answered with most candidates appreciating the link between part **(i)** and part **(ii)** of the question. A small number of candidates obtained the correct quadratic equation in $\tan\theta$ but then incorrectly factorised it to $(\tan\theta-6)(\tan\theta+1)$ resulting in incorrect angles. A small number of candidates had a fully correct method but then spoilt this by giving inaccurate answers or included extra incorrect answers.

Answer: **(ii)** 63.4, 71.6.

Question 6

This question proved to be the most difficult on the paper for many candidates. In part **(i)** a number did not consider the range of the cosine function and use $\cos kt = –1$ which would have given them the correct answer of 120m. Some candidates saw the word 'maximum' and differentiated. Others thought that the maximum height was 60m in spite of part **(iii)** asking when the height was above 90m. In part **(ii)** many candidates substituted $t = 30$ and $h = 0$ but were either unable to proceed further or found $\cos kt = 0$ and failed to consider the alternative answer of 2π. A number showed a good appreciation of the situation by substituting $t = 15$ and $h = 120$. Part **(iii)** again proved challenging and only the most able candidates appreciated that there were two solutions required which then needed to be subtracted. A very small number of candidates correctly found the time from 90m to 120m and then doubled this.

Answer: **(i)** 120; **(iii)** 10.

Question 7

There was a mixed response to this question. Many candidates found it to be a very routine question and quickly obtained full marks. Some though seemed very confused by the geometrical situation and found the equations and intersections of irrelevant lines. Students would in many cases have benefitted from drawing a sketch.

In part **(i)**, many candidates calculated the gradient of AB correctly but then failed to use the gradient of the perpendicular together with the midpoint of AB for the required line. The points (4, 6) and (2, 10) were often observed being substituted into the equation of the line rather than the midpoint. The point (3,11) was often used with the perpendicular gradient rather than the parallel one by weaker candidates.

Answer: **(i)** $y - 4 = \frac{3}{2}(x-7)$; **(ii)** (9,7).

Question 8

Many candidates scored well on this question. In part **(i)** some candidates did not really know how to tackle the question systematically but the majority successfully found that d = – 3 and then used $-22 = a + (n-1)d$ to find that n was 27 and then substituted this into the formula for the sum of n terms. Some candidates succeeded in equating the two expressions for S_n with $d = -3$, i.e. $\frac{n}{2}(a + l) = \{2a + (n-1)d\}$ but many candidates got bogged down with the algebra. Some thought that $d = 3$ instead of -3 and those who did use $d = -3$ sometimes failed to multiply out the bracket correctly, resulting in $n = 25$. Most were able to use either S_n formula but some applied them with a negative or fractional value of n and received no credit for this.
In part **(ii)** most candidates who achieved full marks used the three terms to form expressions for r giving: $\frac{2k}{2k+6} = \frac{k+2}{2k}$ which led to the quadratic $2k^2 - 10k - 12 = 0$. Other candidates formed the more complex appearing expression $(2k+6)\frac{(2k)^2}{(2k+6)} = k+2$ and did not cancel down before expanding and often got into a mess with their algebra, or abandoned the cubic equation they had at the end. Some candidates successfully solved the cubic equation rejecting the inappropriate values for k. Other common errors were $(2k)^2$ becoming $2k^2$ or $(2k+6)^2$ becoming $4k^2 + 36$ .Some weak candidates "cancelled" $\frac{2k}{(2k+6)}$ to obtain $r = \frac{1}{6}$

In part **(iii)** nearly all candidates understood which equation to use for the sum to infinity but some tried to use a value of r which was greater than 1 and received no credit for this.

Answer: **(a)** 459; **(bi)** $k = 6$, **(ii)** 54.

Question 9

This was very well attempted by many candidates, particularly part **(i)**, but there were occasional errors in calculating the modulus or the scalar product.

In part **(ii)** most candidates were able to find **OC** successfully, but quite a few then stopped, and didn't go on to find a unit vector, achieving only 2 of the 4 marks.

There was often no attempt at part **(iii)**. As the lengths of **OA** and **OC** had been found earlier in the question little work was required. Some candidates, though, confused 'isosceles' for 'equilateral 'and others wasted a lot of time attempting, usually unsuccessfully, to show that the angles rather than the sides were equal.

Answer: **(i)** 31.8°; **(ii)** $\frac{1}{6}(4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$

Question 10

As was indicated in the key message above, the phrase: ‘showing all necessary working’ was deliberately inserted in part **(i)** so that candidates who simply obtained the volume of revolution by using the integration function on their calculators with no method shown, received no credit. Most candidates realised this and attempted to show their working although some simply wrote down the answer from their calculator and showed either no working or completely incorrect working. All but the weakest recognised that they needed to use πy^2 to integrate to find the volume and it was very pleasing to see many candidates realising the need to divide by 2 when integrating. Some weak students expanded the brackets and attempted to integrate the terms individually. The substitution of limits was general good, although the negative sign caused some arithmetical issues.

In part **(ii)** many candidates recognised that they should attempt to differentiate, but most did not realise that they should use the given line to identify the gradient of the normal, and thus find the gradient of the tangent, which they needed to equate to their differential. Some who did sometimes struggled with the algebraic manipulations, and many failed to solve the quadratic so that both values of x were calculated, and as a result only achieved one pair of values. Many candidates, even some stronger ones, did not differentiate but incorrectly tried to equate the curve with the normal and then used the discriminant. A small number used this approach correctly, but with the tangent and the curve.

Answer: **(i)** $16\frac{\pi}{3}$; **(ii)** $\frac{5}{2}$ or $-\frac{7}{2}$

Question 11

There was a mixed response to this question. Those with strong algebraic skills found it straightforward but weaker candidates failed to make much progress. In part **(i)** some candidates forgot to take p to the left hand side and others struggled with the resulting expansion. Some managed it correctly but then put $p = \frac{1}{2}$. In part **(ii)** many candidates were easily able to complete the square but weaker candidates struggled to cope with the $2x^2$. As was mentioned in the general comments, a question with parts labelled **(i),(ii)** and **(iii),** etc. implies that there is a link between the parts. Weaker candidates failed to see the link between part **(ii)** and the final 3 parts of the question. In part **(iii)** some candidates put the range equal to a single value and others simply substituted in 0 and 4 and did not see the implications of part **(ii)**. In part **(v)** a common error was to try to rearrange the quadratic without completing the square.

Answer: **(i)** *p<1/2;* **(ii)** $2(x-\frac{3}{2})^2 + \frac{1}{2}$, **(iii)** $\frac{1}{2} \leq g(x) \leq 13$ **(iv)** $\frac{3}{2}$, (v) $\frac{3}{2}+\sqrt{\left(\frac{2x-1}{4}\right)}$

MATHEMATICS

Paper 9709/13
Paper 13

Key Messages

It is important to emphasise that essential working must be shown. Sometimes candidates include their 'rough working' at the end of their script - but this will not gain any credit. Candidates should be encouraged to do all their working within the questions they are submitting in the main body of the script. Other candidates do a great deal of work using calculators - but again, intermediate steps must be shown. In particular, questions which require definite integration require the actual process of integration to be shown before the limits are applied. Some questions include the words "Showing all necessary working", and it must be reported that these words are sometimes being ignored and significant loss of marks is usually the result. Candidates who do show their working, however, are often able to gain some marks for their working even when an error causes the final answer to be incorrect.

General Comments

The paper was generally well received by candidates and many excellent scripts were seen. All candidates seemed to have sufficient time to finish the paper. Some questions (e.g., **Questions 5(i)** and **11(i)**), require candidates to show that a particular result is true. When a result is given it is important that candidates understand that each step needs to be shown clearly so that Examiners are left in no doubt that the result has been obtained legitimately.

It is also worth pointing out that Centres and candidates should be aware that generally a question with parts labelled **(i), (ii), (iii)** etc. implies a certain structure to the question with possibly the results of earlier parts being required in later parts. Questions are often structured in this way to be helpful to candidates. Very often a given answer is supplied (in part **(i)**, for example) to ensure that failure to complete an early part satisfactorily does not prevent candidates from attempting later parts of the question. For a question labelled **(a)**, **(b)** etc., a link between the parts probably does not exist.

Comments on Specific Questions

Question 1

This question was very well answered with the majority of candidates scoring all 3 marks. The most common error was in finding the value of the constant term, - the error usually occurring when a factor of 2 was taken out.

Answer: $2(x - 3)^2 - 11$.

Question 2

This again was a very well answered question with most candidates achieving full marks. Some candidates forgot to divide by the coefficient of x from the linear expression in the bracket with a very few multiplying by it. Very occasionally a candidate thought that that linear expression itself needed to be integrated. Practically all candidates who integrated knew that there needed to be a constant of integration and went on to use the given point to find its value. However, a significant number of candidates thought that they needed to substitute the x-coordinate of the given point into the gradient function. They then used the point and the gradient to find the equation of a line - in effect finding the equation of the tangent to the curve at the given point rather than finding the equation of the curve.

Answer: $y = \dfrac{(2x+1)^{3/2}}{3} - 2$.

Question 3

Part **(i)** was usually done correctly. A few candidates did not fully simplify the coefficients of the terms and a very small number presented their answer in descending powers of x. In part **(ii)**, most candidates realised they needed two terms from the expansion of the two brackets and obtained the correct terms, with a few sign errors seen. The resulting quartic equation, however, was not always handled confidently. Some candidates appeared simply to spot that 2 was a solution and this answer without any working gained no credit. Stronger candidates recognised that the equation could be solved by treating it as a quadratic equation in a^2 and went on to obtain $a^2 = 4$ or -5.

Answers: **(i)** $a^5 - 5a^4x + 10a^3x^2 - 10a^2x^3$; **(ii)** $a = \pm 2$.

Question 4

In part **(i)**, almost all candidates reached $\tan\theta = \dfrac{1}{3}$ but a few candidates left this as their answer without going on to solve for θ. A few candidates thought that (180° – their acute angle) was also a solution. In part **(ii)**, it was anticipated that candidates would use a similar method as that employed in part **(i)** to obtain the equation $\tan^2 2x = \dfrac{1}{3}$ but significant numbers used the identity $\cos^2 x + \sin^2 x = 1$ to obtain correctly either $\sin^2 2x = \dfrac{1}{4}$ or $\cos^2 2x = \dfrac{3}{4}$. A disappointingly large number of candidates then forgot the ± sign when taking the square root and/or forgot to find solutions for $2x$, where $0° < 2x < 360°$ and so obtained only 2 solutions (and some candidates only 1 solution) instead of 4 solutions.

Answers: **(i)** $\theta = 18.4°$; **(ii)** 15º, 75º, 105º, 165º.

Question 5

In part **(i),** almost all candidates realised that they first had to find vectors ***AB*** and ***BC*** and usually this was done accurately. Quite a large proportion of candidates did not appear to realise that all that was needed was to show that ***AB.BC*** = 0 but applied the formula for the cosine of the angle between two vectors. This was, of course, perfectly correct but used up a little more time. Since the answer was given on the question paper, it was not sufficient to simply state that ***AB.BC*** = 0. It was necessary to show, for example, that $2 - 6 + 4 = 0$, or equivalent. Part **(ii)** attracted a large proportion of correct answers.

Answer: **(ii)** 8.6.

Question 6

The majority of candidates realised that part **(i)** of the question was no different to the more usual situation where they are given a function and are asked to find its inverse and that they needed to treat the given inverse function as the original function and find the inverse of it. This they did confidently and accurately with just the occasional sign slip. A few candidates thought they needed to integrate the given function - presumably because they interpreted $f^{-1}(x)$ as $f'(x)$. Very few candidates obtained the correct domain for $f(x)$. Although many achieved a value of –9/4 as relevant to the domain it was rare for this to be used in a correct way. Often $x < -9/4$ was seen or –9/4 was used as the lower value in a range but the upper value was 0 or some other value and not the correct upper value of infinity. Most candidates achieved full marks for part **(ii)**. Errors were then sometimes made in trying to simplify the expression into the required form.

Answers: **(i)** $f(x) = \dfrac{1}{2x+5}$ for $x \geq -\dfrac{9}{4}$; **(ii)** $\dfrac{1}{2}x - \dfrac{5}{2}$.

Question 7

Part **(i)** was usually well answered with many achieving full marks. A common mistake, however, was the failure to put brackets round $3p + 1$ when finding the difference of the y-coordinates, an error often also seen in part **(ii)**. Part **(ii)** was quite well answered by those who used the method of equating the gradient of AB with the gradient of the line perpendicular to the given line $2x + 3y = 9$ (although a surprising number of candidates did not obtain $-\dfrac{2}{3}$ as the gradient of the given line). However, those candidates who used the longer method of employing simultaneous equations were not as successful, with arithmetic and algebraic errors proliferating.

Answers: **(i)** $p = 4$ or $-\dfrac{11}{5}$; **(ii)** $p = 3$.

Question 8

Part **(i)** was generally quite well done. Some candidates appeared not to recognise the notation $f'(x)$ and attempted to find the inverse of f. Other candidates, before differentiation, attempted to simplify the given function by expressing it as a single fraction over a denominator of $(x + 1)^2$. This made differentiation much more difficult (effectively requiring differentiation of a quotient which is outside the syllabus of P1). Attempts at part **(ii)** were surprisingly disappointing. A significant number of candidates substituted one or more particular values of x before stating their conclusion and this approach nullifies the generality of the conclusion. In part **(iii)** most candidates set their first derivative to zero and a high proportion were able to solve the resulting equation successfully and find the coordinates of the stationary point.

Answers: **(i)** $(x + 1)^{-2} - 2(x + 1)^{-3}$; **(ii)** $f'(x) < 0$ hence decreasing function; **(iii)** $(-3, -\dfrac{1}{4})$.

Question 9

In part **(a)** some candidates used the formula for the nth term and set this equal to zero in order to find the value of n required to give a positive term, while other candidates used a less formal approach. Unfortunately, a significant number of candidates misinterpreted the question and stopped at 131, the value of n. There were relatively few correct responses to part **(b)** and these 5 marks proved to be the most difficult on the paper. Only a small proportion of candidates wrote down an inequality to represent a condition for convergence involving their common ratio, and rarely was it completely correct, with zero often being the lower limit rather than –1. A further complication remained for candidates in that cosine is a decreasing function within the given domain and a few candidates, having arrived correctly with critical values of $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$ then gave answers of $\theta < \dfrac{\pi}{6}$ and $\theta > \dfrac{5\pi}{6}$.

Answers: **(a)** 5; **(b)** $\dfrac{\pi}{6} < \theta < \dfrac{5\pi}{6}$.

Question 10

Part **(i)** provided a rich source of marks as the majority of candidates dealt efficiently with the relatively straightforward differentiation and found the *x*-coordinate successfully. An error that some weaker candidates made was to put d*y*/d*x* = 0 and solve for *x*. Part **(ii)**, however, discriminated well. The simplest, and most successful, approach was to find the area of the appropriate triangle (using half-base × height) and subtract from it the area under the curve (found by integrating between limits 2 and 3). Many candidates attempted to find the area of the triangle also by integration, which requires rather more work and also recognition that the limits for this are 2 and $\frac{7}{2}$. This approach was more subject to error - particularly with the employment of the different limits. A very common error was to combine the two equations before integrating and only a handful of candidates were able to compensate for the fact that the two integrations essentially require different limits.

Answers: **(i)** $y = -6x + 21$, $x = 3\frac{1}{2}$; **(ii)** $1\frac{3}{4}$

Question 11

In part **(i)** there were many excellent solutions, using a variety of approaches, but the point that candidates must bear in mind is that the major requirement was to show the stated result in a clear unambiguous manner. Indeed, many of the stronger candidates missed out key steps which they clearly felt were obvious, but there is always an inherent danger when the final answer is stated in the question. Candidates should also note that whenever they choose not to draw a diagram in their solution, the majority probably annotating the diagram on the question paper, the Examiner cannot see this diagram when marking and it is essential to be absolutely specific about the elements involved. For example, Examiners would have liked to have seen statements such as “Area of triangle *OAC* = ...”Area of sector *OAB* = ...” etc., but this clarity was often missing. A number of candidates chose to begin with the stated result and produce a justification from it; this is perfectly acceptable but several made the mistake of simply returning to their starting point in what could have been a circular argument. A minority of candidates used the stem prior to part **(ii)** in part **(i)**; it is important that candidates understand that the positioning of the stem is for a very good reason in that it is only required in the subsequent parts of the question.

Part **(ii)** required the evaluation and comparison of two perimeters using a given value for the angle; it was a little disappointing in a question on radian measure to see a significant proportion of candidates using degrees as the unit. Also, if a quantity is given to 4 decimal places in the question, it is expected that the two perimeters are calculated to the same degree of accuracy. A number of candidates obtained correct expressions for the perimeters of the two regions but then proceeded to cancel elements from either side of the ratio.

Answers: **(ii)** 1.1:1; **(iii)** 54.3º.

MATHEMATICS

Paper 9709/21
Paper 21

Key Messages

Centres should remind candidates of the importance of reading the instructions on the front of the question paper. It is expected that Centres provide candidates with paper/answer booklets to use for their solutions. It is **not** expected that candidates write their responses on the question paper. Attention should again be drawn to the level of accuracy required for answers, especially where degrees and radians are concerned. Candidates should also be reminded to read each question carefully and ensure that they have answered fully and in the required form. Answers which required an answer in radians were often given in degrees. Candidates should also remember that any questions involving both trigonometry and calculus will always make use of angles in radians.

It was evident that many candidates still do not appreciate the meaning of the word ‘exact’ in a mathematical context. Centres should endeavour to make sure that this is covered as a matter of urgency.

General Comments

There were many well prepared candidates were able to make good attempts at all the questions, showing their understanding of the topics covered in the syllabus.

Comments on Specific Questions

Question 1

(i) Most candidates were able to make a good start to the paper and gain some credit. There were many correct solutions with very few spoiled by additional ones. The most popular method was to subtract the two squared functions leading to the linear equation.

(ii) It was evident that many candidates do not appreciate the meaning of the word ‘Hence’. Those, relatively few, candidates who did and then recognised the connection with **(i)** were usually able to solve the given equation successfully, A common error was to try to start again and use logs on the whole equation, usually with little success.

Answer: **(i)** $\frac{7}{6}$, **(ii)** 0.222

Question 2

Very few candidates were able to complete this question successfully; the most common error being that candidates did not read ‘ln *y*’ on the diagram on the vertical axis, or if they did, not appreciate the impact of it. Other frequent errors involved an inability to express the equation in log form accurately. Some candidates were able to calculate the gradient of the line but were then unable to relate it to the constant *p*. Questions of this type have been problematic in the past and as a syllabus requirement more practice by candidates of this topic is clearly necessary.

Answer: $p = 0.44$, $A = 3.2$

Question 3

Most candidates were aware of the correct approach to take, but while the great majority of candidates were able to attempt differentiation of $\sin x$ many failed to differentiate $\cos 2x$ correctly. Many candidates were able to obtain a correct tangent equation but failed to comply with the requirement for 3 sf in the final answer.

Answer: $y = 0.866x - 2.53$

Question 4

(i) The majority of candidates were able to recognise the need to use both the remainder theorem and the factor theorem, usually very successfully. Some candidates, however attempted to use synthetic division to find the remainder and often failed to recognise the need for $0x$ in their table, which lead to incorrect solutions.

(ii) Most candidates were able to calculate $\mathrm{g}(x) - \mathrm{f}(x)$ for their functions but few found the maximum value by recognising that this occurred when $x = 0$. The most common error was to solve the equation $\mathrm{g}(x) - \mathrm{f}(x) = 0$ suggesting a failure to read the question fully.

Answer: **(i)** $a = 5$, $b = -12$, **(ii)** 7

Question 5

(i) Integration of an exponential function was attempted by most with varying degrees of success. Common errors included failure to divide by $\frac{1}{2}$, failure to integrate the term 1 and failing to equate e^0 to 0. Some candidates contrived to obtain the given result through incorrect work. If a candidate does not obtain a given result and cannot identify an error, it is often better to leave a response unaltered as there may be method marks available which would not otherwise be available once the solution has been 'changed'.

(ii) Candidates demonstrated further difficulties in using logs accurately and in rearranging an expression into an iterative formula. Use of iterative formulae is a syllabus topic that most candidates are able to do well and this part of the question was no exception. Most candidates were able to use an iterative formula but some did demonstrate inaccuracies in their rounding to 5 decimal places and in giving a final solution correct to 3 decimal places.

Answer: **(ii)** 1.732

Question 6

(i) Most candidates were able to make use of the correct trigonometric relationships and complete the proof successfully.

(ii) (a) Most candidates recognised the need to use the result from part **(i)** to attempt to solve the given equation. Frequent errors were to give only one solution in the required range or to give answers in degree form. Candidates should consider the range of values for a given angle to determine what unit their angle needs to be given.

(b) Very few candidates were able to apply the result from part **(i)** to help with the integration of $\sec^2 2x$. Even fewer candidates were able to integrate this correctly and obtain $\frac{1}{2}\tan 2x$.
Candidates again indicated lack of attention to detail in reading the question by confusing the use of θ and x. An exact answer was required as this type of question may be answered using the appropriate function on a calculator, which then does not show an understanding of, or application of the syllabus objectives.

Answer: **(ii) (a)** 1.11 rad, 2.13 rad **(b)** $\frac{1}{2}\sqrt{3}$

Question 7

(i) Most candidates were able to attempt differentiation of an implicit function, including the product, and to rearrange their expression successfully. It did appear that some candidates had ‘worked backwards’ from the given result. This is another topic which candidates are usually very well prepared for and this session was no exception.

(ii) Most candidates recognised the need to use $4y = 0$ but few realised the need to substitute this back into the equation of the curve in order to show contradiction.

(iii) Very few candidates recognised that the denominator should be 0. Of those that did, even fewer attempted to make a suitable substitution back into the equation of the curve. Complete and correct solutions were a rarity.

Answer: $(-3, -2)$

MATHEMATICS

Paper 9709/22
Paper 22

Key Messages

Centres should remind candidates of the importance of reading the instructions on the front of the question paper. Work from each candidate should be fastened securely and not left as separate loose sheets. Attention should again be drawn to the level of accuracy required for answers, especially where degrees and radians are concerned. Candidates should also be reminded to read each question carefully and ensure that they have answered fully and in the required form. Answers which required an answer in radians were often given in degrees. Candidates should also remember that any questions involving both trigonometry and calculus will always make use of angles in radians.

It was clearly evident that many candidates still do not appreciate the meaning of the word 'exact' in a mathematical context. Centres should endeavour to make sure that this is covered as a matter of urgency.

General Comments

Well prepared candidates were able to make good attempts at all the questions, showing their understanding of the topics covered in the syllabus.

Comments on Specific Questions

Question 1

(i) Most candidates realised that they needed to make use of logarithms and were able to obtain a correct answer to the required level of accuracy.

(ii) The question was often misunderstood by candidates with incorrect responses of $-21.6 < n < 21.6$ being all too common. Very few correct solutions were seen. Those candidates that did realise that they were required to find the number of integers in the above inequality all often forgot to count zero as an integer, thus obtaining an incorrect answer of 42.

Answer: **(i)** 21.6, **(ii)** 43

Question 2

(i) Most candidates realised that they nneded to make use of the factor theorem and correctly attempted to use a substitution of $x = -2$ into the given expression. There were problem with the manipulation of the term involving $-(a+1)x$ which often lead to an incorrect result.

(ii) For those candidates who had failed to obtain an incorrect value for a, credit was given for a correct method using their value of a. Some candidates failed to factorise fully with a response of $(x+2)(4x^2-9)$ being quite common. Other responses included $(x+2)\left(x+\frac{3}{2}\right)\left(x-\frac{3}{2}\right)$ which meant that candidates had made use of their calculator to solve the equation $4x^3+8x^2-9x-18=0$ and then work 'in reverse' from the solutions to get factors. Candidates should be reminded that the calculator function for solving polynomial equations is best used as a check of work. A correctly factorised response was required for full marks.

Answer: **(i)** 8, **(ii)** $(x+2)(2x-3)(2x+3)$

Question 3

(i) Most candidates were able to solve the given equation, making use of the correct trigonometric identity. The fact that the angle θ was given as an acute angle was usually disregarded with $\tan\theta = -4$ being included incorrectly.

(ii) Many candidates misunderstood what was required of them by failing to relate the angle θ in part **(i)** to the angle θ in $\tan(\theta + 135°)$. All that was required was a substitution into the appropriate double angle formula and manipulation to get an exact result. Many candidates attempted to use the correct double angle formula and then attempt to solve their expression equated to zero. It was also evident that some candidates are unsure of what an exact value is.

Answer: **(i)** $\frac{5}{2}$, **(ii)** $\frac{3}{7}$

Question 4

(i) A good response from most candidates who were able to differentiate correctly and equate their result to zero. There were occasions when some of the ensuing algebraic manipulation was incorrect as candidates strived to obtain the given answer.

(ii) It was pleasing to see many completely correct solutions, with no real problems. Having a given answer meant that candidates were able to identify errors in their working and rectify them in order to gain full marks.

Question 5

(i) Very few completely correct pairs of graphs were seen. Typical errors included drawing the line $y = 3x$, rather than $y = |3x|$, $y = 16 - x^4$ for $x \geq 0$ only and on occasions each graph on a separate set of coordinate axes. It was expected that candidates would make reference to the points of intersection of the 2 graphs or make some indication on their graph as the question asked them to show that the given equation had two real roots. This was often not done.

(ii) Most candidates were able to produce a correct set of iterations, making appropriate use of the 'answer' function on their calculator, going on to give the final answer to the required level of accuracy.

(iii) Again, many candidates failed to relate this part of the question to the preceding parts. The word 'Hence' was used in this part of the question as it was intended that candidates made use of their sketch in part **(i)** and their result in part **(ii)**. However many candidates failed to appreciate the meaning of the word 'Hence' and often an attempt to actually solve the equations by various methods followed. The *y* value that candidates obtained depended on which equation they used, there being a slight difference due to the nature of the question.

Answer: **(ii)** 1.804, **(iii)** $(1.804, 5.409)$, $(-1.804, 5.409)$

Question 6

(i) Most candidates were able to solve the given equation and reach the point $\sin x = -\frac{1}{2}$. However from this point on, many candidates were unable to solve this equation correctly. It was very common to see answers of 30°, -30° and 210°. Candidates should be aware that when using calculus, angles have to be in radians for the derivatives that are used to hold true. Correct response of $\frac{7\pi}{6}$ were rare.

(ii) Correct use of the appropriate double angle formula was common, although it did help candidates to have the given equation to work towards. Errors could be identified and corrected. Most were then able to integrate correctly but failed to gain full marks by using angles in degrees rather than radians when finding the definite integral required. Again it was evident that many candidates do not fully understand the meaning of the word 'exact' in a mathematical context.

Answer: **(i)** $\frac{7\pi}{6}$, **(ii)** $\frac{35\pi}{6} + \frac{7\sqrt{3}}{2} + 8$

Question 7

(a) Most candidates were able to make a good attempt at implicit differentiation. The term involving $\ln y$ seemed to cause more problems than the other terms. This appears to be a syllabus topic which most candidates are quite knowledgeable.

(b) A correct method of differentiation making use of the parametric equations was pleasingly common. There were occasional slips with algebraic simplification, but most candidates were able to reach a correct expression. Problems occurred if candidates failed to realise that they needed to find the numerical value of the parameter at the point in question and then they were unable to progress any further. Yet again, some candidates failed to appreciate the meaning of the word 'exact' in a mathematical context.

Answer: **(a)** $-\frac{9}{10}$, **(b)** $-\frac{5}{21}$

MATHEMATICS

Paper 9709/23
Paper 23

Key Messages

Centres should remind candidates of the importance of reading the instructions on the front of the question paper. Work from each candidate should be fastened securely and not left as separate loose sheets. Attention should again be drawn to the level of accuracy required for answers, especially where degrees and radians are concerned. Candidates should also be reminded to read each question carefully and ensure that they have answered fully and in the required form. Answers which required an answer in radians were often given in degrees. Candidates should also remember that any questions involving both trigonometry and calculus will always make use of angles in radians.

It was clearly evident that many candidates still do not appreciate the meaning of the word 'exact' in a mathematical context. Centres should endeavour to make sure that this is covered as a matter of urgency.

General Comments

Well prepared candidates were able to make good attempts at all the questions, showing their understanding of the topics covered in the syllabus.

Comments on Specific Questions

Question 1

(i) Most candidates realised that they needed to make use of logarithms and were able to obtain a correct answer to the required level of accuracy.

(ii) The question was often misunderstood by candidates with incorrect responses of $-21.6 < n < 21.6$ being all too common. Very few correct solutions were seen. Those candidates that did realise that they were required to find the number of integers in the above inequality all often forgot to count zero as an integer, thus obtaining an incorrect answer of 42.

Answer: **(i)** 21.6, **(ii)** 43

Question 2

(i) Most candidates realised that they nneded to make use of the factor theorem and correctly attempted to use a substitution of $x = -2$ into the given expression. There were problem with the manipulation of the term involving $-(a+1)x$ which often lead to an incorrect result.

(ii) For those candidates who had failed to obtain an incorrect value for a, credit was given for a correct method using their value of a. Some candidates failed to factorise fully with a response of $(x+2)(4x^2-9)$ being quite common. Other responses included $(x+2)\left(x+\frac{3}{2}\right)\left(x-\frac{3}{2}\right)$ which meant that candidates had made use of their calculator to solve the equation $4x^3+8x^2-9x-18=0$ and then work 'in reverse' from the solutions to get factors. Candidates should be reminded that the calculator function for solving polynomial equations is best used as a check of work. A correctly factorised response was required for full marks.

Answer: **(i)** 8, **(ii)** $(x+2)(2x-3)(2x+3)$

Question 3

(i) Most candidates were able to solve the given equation, making use of the correct trigonometric identity. The fact that the angle θ was given as an acute angle was usually disregarded with $\tan\theta = -4$ being included incorrectly.

(ii) Many candidates misunderstood what was required of them by failing to relate the angle θ in part **(i)** to the angle θ in $\tan(\theta + 135°)$. All that was required was a substitution into the appropriate double angle formula and manipulation to get an exact result. Many candidates attempted to use the correct double angle formula and then attempt to solve their expression equated to zero. It was also evident that some candidates are unsure of what an exact value is.

Answer: **(i)** $\frac{5}{2}$, **(ii)** $\frac{3}{7}$

Question 4

(i) A good response from most candidates who were able to differentiate correctly and equate their result to zero. There were occasions when some of the ensuing algebraic manipulation was incorrect as candidates strived to obtain the given answer.

(ii) It was pleasing to see many completely correct solutions, with no real problems. Having a given answer meant that candidates were able to identify errors in their working and rectify them in order to gain full marks.

Question 5

(i) Very few completely correct pairs of graphs were seen. Typical errors included drawing the line $y = 3x$, rather than $y = |3x|$, $y = 16 - x^4$ for $x \geq 0$ only and on occasions each graph on a separate set of coordinate axes. It was expected that candidates would make reference to the points of intersection of the 2 graphs or make some indication on their graph as the question asked them to show that the given equation had two real roots. This was often not done.

(ii) Most candidates were able to produce a correct set of iterations, making appropriate use of the 'answer' function on their calculator, going on to give the final answer to the required level of accuracy.

(iii) Again, many candidates failed to relate this part of the question to the preceding parts. The word 'Hence' was used in this part of the question as it was intended that candidates made use of their sketch in part **(i)** and their result in part **(ii)**. However many candidates failed to appreciate the meaning of the word 'Hence' and often an attempt to actually solve the equations by various methods followed. The *y* value that candidates obtained depended on which equation they used, there being a slight difference due to the nature of the question.

Answer: **(ii)** 1.804, **(iii)** $(1.804, 5.409)$, $(-1.804, 5.409)$

Question 6

(i) Most candidates were able to solve the given equation and reach the point $\sin x = -\frac{1}{2}$. However from this point on, many candidates were unable to solve this equation correctly. It was very common to see answers of 30^o, -30^o and 210^o. Candidates should be aware that when using calculus, angles have to be in radians for the derivatives that are used to hold true. Correct response of $\frac{7\pi}{6}$ were rare.

(ii) Correct use of the appropriate double angle formula was common, although it did help candidates to have the given equation to work towards. Errors could be identified and corrected. Most were then able to integrate correctly but failed to gain full marks by using angles in degrees rather than radians when finding the definite integral required. Again it was evident that many candidates do not fully understand the meaning of the word 'exact' in a mathematical context.

Answer: **(i)** $\frac{7\pi}{6}$, **(ii)** $\frac{35\pi}{6} + \frac{7\sqrt{3}}{2} + 8$

Question 7

(a) Most candidates were able to make a good attempt at implicit differentiation. The term involving $\ln y$ seemed to cause more problems than the other terms. This appears to be a syllabus topic which most candidates are quite knowledgeable.

(b) A correct method of differentiation making use of the parametric equations was pleasingly common. There were occasional slips with algebraic simplification, but most candidates were able to reach a correct expression. Problems occurred if candidates failed to realise that they needed to find the numerical value of the parameter at the point in question and then they were unable to progress any further. Yet again, some candidates failed to appreciate the meaning of the word 'exact' in a mathematical context.

Answer: **(a)** $-\frac{9}{10}$, **(b)** $-\frac{5}{21}$

MATHEMATICS

Paper 9709/31
Paper 31

General Comments

The standard of work on this paper varied considerably. The very able candidates were able to make reasonable attempts at all the questions except for **Question 8(ii)** and **(iii)**. The questions that these candidates found relatively easy were **Question 1**, **Question 3**, **Question 4**, **Question 6**, **Question 8(i)**, **Question 9(i)** and **Question 10(i)**. Those questions that they found difficult were **Question 2**, **Question 5**, **Question 7**, **Question 8(ii)** and **(iii)**, **Question 9(ii)** and **Question 10(ii)**, both **(a)** and **(b)**.

It is recommended that candidates taking this paper should have attempted several of the past papers before they take this examination.

In the 2015 report for this paper it was stressed that 'when attempting a question candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answer, whether they are working towards a given answer or an answer that is not given.' The more able candidates appeared to have responded positively to this message.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

Candidates usually could apply the law for the logarithm of a power correctly, however then most failed to apply brackets to the term $2x + 1$. Whilst logarithms to any base are acceptable, the following algebra is not. Instead of solving the resulting linear equation in x by combining 5ln2 with − 2ln3, or their numerical equivalents, many candidates opted to divide through by $5x$ then invert each of the 3 terms individually in order to obtain x back in the numerator.

Answer: 0.866

Question 2

Many candidates did not understand the meaning of the function 'modulus' and it was very common to see ordinates with the values –9 and –7. Whilst the step size of the interval and the general formula were usually quoted correctly, the substitution of the ordinates into their formula proved troublesome.

Answer: 21

Question 3

Many candidates decided to express the first term in the expression as $\dfrac{1}{(1-2x^2)^2}$ and to then expand this denominator, hence making no progress. The positive index of the second term avoided this problem; however it was still very common to see the omission of negative signs and x^2, or even x, instead of $6x^2$ raised to a power.

Answer: 16

Question 4

The usual errors in this question occurred in the differentiation of the term 3cos2x. Common to see this as 3sin2x, 3sin$x$ or 6sin$x$. Hence only with the first of these expressions was any further progress really possible, and even then none of the final 3 accuracy marks were available.

Answers: 0.623, 2.52, 1.57 ($\frac{1}{2}\pi$ allowed)

Question 5

(i) Few candidates realised that $\tan^2 2x$ needed to be replaced by $\sec^2 2x - 1$ to make any progress, and even if they did this was often incorrectly integrated as $\frac{1}{2}\tan x$.

(ii) Many candidates realised that it was necessary to expand $\sin(x + \frac{1}{6}\pi)$. However, this was rarely followed with the division by sin$x$ to leave the integral of a constant term and a multiple of cot$x$.

Answers: **(i)** $3x + \frac{1}{2}\tan 2x + c$ **(ii)** $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$

Question 6

(i) The direction vector was usually obtained correctly. However, few candidates realised it was necessary to show that the two directional vectors were not parallel. The straight line l_1 was usually obtained correctly but when solving to find whether there was a point of intersection, arithmetical errors were common.

(ii) Few candidates knew the vector representing the x-axis. Some were close, but had (0, b, 0) or (0, 0, c).

Answer: **(ii)** 1.39 radians or 79.5°

Question 7

The most able candidates often produced a completely correct solution, unfortunately some of the candidates failed to realise that the use of partial fractions was necessary.

Answer: $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$

Question 8

(i) Many correct solutions, although some candidates had to use the given answer to work backwards to 22 + 4i. Whilst this is acceptable it is not to be encouraged since this approach is only possible if the answer is given.

(ii) The values $p = -6$ or 2 were rare.

(iii) Despite producing an appropriate sketch only the odd candidate realised that the centre of the circle on the x-axis and the fact that the circle passed through the origin meant that the radius of the circle must equal the x-coordinate of the centre. Hence this single parameter, (the x-coordinate of centre of circle), could then easily be determined using the fact that the circle also passes through point S represented by the complex number w.

Answers: **(ii)** $-6 \le p \le 2$ **(iii)** $|z - 5| = 5$

Question 9

(i) Many candidates were usually able to show this result, but like **Question 8(i)** they often used the given answer. Again not a procedure to be encouraged for the reason given in **Question 8(i)**. A few candidates integrated instead of differentiating.

(ii) Here again many knew what to do, but instead of writing down the standard integration by parts, followed by another standard integration by parts, they produced a table of two columns and three rows. Various derivatives and integrals were displayed, with arrows linking one element to another. Whilst such memory methods may help the candidate they don't clearly display the real operations required for the examiner to mark. Fortunately in this case there was a clear pattern to the answer so by allowing the method marks to be awarded when there were sign errors in the various terms the issue was successfully resolved. However, again not a procedure to encouraged, since many sign errors were prevalent throughout. In addition with a more difficult example, following exactly what the candidate is doing may not be so easy.

Answer: **(ii)** $2e^2 - 10$

Question 10

(i) Many candidates made several basic errors in this part. Such as omitting the factor 2 from the x-derivative with respect to t, or assuming that the value of t at the origin was zero, instead of solving $x(t) = 0$ or $y(t) = 0$.

(ii) (a) Without part **(i)** correct, obtaining the answer given in this part was not possible. When candidates were successful in **(i)** they usually obtained the given answer in **(ii)**.

(b) This was a different type of numerical iterative question from that normally set. Most candidates knew the general procedure but failed to realise what was required. The question clearly asks for the coordinates correct to two decimal places. Unfortunately, using a value of p correct to two decimal places does not necessarily produce two decimal place accuracy in any function of p, namely the coordinates here. Hence although candidates had their values of p correct to 4 decimal places they rounded this to two decimal places, and then used this result to find the coordinates. Using p =1.92 fails to produce the required accuracy in the coordinates, whilst using p = 1.924 does so.

Answer: **(i)** $\frac{5}{2}$ **(ii)(b)** $(-5.15, -7.97)$

MATHEMATICS

Paper 9709/32
Paper 32

General Comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. All questions discriminated successfully and yet were accessible to well-prepared candidates. The questions or parts of questions that were generally done well were **Question 3** (differentiation), **Question 4** (trigonometry), **Question 5** (iteration) and **Question 8 (i)** (partial fractions). Those that were done least well were **Question 1** (trapezium rule), **Question 6** (integration), **Question 7** (complex numbers) and **Question 9** (differential equation).

In general the presentation of work was good, though there were some unhelpfully untidy scripts. In each of the first five questions there are requirements as to the accuracy to which answers are to be given. In **Question 5 (iii)** nearly all candidates fulfilled the two requirements about accuracy. In the other questions it was quite common for candidates to fail to give answers to the required accuracy. For example in **Question 2**, where the answer 0.0466 to 3 significant figures was wanted, candidates often spoiled a correct solution by giving the answer as 0.05 or 0.047.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only ‘correct answer’.

Comments on Specific Questions

Question 1

In general this was poorly answered. Some candidates appeared to be unfamiliar with the trapezium rule. Common errors seen in the attempts to apply the trapezium rule included the use of **(a)** abscissae instead of ordinates, **(b)** incorrect weightings of the ordinates, **(c)** 3 or 5 ordinates instead of 4, **(d)** an incorrect value for the interval width, and **(e)** ordinates calculated at unequal intervals. In this case the fact that the value of the first ordinate y_0 was zero, seemed to cause some candidates to work with $y_1 + 2y_2 + y_3$ rather than $y_0 + 2(y_1 + y_2) + y_3$ when applying the rule.

Answer: 0.72

Question 2

This was moderately well answered. Replacing 4^{x+2} by u^2 rather than 4^2u^2 was a common error when making the initial substitution $u = 4^x$. Some solutions stopped when u had been found. Those that went on did not always present their answer to the correct degree of accuracy.

Answer: 0.0466

Question 3

This was done well in general by a variety of methods. The most popular approach began by differentiating y by the product rule, equating the answer to zero, and then obtaining an equation in one trigonometric function, e.g. $6\cos^2 x = 1$. Another method began by expressing y terms of $\cos x$ before differentiating.

Answer: 1.15

Question 4

Part **(i)** was generally well answered. The two commonest errors were **(a)** failure to give the *exact* value of R, usually because they used their inexact value for α to calculate R, and **(b)** stating angle α correct to one decimal place instead of two.

Part **(ii)** proved quite discriminating as candidates had to devise a method for obtaining an answer in the given interval. Following correct work in part **(i)**, taking $\sin^{-1}(1/\sqrt{13})$ to be 16.10° led to a negative root – 17.59°. Faced with this, some candidates added 180°, some changed the sign of this root and others changed the function to $\sin(\theta - 33.69)$. Nevertheless a substantial number used the supplement of 16.10° and obtained the correct answer.

Answer: **(i)** 33.69° **(ii)** 130.2°

Question 5

Part **(i)** was well answered by those who did not omit it altogether. A few used the area formula rather than the arc formula. Part **(ii)** was generally done well. Most candidates looked for a sign change in a suitable function or in the difference of two appropriate expressions. A minority used the unsuitable function $\pi - \tan x$. There were many correct answers to part **(iii)** and only a few candidates failed to have their calculators in radian mode.

Answer: 1.11

Question 6

(i) Fully correct solutions were not common. Often a solution began with a correct relation between differentials, such as $\mathrm{d}u = -\frac{1}{2\sqrt{x}}\mathrm{d}x$, or $\mathrm{d}x = -2(2-u)\mathrm{d}u$, but in the following work the minus sign was omitted or erased for no good reason. Some candidates did explain that reversing limits changes the sign of an integral or incorporated that step in their work, but the impression gained was that candidates were generally unaware of the principle and, to reach the given answer, had to discard the minus.

(ii) Those who divided and converted the integrand to $2\left(\frac{4}{u} - 4 + u\right)$, or equivalent, were usually successful. A surprising number did not think to divide and embarked on the hazardous task of integrating by parts. Only a few could keep control of the algebra and integration involved in that approach.

Question 7

(i) This part was only fairly well answered. Many compared real and imaginary parts correctly, eliminated an unknown and reached an equation in a or b, but some made slips on the way. Those that solved correctly for a or for b surprisingly often failed to present correct roots. For example, having found $a = \pm\sqrt{3}$ the roots were sometimes stated to be $\sqrt{3} + 2\mathrm{i}$ and $\sqrt{3} - 2\mathrm{i}$. The use of a calculator was expressly disallowed in this part of the question so those that used calculators scored nothing.

(ii) There were some good solutions to this part but also many poor ones. Candidates usually sketched an incorrect circle. Common errors were **(a)** the failure to identify the Centre correctly, e.g. placing it at the point representing $1 + 4\sqrt{3}\mathrm{i}$, **(b)** giving the circle a radius of 1 incompatible with the scale on the axes, and **(c)** presenting two circles. Candidates need to be reminded that in an Argand diagram the scales on the axes should be the same. To complete the question a little geometry was needed to identify the precise point on the circle where the oblique tangent from the origin touches the circle. Then correct trigonometry was needed to calculate the components of the angle in question. Only a few succeeded to meet both these challenges.

Answers: **(i)** $\pm(\sqrt{3} + 2\mathrm{i})$; **(ii)** 1.86

Question 8

Part **(i)** was very well answered. Almost all stated a correct form of partial fractions such as $\dfrac{A}{3-2x}+\dfrac{Bx+C}{x^2+4}$ and most solutions were either fully correct or evaluated all but one constant correctly. In part **(ii)** most had a good idea of the overall method but the fact that $B = -1$ and $C = -2$ caused problems for some. For example, they took the numerator of the second partial fraction to be $x - 2$. In forming the expansions, errors such as taking $(3-2x)^{-1}$ to be equivalent to $3(1-\frac{2}{3}x)^{-1}$ or $(x^2+4)^{-1}$ to be equivalent to $4(1+\frac{1}{4}x^2)^{-1}$ were quite frequently encountered.

Answers: **(i)** $\dfrac{3}{3-2x}-\dfrac{x+2}{x^2+4}$; **(ii)** $\dfrac{1}{2}+\dfrac{5}{12}x+\dfrac{41}{72}x^2$

Question 9

Parts **(i)** and **(ii)** were correctly done by a fair number of candidates. But a large number of attempts failed to make a sound start to the integration and scored very little. There were a few sound attempts at part **(iii)**. The best showed that since $e^{-t} \to 0$ the limiting value of x is 47.8 and this is less than 48.

Answer: **(i)** $\ln x - \ln 10 = -\ln(k + e^{-t}) + \ln(k+1)$

Question 10

(i) Most candidates obtained a correct equation for the line *AB* and equated its components to those of the given line *l*. The ensuing work was often well done and only marred by arithmetic or algebraic slips. The equation of *l* involved the parameter μ. There were a few candidates who made the error of using the same parameter in their equation for *AB*. Those who just found the vector $\overrightarrow{AB}$ and then equated its components to those of *l* scored nothing for this part.

(ii) The majority tried to use a vector product to find a normal to the required plane. A suitable product will contain two vectors parallel to the plane. Most products used the direction vector of *l*, $3\mathbf{i} + \mathbf{j} - \mathbf{k}$. For a satisfactory product candidates then needed to find and use a vector such as $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ which is parallel to the plane and not parallel to *l*. Such vectors can be found by subtracting the position vectors of *A* and any point on *l*. But the majority of attempts used unsatisfactory second vectors such as $\overrightarrow{OA}$ or $\overrightarrow{AB}$. The same comment applies to those who used scalar products equated to zero. Thus the equation $3a + b - c = 0$ was often seen, but only a minority accompanied this with a second relevant equation such as $a - 2b + c = 0$ or $2a + 3b - 2c = 0$. Very few made a simple sketch showing the line *l* and the point *A* lying in a plane. Such a sketch might have helped candidates to find a pair of vectors perpendicular to the normal of the plane.

Answer: **(ii)** $x + 4y + 7z = 19$

MATHEMATICS

Paper 9709/33
Paper 33

General Comments

In general the presentation of the work was good and most candidates attempted all the questions. When attempting a question it is essential that sufficient working is shown to indicate how they arrive at their answer, whether they are working towards a given answer or an answer that is not given. For example **Question 10(ii)**, clearly showing the substitution of their limits, or **Question 3**, clearly displaying the substitution of their values in the quadratic equation solution formula. However, some candidates believed **incorrectly** that showing their working meant solving the quadratic, or even cubic, equation on their calculator and then using these values to construct the factors of the equation being solved.

Candidates are expected to use only an electronic calculator and full method must always be shown.

Two final general points:

- Candidates should not use a column system for submitting their solutions, (basically dividing the page into two). Not only is it difficult for an examiner to mark such scripts, but there is also the possibility that something will be missed in the marking process.

- Centres that provide candidates with more individual sheets of paper than they need. Then all the excess blank sheets are submitted within the candidate's script. Whilst this is often unavoidable when using booklets, it should not happen when using individual sheets of paper.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions.

Question 1

The solution to this question required, (i) application of the laws of logarithms for product/quotient and power, (ii) reduction of a logarithmic equation to an equation free of logarithms by taking the inverse, (iii) solution of a three term quadratic to obtain two roots then (iv) the selection of only the positive root to three significant figures, based on knowing that the logarithmic function is undefined for negative values

Answer: $x = 1.13$

Question 2

This question required the solution of a modular inequality. The candidate needed to (i) remove the modulus, (ii) obtain a critical value, here $\frac{5}{3}$, and (iii) interpret the value to obtain the appropriate solution. Those candidates who first sketched the graphs almost always scored full marks. However, the majority of candidates adopted a purely algebraic approach either by squaring both sides of the inequality or by solving two linear inequalities. Only a minority of candidates who approached the problem in this purely algebraic fashion were able to obtain a complete solution. The method of squaring both sides of the inequality to remove the modulus sign is not the best approach for this type of inequality since it introduces the additional critical point $x = 1$ and leaves the candidate with the problem of interpreting the region in which the original modular inequality is satisfied.

Answer: $nx < \frac{5}{3}$

Question 3

The most straightforward approach was to express cot 2x and cot x in terms of tan 2x and tan x, respectively, then use the double angle formula for tan 2x to form an equation in tan x. The resulting quadratic equation in tan x then led to the required two solutions. The candidates who opted to try to get everything in terms of sin$x$ and cos$x$ nearly always quit this question before they reached a position that could lead to any equation that they could solve. Unfortunately, too many candidates, having solved correctly the quadratic equation in tan x, were unable to convert these values into the correct x values, with 81.2° appearing far too often.

Answers: 24.9° and 98.8°

Question 4

This question required the use of the product or quotient rule for differentiation, with the resulting derivative then set to zero. A little cancellation and the solution of the equation $ae^{3x} = b$, or similar such equation, produced the required x coordinate. With the y coordinate following via the substitution of the x coordinate in the given equation. Using the quotient rule usually proved more successful than using the product rule since $(4 + e^{3x})$ to a negative power proved both difficult to differentiate and to handle algebraically. Several candidates opted to multiply the equation up before commencing their differentiation and this was usually undertaken correctly. In fact it is probably the simplest approach of the various methods available. However, the usual errors were present, such as omission of denominator in quotient rule, positive sign in quotient rule formula, reverse signs in numerator of quotient rule formula. Too many candidates had the error $e^{2x}.e^{3x} = e^{6x^2}$. Common errors at the end were to see no y coordinate, coordinates as decimals, or even trying to turn decimals into exact values.

Answer: $(\ln 2, \frac{1}{3})$

Question 5

(i) Common to see parameter a lost in the differentiation.

(ii) Some candidates opted to look at the answer given and try to manipulate it, instead of forgetting about the answer given and using their standard method to establish the equation of the tangent with help from their $\frac{dy}{dx}$ found in **(i)**. Having established their tangential equation it is at that stage that candidates should look at the given answer and provide all the detail required to take their result into the given answer form. The given answer was only there so that candidates could find *OP* and *OQ* correctly should their **(i)** or **(ii)** be incorrect.

(iii) Candidates were expected to find from **(ii)** the x and y coordinates for *OP* and *OQ*, respectively, then add these together, and with a little simplification acquire the given answer

Answers: **(i)** $-\tan^2 t$

Question 6

(i) This proved to be a high scoring section, although a few candidates muddled their integration by parts, or failed to show the detail required when substituting the limits to establish the given answer.

(ii) Only the odd candidate realised that '*a*', the solution of the equation in **(i)**, was between unity and another relevant value between unity and $\frac{\pi}{2}$, and all that was required was to show a sign change in the appropriate function between these two values.

(iii) Most candidates had no trouble with this section, although the odd candidate did unfortunately have their calculator set in degree mode.

Answers: **(iii)** 1.2461

Question 7

(i) Most candidates separated correctly, but then failed to integrate either $M^{-\frac{1}{2}}$ or cos(0.02*t*) successfully. $M^{\frac{3}{2}}$, 0.02 sin(0.02*t*) and sin(*t*) were all commonly seen incorrect expressions. However, although candidates introduced a constant of integration, they were unable to evaluate it correctly due to their earlier integration error(s).

(ii) Again candidates knew that having found their constant of integration they could determine the value of *k* by introducing the additional information that $M = 196$ when $t = 50$. Obviously earlier integration errors, plus arithmetical and algebraic errors for those candidates who had successfully integrated, meant that few candidates obtained the correct value of *k*. Like **Question 6** some candidates still had their calculator in the wrong mode.

(iii) The errors arising in **(i)** and **(ii)** meant that few candidates were in a position to obtain the least possible number of micro-organisms. The easiest way was to set sin(0.02*t*) to –1 in the expression for *M*, the latter having been obtained by squaring $M^{\frac{1}{2}}$. Some candidates decided to expand the RHS of *M* by incorrectly squaring term by term. However, a few candidates were successful using calculus to find the minimum point of either $M^{\frac{1}{2}}$ or M.

Answers: **(i)** $2\sqrt{M} = 50k\sin(0.02t) + 20$ **(ii)** 0.19 **(iii)** 27.6 or 28

Question 8

(i) Some candidates genuinely misread $\frac{\mathrm{i}}{u}$ as $\frac{1}{u}$, however many others started with the correct expression and then miscopied this so that it became $\frac{1}{u}$. However, there were many correct answers to this section.

(ii) Unfortunately, again many candidates were muddled and used their answer from **(i)** as *u*, instead of its given value. This had consequences as it meant **(iii)** became impossible, because it was expected candidates would use their diagram from **(ii)** to determine the arguments in **(iii)**. In addition, too many candidates had the centre of their circle of radius 2 at $z = -\mathrm{i}$ instead of at $z = \mathrm{i}$.

(iii) Only with a correct diagram in **(ii)** were marks in **(iii)** easily obtained. However, there was the odd candidate who correctly solved the modular equations algebraically, although the intention of the question was that the solutions in **(iii)** would follow directly from the sketches in **(ii)**.

Answers: **(i)** $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$ **(iii)** $-\frac{1}{2}\pi$ (270°) and 0.464 radians (26.6°)

Question 9

(i) Most candidates knew how to identify the normal vectors of the two planes and find the required scalar product. A small number of candidates wrongly divided by $2\sqrt{14}$ in their attempt to find the cosine of the required angle, and there were also a few candidates who wrongly used sine of the required angle in their calculation. The most common error was in not stating the correct acute angle 85.9° and the answers 94.1° and 4.1° were quite frequently seen. Unfortunately, when performing the calculation in radians, it was not always realised that 1.64 is an obtuse angle.

(ii) The vast majority of candidates were able to find a point on the required line by setting one of *x*, *y* or *z* to a specific value. There were two methods of finding the direction vector of the line which were most commonly seen. The first of these involved calculating the vector product of the two normal vectors to the planes, which was usually well done except for a few numerical errors. The second method which was almost as popular, was to obtain two points on the line and then to find the vector joining them. Other methods were not as successfully executed

Answers: **(i)** 85.9° (1.50) **(ii)** $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$

Question 10

(i) The most commonly used form of partial fractions was $\frac{A}{2x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$. Those candidates who used the specific values $x = \frac{1}{2}$ and $x = -2$ were able to find two of the constants immediately and it was only necessary to use a third value, often chosen as $x = 0$, to find the remaining constant. Some candidates however, chose to use a much longer method of finding the third constant. This involved equating the partial fraction with the unknown constant to the other two partial fractions subtracted from f(x) and comparing the two sides of this equation. The other commonly seen method was to compare powers of x and obtain three linear equations in A, B and C, which they could then solve, often using their calculator. The other valid form of partial fractions $\frac{A}{2x-1}+\frac{Dx+E}{(x+2)^2}$ was not as frequently seen but as in the previous case the values of the constants were usually correctly calculated. Full marks were often gained on this section.

(ii) When the form $\frac{A}{2x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ was chosen, this section was usually successfully attempted and totally correct solutions were often seen. The most frequent errors which occurred were in obtaining the integral of $\frac{2}{2x-1}$ as $2\ln(2x-1)$ and in wrongly integrating the term $\frac{C}{(x+2)^2}$ to obtain a ln function.

Answers: **(i)** $\frac{2}{2x-1}+\frac{-1}{x+2}+\frac{3}{(x+2)^2}$

MATHEMATICS

Paper 9709/41
Paper 41

General Comments

There were some excellent candidates who produced very good answers on this paper. However in some cases the layout of the workings could be better.

Some candidates lost marks due to not giving answers to 3sf as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3sf then their working should be performed to at least 4sf. In questions 2, 3 and 5 either the sine or tangent of an angle was given in the question. In these questions it was not necessary to determine the actual angle to 1 dp as this often lead to premature approximation and frequently also to loss of accuracy marks.

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used. This is the case here with the given answer in **Question 1(i)**.

Comments on Specific Questions

Question 1

(i) Candidates usually applied Newton's second law to the system in the vertical direction but often did not lay out their solution clearly. When an answer is given as in this question it is vital to show all working clearly.

(ii) Most candidates who attempted part **(i)** attempted to use the formula for the work done by a force F which moves a distance d as WD = $Fd \cos \theta$ where θ is the angle between the direction of the force and the direction of movement. Some forgot to include the $\cos \theta$ term but most candidates who attempted this question achieved the correct answer.

Answer: **(ii)** Work done by the pulling force is 120 J

Question 2

Most candidates who attempted this question correctly resolved forces vertically and horizontally. In many cases the layout of the answers could be improved. Candidates would be advised to clearly describe their approach to the problem at each stage of their working, such as stating that they are resolving forces vertically or horizontally. Most were able to find the values of $F \sin \theta$ and $F \cos \theta$. Some left this as their answer but most went on to attempt to solve these simultaneous equations. A good number produced the correct answer but many lost a mark because of determining the angle itself and losing accuracy rather than using the sine and cosine values from the angle that was given in the question.

Answer: F = 52 N or $\tan \theta = 2.4$

Question 3

This question proved to be tricky for lots of candidates as the weight of the particle was given in newtons rather than its mass in kg. This caused dimensional problems when writing down Newton's second law. For those who used the Newton approach there were two forces acting, namely the component of the weight down the plane and the frictional force. It is worth pointing out that in cases such as this the method mark is only awarded if the equation is dimensionally correct and often this mark was lost by candidates who equated the forces acting to $6.1a$ rather than to $0.61a$. Some good answers were seen using this method but again some lost marks due to finding the angle rather than using the given value of the tangent and corresponding exact values of sine and cosine. This approach gave a value for the acceleration ($a = 40/61$) and the solution is completed by using $v^2 = u^2 + 2as$ in order to find the required distance.

An alternative approach is to use the work/energy equation. Candidates who followed this method found the KE loss and the PE loss as the block moved through a distance x down the plane. The solution to the problem was found by using the work/energy equation as WD against Resistance = PE loss + KE loss. Some candidates did this correctly and others made sign errors in this equation.

Answer: Distance moved by the block from *A* before it comes to rest is 3.05 m

Question 4

(i) Most candidates attempted this question by finding the kinetic energy gain and the potential energy lost from the basic definitions and most were successful in spite of some very large numbers being involved.

(ii) In this part of the question it was necessary to use the work/energy equation since only the total work done by the driving force was given which did not imply a constant driving force throughout the motion. Some candidates found this to be a tricky question but several produced very good answers by using the equation WD by driving force + PE loss = KE gain + WD against resistance

Again some sign errors were seen which inevitably lead to an incorrect solution.

Some candidates did attempt to solve this with the assumption that the driving force is constant but very few took this to a conclusion as it became quite complex and was in fact not a correct method for this problem.

Answer: **(i)** Gain in Kinetic Energy = 4032000 J = 4032 kJ
Loss of Potential Energy = 42000000 sin θ J = 42000 sin θ kJ
(ii) $\theta = 3.3$

Question 5

Many candidates found this question to be particularly difficult. Candidates needed to use the fact that the driving force is given by P/v and then to use this in setting up the equations of motion both uphill and downhill. Some candidates tried to use constant acceleration formulae here probably because both accelerations up and down were the same but in fact the values given were instantaneous values. In both the uphill and downhill cases there were three forces acting, the driving force, P/v, the component of the weight down the plane and the resistance force, R. Newton's second law has to be applied in both cases with the weight component being against the uphill motion but in the same direction as the driving force for the downhill motion. Some good solutions were seen but many failed to include all of the forces, in particular forgetting to include the weight component. Once the equations had been set up the resulting simultaneous equations had to be solved.

Answer: $P = 720$ $R = 51$

Question 6

(i) Although the acceleration was given in the question as a function of t, many candidates wrongly assumed that they could use constant acceleration formulae to determine the required velocities for A. Some good answers were seen where candidates integrated the given expression for a to determine the velocity v as a function of t and substituted the given values of t to determine the required velocities at $t = 200$ and $t = 500$.

(ii) Most candidates who found the correct values in part **(i)** went on to complete this part of the question. Many good and complete answers were seen. It required a further integration to determine the displacement of A. For the displacement of B either the use of constant acceleration formulae or the evaluation of the area under a v-t graph was required. Finally these values had to be subtracted to determine the distance between A and B. If candidates failed to find the correct answer in part **(i)** then often their answers made it almost impossible to make any significant progress in part **(ii)**

Answer: **(i)** Velocity of A when t =200 is 6 ms^{-1} Velocity of A when t = 500 is 0 ms^{-1}
(ii) Distance between A and B when t = 500 is 2083 m −1500 m = 583 m

Question 7

(i) The nature of this question meant that the first task was to find the acceleration of the two particles over the first 0.25 seconds by using Newton's second law applied to the particles, horizontally for A and vertically for B. Many wrote down the correct equations although some also wrongly included the weight in their horizontal equation for particle A. Before the question could be answered it was necessary to find the velocity of the particles at time $t = 0.25$ ($v = 1.6$ms^{-1}) using $v = u + at$ with the value of a that has been found. It was also necessary to find the distance travelled by the particles in this time ($s = 0.2$ m) using one of the constant acceleration equations for s. Many tried to find the velocity of B at the floor without going through this stage which is not possible.

(ii) This part was not generally well done. Candidates needed to consider particle A after the break when the only horizontal force acting on it was friction and this lead to there being a retardation of 2 ms^{-2} and this had to be used in the equation $v^2 = u^2 + 2as$ along with $u = 1.6$ and $v = 0$ and the equation solved for s, the distance travelled by A after the break. Finally the initial 0.2 m which A had travelled before the break has to be added to this value. Only a few complete answers were seen for this part. There is an alternative energy method which could be applied but this was not seen.

Answer: **(i)** Speed of B immediately before it hits the floor = 2.93 ms^{-1}
(ii) Total distance travelled by A is 0.84 m

MATHEMATICS

Paper 9709/42
Paper 42

General Comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Candidates should be advised not to divide their written answer page vertically as this often makes it much more difficult to follow the candidate's work and their arguments.

Some candidates lost marks due to not giving answers to 3sf as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in questions 2, 3 and 7. Candidates should be reminded that if an answer is required to 3sf then their working should be performed to at least 4sf.

In questions 2 and 3 the sine of a required angle was given and in **Question 7** a **3-4-5** right-angled triangle was shown in the diagram. In all cases it was not necessary to calculate the angle itself as the sines and cosines required could be evaluated exactly. However, many candidates often proceeded to find the relevant angles to 1 decimal place and immediately lost accuracy and in some cases marks.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases such as **Question 7 (ii)** it is impossible to achieve the correct given answer unless this value is used.

Comments on Specific Questions

Question 1

(i) Most candidates attempted to use the constant acceleration equations in some form to determine the distance travelled. Some lost marks by misquoting the formulae or not substituting the given values correctly. Overall this question was well done by the majority of candidates.

(ii) This part of the question involved the use of the definition of work done when a force, F, moves through a distance d as Work Done = $Fd \cos \theta$ where θ is the angle between the applied force and the direction of motion. Some candidates forgot to include the $\cos \theta$ term. Again overall this part was well done.

Answer: **(i)** Distance travelled = 7.75 m **(ii)** Work done by the tension in the string = 31 J

Question 2

(i) Almost all candidates performed well on this part of the question, using the definition that the driving force can be expressed as $\frac{P}{v}$ where P is the power and v is the speed and then applying Newton's second law to the system.

(ii) A good number of candidates scored well on this part of the question. A common mistake was to forget to include the effect of the component of the weight down the plane acting against the motion of the cyclist. Another error which was made was to include the weight term but with the mass used rather than the weight. Note that in all questions such as this the method mark is only scored for a dimensionally correct equation.

Answer: **(i)** $P = 480$ **(ii)** The acceleration of the cyclist is $\frac{97}{80} = 1.21\ \text{ms}^{-2}$

Question 3

(i) The definition of kinetic energy as $\frac{1}{2}mv^2$ was required as the object moves from being at rest to a speed of 4.5 ms^{-1}. The potential energy loss involved evaluating the fall in height, h, of the object using the given value of the angle as $\sin^{-1}(1/8)$ and then using PE = mgh. Some candidates evaluated both expressions correctly but then lost marks by saying that the loss in PE was the difference between PE and KE.

(ii) The most straightforward method of approaching this part was to use the information found in part **(i)**. The work done against the constant resisting force can be linked to the gain in KE and the loss in PE by using the formula: WD against resistance = PE loss – KE gain. Many used this correctly and then subsequently divided their WD by 12 to obtain the resisting force. A common error was to use the formula but with incorrect signs. Another error that was seen was when candidates equated the resistance force itself to the difference between PE and KE, which is a dimensionally incorrect statement.

An alternative but equally good method was to determine the acceleration of the object using the constant acceleration equations, usually $v^2 = u^2 + 2as$, and then apply Newton's second law to the object where the forces acting are the component of the weight and the resisting force. Many scored well with this method but often found the acceleration to 2sf and lost marks due to this premature approximation.

Answer: **(i)** Increase in kinetic energy of P = 81 J Decrease in potential energy of P = 120 J
(ii) The magnitude of the constant resisting force is 3.25 N

Question 4

(i) Most candidates integrated correctly to find the velocity, often retaining the + c in their answer even though it was given that the particle starts from rest at O. Again most successfully integrated their expression for velocity to determine displacement but often left + c or even +ct + d in addition to the correct expression and hence did not find the correct displacement. Overall most candidates performed well on this part. It was good to see that very few candidates wrongly attempted to use the constant acceleration equations.

(ii) In this part it was necessary to set the value of the displacement to zero to determine when the particle returns to O. The majority of candidates attempted this. Some candidates used velocity set to zero and even though it was possible in this particular case to obtain the correct answer by this method it was still necessary in the end to check that the displacement was also zero. Some used acceleration = 0 which is incorrect. A significant number of candidates scored well here.

Answer: **(i)** The displacement of P from O is = $0.00625t^4 - 0.25t^3 + 2.5t^2$
or equivalent forms such as displacement of P from O = $\frac{1}{160}t^4 - \frac{1}{4}t^3 + \frac{5}{2}t^2$
(ii) Time taken for P to return to the point O is 20 s

Question 5

(i) Many candidates scored very well on this question. The error which occurred most frequently was that in the attempt to find the time of flight of particle Q using the constant acceleration equations with $a = -g$ candidates often only considered the upward motion but should have doubled their answer to find the total time before Q returned to its original position. Once the time of flight was obtained then the acceleration of particle P could be found directly by using the equation $v = u + at$.

Some approached the problem by finding that P travelled for a time $\frac{30}{a}$, where a is the required acceleration of P and then used this in the equation of motion for Q. Either method was perfectly sound.

(ii) Once the acceleration had been found then there are several different constant acceleration equations that can be used to determine the distance OA and most candidates used one of these to find their answer.

Answer: **(i)** The acceleration of P is 7.5 ms^{-2} **(ii)** The distance OA is 60 m

Question 6

This was an interesting question being based on a diagram and a v-t graph and it needed some thought but in spite of this it was particularly well done by many candidates.

(i) Most successfully found the value of h by considering the area under the v-t graph or by using the constant acceleration equations

(ii) Most candidates wrote down the equations of motion for each of the masses. An error which frequently occurred was when writing the equation for the mass $1 - m$ some candidates equated the forces to ma rather than to $(1 - m)a$. A few candidates used the formula which they remembered for linked particles to give an equation not involving the tension, T, in the string which is equivalent to eliminating T from the two Newton's second law equations. At some stage it was necessary to find the acceleration from the gradient of the v-t graph. Most did this correctly. Having determined m, one of the equations involving T was needed to determine the tension in the string.

(iii) For this part of the question it was necessary to use the v-t graph and the fact that the slope of the graph after $t = 0.5$ is $-g$. Various different approaches were made but essentially the value of v goes from +2 to −2 with an acceleration of −10 and the constant acceleration equations can be used to determine the time taken for this to happen or the problem could be solved geometrically using the diagram. Finally the question asked for the total time from the instant that P was released and so an extra 0.5 seconds had to be added to the value found. Some candidates forgot to do this.

Answer: **(i)** $h = 0.5$ m **(ii)** $m = 0.3$ Tension = 4.2 N **(iii)** Total time taken = 0.9 seconds

Question 7

This question proved to be the most difficult on the paper for most candidates.

(i) There are many different approaches that can be made to this part of the question. These include resolving forces at *J* horizontally and vertically, using the triangle of forces, using Lami's theorem or probably the best method is to resolve along the directions of the two strings since they are perpendicular and so each equation immediately gives a value for one of the tensions. A good diagram with all forces including tensions shown is a very good way to start this question and candidates would be well advised to begin their attempts at such questions in this way. Often it was difficult to distinguish from the notation used by candidates as to which tensions were in *AJ* or *JR*. Some candidates wrongly assumed that the tensions in strings *AJ* and *JR* were the same. Candidates would be strongly advised to state the meaning of any symbols used in questions such as this.

(ii) For this part of the question, candidates had to use the tension found in *JR* in part **(i)** and use it when writing down the fact that the ring *R* was in limiting equilibrium. Four forces are acting on *R*, namely, the tension in *JR*, the weight of the 0.2 kg particle, the frictional force and the normal reaction at the wall. Candidates usually resolved these forces horizontally and vertically and linked them by using $F = \mu R$. Many wrongly thought that *R* was merely $2g$.

(iii) There is now an extra force acting on *R* due to the weight of the particle of mass *m*. Very few candidates managed to complete this part successfully. Since the normal reaction is unchanged from the previous part it is only necessary to consider the vertical forces acting on *R* in which the weight of the two particles is balanced by the frictional force and the component of the tension in *JR*. Several candidates continued to include the tension in *AJ* at this stage but it did not play any direct part when considering the forces on *R*.

Answer: **(i)** Tension in *AJ* is 4.48 N Tension in *JR* is 3.36 N **(ii)** given $\mu = 0.341$ **(iii)**$m = 0.1376$ (answers of either $m = 0.137$ or $m = 0.138$ were accepted)

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

- Candidates are reminded that non-exact numerical answers should be given correct to 3 significant figures.
- When asked to **sketch** a graph, candidates are not expected to use graph paper or to plot points.
- When using a work / energy equation, candidates should check that they consider work done as well as potential energy and kinetic energy e.g. **Question 2(ii)** and **Question 6(ii)**.

General comments

On the whole, the standard of work for this paper was very high with many excellent scripts. Although the full range of marks were seen, the majority of candidates were able to attempt all questions. The best scripts contained clear, accurate solutions supported by appropriate diagrams and sufficient working. **Question 2** was the least well-answered question, whilst **Questions 1**, **3** and **4** were often found to be straightforward gaining full marks.

Comments on specific questions

Question 1

The majority of candidates gained full marks either by finding the work done and the power applied independently or by using $P = \dfrac{\text{WD}}{t}$ or WD = Pt for one part of the question. A few candidates found only one of the two quantities required. Occasional errors included the use of an incorrect formula such as $\text{WD} = \dfrac{P}{t}$ or an incorrect substitution into a correct formula such as WD = Fs = 500 x 40 instead of using the correct substitution WD = 500 x 40 x 2.75. Of those who used WD = Fs, a few calculated the distance to be 55 m instead of 110 m, assuming constant acceleration instead of constant speed.

Answer: Work Done = 55000 J Power = 1375 W

Question 2

(i) Very few candidates answered this part of the question successfully. Many appeared to believe that particle *B* reached the floor at the same time as particle *A* reached the pulley and consequently described or explained the motion of the two connected particles until *B* reached the floor. The question stated that '*A* **subsequently** reaches the pulley with a speed of 3 ms^{-1}', and candidates were expected to refer to the lack of friction and also to the change in motion for *A* due to the loss of tension when *B* reached the floor.

(ii) This part of the question differentiated well. Candidates attempted to solve the problem either by using Newton's Second Law or by considering energy. The most common error for those using Newton's Law was to state the equation of motion for *A* as $T - 0.35\,g = 0.35\,a$ instead of $T = 0.35\,a$ as if considering a pulley system with both *A* and *B* hanging vertically. A complete force diagram could have helped to avoid this error. A few candidates used $a = g$ when applying the constant acceleration formulae to determine h. Those who used an energy method frequently equated the potential energy loss for particle *B* to either the kinetic energy gain for *B*, which leads erroneously to $h = 0.45$, or to the kinetic energy gain for *A* leading erroneously to $h = 1.05$.

Answer: **(ii)** $h = 1.5$

Question 3

While this question was frequently well answered by candidates, there was a minority who were unable to set up any appropriate equations to work with. Those who applied Newton's Second Law with driving force equal to P/v to the two situations were usually able to solve the resulting simultaneous equations. Occasionally errors were made in solving the equations or in applying $P = mav$ or $P = (ma - R)v$ instead of $P = (ma + R)v$. The first ma value of 3440 gave rise to a numerical error for some candidates who reused this as 3340 at a later stage. A final answer of $P = 17.9$ was not accepted since the question stated 'P W'.

Answer: $P = 17900$ $R = 537.5$

Question 4

Many candidates calculated the driving force accurately using either a work / energy method or a constant acceleration formula combined with Newton's Second Law.

The work / energy equations included a variety of errors such as an incorrect sign (e.g. 1920000 instead of –1920000), an incorrect dimension (e.g. 7500 instead of 7500 x 500) or an extra term, with the work done by the component of weight down the hill (12000 g x 25/500 x 500) included as well as the potential energy loss. Those who calculated the angle of inclination of the hill and used e.g. 12000 $g\sin 2.9^{\circ}$ x 500 instead of the exact value of 12000 g x 25 lost some accuracy in their final answer. A few candidates appeared to believe that the lorry was in vertical motion (PE = 12000 g x 500 or PE = 12000 g x (25 + 500)), whilst a few believed that 500 m represented the horizontal distance between the bottom and the top of the hill. Some candidates calculated the work done by the driving force but then omitted to divide by 500 to obtain the driving force of the lorry.

Using the Newton's Law method the common errors were to omit the component of weight down the hill from the equation of motion or to use $a = 0.32$ ms^{-2} instead of $a = -0.32$ ms^{-2} suggesting acceleration rather than deceleration up the hill.

Answer: Driving Force = 9660 N

Question 5

This question was usually well answered with any loss of marks commonly due to an incomplete or incorrect description of the direction of the resultant.

(i) Although some candidates were unclear about what was required and simply listed the components of each force, the majority confidently resolved in perpendicular directions and used Pythagoras' Theorem and trigonometry to obtain the required results. Candidates were expected to make reference to a direction as well as stating an angle e.g. 60.9° with the 4 N force. A very few candidates opted to combine the forces two at a time using the cosine and sine rules which although a possible method was lengthy and seldom completed successfully.

(ii) Although this part of the question said 'state ...', candidates frequently restarted with a new set of calculations. It was hoped that candidates would relate the two situations and write down the magnitude and direction of the resultant without further working. Those who made an error in part **(i)** were thus usually still able to gain full credit if they used their previous answers.

Answer: **(i)** Magnitude of the resultant force = 34.8 N Direction is 60.9° with 4 N force
(ii) Magnitude of the resultant force **=** 34.8 N Direction is 29.1° with 16 N force

Question 6

This question differentiated well with most candidates gaining some marks in part **(i)** whilst part **(ii)** was found to be more challenging.

(i) In applying $\mu = F/R$, the majority of candidates had the correct value for R whilst more errors were made in evaluating the frictional force. Some used $F = 20$ N without considering the component of weight down the plane whilst others had the component of weight and the frictional force acting in the same direction, stating $20 - 5\,g\sin 10^{\circ} = F$ instead of $20 + 5\,g\sin 10^{\circ} = F$.

(ii) Although candidates generally attempted to apply $v^2 = u^2 + 2\,as$, there was some difficulty in finding the acceleration after the 20 N pulling force was removed. The effect of friction was sometimes ignored leaving $a = g\sin 10^{\circ}$ despite the deceleration of the box. The less popular method of considering work and energy often had a term missing from the equation with the use of e.g. KE loss = PE loss rather than KE loss + PE loss = WD against friction.

Answer: **(i)** $\mu = 0.582$ **(ii)** Distance moved by the box = 0.781 m

Question 7

Most candidates were able to answer some parts of this question but many were challenged by the graph in part **(ii)** which then sometimes resulted in the use of an inappropriate method of solution in part **(iii)**.

(i) Since this was a 'show that ...' question with the answer provided, candidates were expected to give sufficient evidence for $v = 0$ at the three given times. Some simply stated the result that they were asked to demonstrate. Those who tested the three values of t, nearly always achieved the given result. Those who started to factorise were sometimes too brief in their explanation e.g. $t(0.0001t^2 - 0.15t + 0.5)=0 \rightarrow t = 0$, 50 or 100.

(ii) The majority of candidates knew that differentiation was needed and were able to solve $a(t)=0$. The resulting values of v were sometimes given correct to 2 significant figures as ±4.8 instead of ±4.81 and occasionally v was not evaluated at all. It was common to see the graph drawn as three straight line segments joining (0,0), (21.1,4.81), (78.9,−4.81) and (100,0) rather than a curve with maximum and minimum turning points. Candidates need to be aware that a sketch does not require either graph paper or fully scaled axes and point plotting.

(iii) On the whole candidates knew that the distance could be found using integration and this was often applied accurately and with appropriate limits of 0 and 50. Occasionally candidates doubled this answer to give the total distance travelled rather than the maximum distance from O. A minority were unclear about the limits of integration and 21.1, 78.9 and 100 were all occasionally used as one of the limits. Unfortunately some candidates were misled by their graph and calculated the area of one or two triangles instead of using integration.

Answer: **(ii)** The values of v when $a = 0$ are 4.81 and –4.81 **(iii)** Greatest distance of P from O is 156 m

MATHEMATICS

Paper 9709/51
Mathematics

General Comments

The presentation of the work by some candidates could be improved.

Some candidates lost marks due to not giving answers to three significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to three significant figures, then their working should be performed to at least four significant figures.

Most candidates are now using g = 10 as requested on the question paper.

Candidates should refer to the formula booklet provided if in doubt about a formula.

The easier questions proved to be **Questions 2**, **4** and **5(i)**.

The harder questions proved to be **Questions 1**, **3**, **5(iii)**, **6** and **7**.

Comments on Specific Questions

Question 1

Many candidates were unable to set up the required three term energy equation needed.

It was necessary to use loss of elastic energy = gain in kinetic energy.

Answer: Modulus of elasticity = 16 N

Question 2

This question was quite well done.

(i) A number of approaches were possible. The most popular one was to differentiate to find $\frac{dy}{dx}$ and then equate it to zero to find x. This value was then substituted into the trajectory equation.

(ii) This part was solved by doubling the value of x found in part **(i)**.

Answers: **(i)** Greatest height = 2.4 m **(ii)** Distance from O = 8 m

Question 3

This question proved to be rather difficult.

(i) By using Newton's Second Law in the direction PO, the required value for the length of the string could be found.

(ii) Candidates needed to resolve vertically to find θ. Having found θ, the speed was calculated by using $v = r\omega$.

Answers: **(i)** Length of string = 1.2 m **(ii)** Speed of P = 5.66 m s^{-1}

Question 4

(i) Candidates recognised that it was necessary to find the horizontal and vertical velocities as the ball hit the ground. They also recognised the use of Pythagoras's theorem to calculate the speed and were also able to find the required angle. Quite a number of candidates made careless errors.

(ii) This part of the question was solved by using the equation $s = ut + \frac{1}{2}at^2$ for the vertical motion.

Answers: **(i)** Speed = 26(.0) m s^{-1}, Direction = 60° to the horizontal **(ii)** Height of O above the ground = 22.5 m.

Question 5

(i) This part of the question was generally well done.

(ii) (a) Candidates were able to find the extension when the string broke but did not then go on to find the height.

(b) This part of the question proved to be very difficult. A five term energy equation was required to find the speed of P when the string broke. The particle then moves with the acceleration due to gravity.

Answers: **(i)** Extension of P in the equilibrium position = 0.15 m **(ii)(a)** Height of P above the ground when the string breaks = 1.5 m **(b)** Height of O above the ground = 7.25 m

Question 6

This question proved to be very difficult for many candidates.

(i) Newton's Second Law should be used to give $0.1v\frac{dv}{dx} = -0.2e^{-x}$ and hence $v\frac{dv}{dx} = -2e^{-x}$.

(ii) The candidates now needed to solve the differential equation found in part **(i)** and then substitute the correct limits.

(iii) The candidate should now let x tend to infinity in the equation found in part **(ii)**.

Answers: **(i)** $k=-2$ **(ii)** Value of x when velocity of P is 2 m s^{-1} = 0.236 **(iii)** Speed does not fall below 0.917 m s^{-1}

Question 7

This question proved to be extremely difficult.

(i) The centre of mass can be calculated by taking moments about BD.

(ii) (a) The candidate has to now take moments about the point B, the point about which the prism would topple.

(b) In this part of the question it was necessary to resolve along and perpendicular to the plane in order to find the friction force F and the normal reaction R. $F = \mu R$ is then used.

(iii) The candidate has now to find the actual friction force and the maximum friction force. The two are then compared and a conclusion made.

Answers: **(i)** Centre of mass from BD = 0.143 m **(ii)(a)** Smallest value of P for no toppling = 2.5(0) **(ii)(b)** Coefficient of friction = 0.787

MATHEMATICS

Paper 9709/52
Paper 52

General Comments

The paper was fair and a test for all abilities.

The work was generally well presented and easy to follow. There were a few exceptions to this.

Some candidates pre-approximate their values while working in the middle phase of their solutions. This eventually produces errors in their final answers. If a three significant figures answer is required, it is recommended that at least four significant figures are used in all intermediate working.

Most candidates now use g = 10 as instructed on the question paper.

Candidates should refer to the formula booklet provided if in doubt about a formula.

The easier questions proved to be **Questions 1**, **2(ii)**, **5(i)** and **7(i)**.

The harder questions proved to be **Questions 3(i)**, **5(ii)** and **6**.

Comments on Specific Questions

Question 1

This question was generally well done with many candidates scoring all three marks. A few candidates quoted the wrong direction or gave no direction at all.

Answer: Frictional force = 2.16 N towards the centre.

Question 2

(i) This part of the question was usually well done.

(ii) Many candidates attempted to set up an energy equation. Sign errors occurred in the equation at times and a number of candidates used $\frac{\lambda x}{2l}$ instead of $\frac{\lambda x^2}{2l}$ for the elastic energy term.

Answers: **(i)** Mass of P = 1.8 kg **(ii)** Speed of P when the string becomes slack = 1.53 m s^{-1}

Question 3

(i) In this part of the question candidates needed to take moments about AC.

(ii) Candidates needed to take moments about A in order not to include the unknown force at A. This gave the value of F. Finally by resolving vertically for the whole system, the required force was calculated.

Answers: **(i)** Distance of centre of mass from AC= 0.12 m **(ii)** F = 3 and magnitude of force at the hinge = 7 N upwards

Question 4

A good clear diagram would certainly help the candidate to solve this question.

(i) Some basic trigonometry was needed in order to find angle BAP and also the radius of the circle before any attempt could be made to find the required speed. The candidate was then required to resolve vertically and also to use Newton's Second Law. The speed of P could then be found.

(ii) By using Newton's Second Law and the tension in BP equal to zero it was possible to find the minimum value of the angular speed.

Answers: **(i)** Speed of P = 1.86 m s^{-1} **(ii)** Angular speed of P must exceed 5 rad s^{-1}

Question 5

(i) This part of the question was generally well done with many candidates scoring all three marks.

(ii) This part of the question proved to be far too difficult for many of the candidates. The candidates were required to take moments about B (the point of toppling). Candidates found it very hard to find the correct distance in order to find a moment for the weight. An easier way was to use the weight components parallel and perpendicular to the plane and then to take moments about B.

Answers: **(i)** Tension in the string = 12 N **(ii)** Weight of cube = 56.8 N

Question 6

This question proved to be the hardest question on the paper.

(i) Candidates were required to consider horizontal and vertical motion to set up two equations in U and θ. These equations should be $U\cos\theta = 18\cos 30$ and $U\sin\theta - 2g = -18\sin 30$. By using trigonometry these equations can be solved to find U and θ.

(ii) In this part of the question the horizontal and vertical displacements X and Y were needed in terms of V. Having found these, tan60 = Y/X could be used to find V.

Answers: **(i)** $U = 19.1$ and $\theta = 35.2$ **(ii)** V = 2

Question 7

(i) This part of the question was quite well done. A common error was seeing the normal reaction R = 0.2g + 0.4x2sin30 instead of 0.2g – 0.4x2sin30.

(ii) To solve this part of the question Newton's Second Law should be used giving $0.2\frac{dv}{dt} = 0.4t\cos 30 - \mu R$. This was not well done by many candidates.

(iii) Most candidates were able to solve the differential equation. Unfortunately the incorrect limits were often used. Only a few candidates arrived at the correct answer.

Answers: **(i)** $\mu = 0.433$ and $t = 10$ **(ii)** $\frac{dv}{dt} = 2.165t - 0.4330$ **(iii)** Speed of P when it loses contact = 69.3 m s^{-1}

MATHEMATICS

Paper 9709/53
Paper 53

General Comments

This paper was fair and was a test for all abilities. Most candidates work was neat and well presented.

It is pleasing to see that most candidates are using g = 10 as instructed on the question paper.

Some candidates lost marks due to not giving answers to three significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to three significant figures then their working should be performed to at least four significant figures.

The easier questions proved to be **Questions 1**, **5(i)** and **6(i)**.

The harder questions proved to be **Questions 2(i)**, **3(ii)**, **6(iii)** and **7**.

Comments on Specific Questions

Question 1

This question was generally well done.

(i) A few candidates used the incorrect formula for the centre of mass of the lamina. They used the centre of mass of an arc.

(ii) Most candidates recognised that they needed to take moments about B so that they did not involve any forces acting at B.

Answers: **(i)** Centre of mass from AB = 0.17(0) m **(ii)** Mass = 2.66 kg

Question 2

Too many candidates failed to score on this question.

(i) This part of the question required the candidates to find v_x and v_y, the horizontal and vertical velocities at time t = 1.5. By using $\tan 45 = \frac{V_y}{V_x}$ the value of V was found.

(ii) By considering horizontal and vertical motion the required displacements could be calculated.

Answers: **(i)** V = 41(.0) **(ii)** Horizontal displacement = 30.7 m, vertical displacement = 42(.0) m

Question 3

(i) This part of the question was well done by many candidates.

(ii) An energy equation with five terms was required. Too many candidates only had four terms in their equation. This part of the question proved to be far too difficult.

Answers: **(i)** Extension of string = 0.05 m **(ii)** e = 0.508

Question 4

This question was quite well done.

(i) Most candidates scored well on this part of the question.

(ii) Some candidates when finding the vertical velocity at the ground made a sign error in the equation. The overall method was clearly understood by most of the candidates.

Answers: **(i)** B strikes the ground after 3 seconds **(ii)** Speed = 29.5 m s^{-1}, direction = 31.7° to the horizontal.

Question 5

The above average candidates scored well on this question.

(i) To solve this part of the question it was necessary to take moments about D. By doing this the only forces involved were the tension in the string and the weight of the prism.

(ii) This part of the question proved to be too difficult for many of the candidates. The extension had to be found when the tension was 60 N. With this extension a three term energy equation could be set up to find the required velocity.

Answers: **(i)** Tension in the string = 60 N **(ii)** Speed of particle when the prism topples = 3 m s^{-1}

Question 6

This question was usually well done.

(i) Very few candidates failed to set up the required differential equation.

(ii) Most candidates were able to solve the differential equation.

(iii) A minority of candidates did not realise that they needed to integrate again to find the displacement.

Answers: **(i)** $5\frac{\mathrm{d}v}{\mathrm{d}t} = 12.5 - v$ **(ii)** $v = 12.5 - e^{-0.2t}$ **(iii)** Displacement = $12.5t + 5e^{-0.2t} - 5$

Question 7

The above average candidates scored well on this question.

(i) This part of the question was solved by resolving vertically to find the tension and then saying that the horizontal component of the tension was greater than or equal to $mr\omega^2$

(ii) This part of the question proved to be extremely difficult for many candidates. To solve this it was necessary to set up 2 equations in the tension and θ (the angle between the string and the vertical).The two equations were found by resolving vertically and by using Newton's Second Law horizontally. From these equations the tension and θ could be calculated. With θ, the height could be found.

Answers: **(i)** Angular speed of P $\leq$ 8.45 rad s^{-1} **(ii)** Tension in string = 11.5 N, h = 0.244

MATHEMATICS

Paper 9709/61
Paper 61

General Comments

There has been a noticeable improvement in presentation, with candidates setting out their solutions in a clear format. The majority of candidates were not familiar with many aspects of the syllabus, particularly the binomial and normal distributions. There was very little evidence of candidates requiring more time and it was pleasing to see solutions evaluated to the required accuracy in an increasing number of presentations.

Comments on Specific Questions

Question 1

There was a very disappointing response to this question on the normal distribution. Very few candidates were able to determine the sought-after probability and then use the tables in reverse to ascertain the z-value. Those who did use 0.975 and obtained 1.96 often omitted the –ve sign. All too often, candidates had a standardised expression equal to a probability. A simple sketch of the given data would help candidates to eliminate the errors detailed above and assist in determining the correct approach towards a solution.

Answer: 4.60

Question 2

Frequency density was not a familiar concept to the majority of candidates with many plotting frequency against length. Marks were lost by poor plotting, omission of scale, labels, inconsistent tops of bars and vertical lines all too often drawn free-hand. Use of a ruler would be a great benefit and assist in a more accurate presentation.

Answers: **(i)** 5.5–7.0 cm **(ii)** F.D. 5.33, 25, 28, 20.7, 6

Question 3

There were very few reasonable attempts at this question, with little evidence that candidates understood the underlying principles of independence and mutual exclusion. Those candidates who attempted the probabilities of events A and B often overlooked, say, (1, 3) and (3,1).

Answers: **(i)** Independent since P(A) × P(B) = P(A∩B) i.e. $\frac{16}{36}\times\frac{27}{36}=\frac{12}{36}$

(ii) Not mutually exclusive because P(A∩B) ≠ 0, or give a counter example, say, (3,2).

Question 4

It was disappointing that many candidates were unable to transfer the given tree-diagram into the correct equation. The probability of (1 – x) was not realised by many candidates and the calculation (0.801 – 0.9) was a common source of error.

It was very pleasing that there were many good solutions to the required probability in **(ii)**

Answers: **(i)** 0.15 **(ii)** 0.427

Question 5

(i) Few candidates understood the basic principle of combining the 27 weights to obtain their mean.

(ii) Many good attempts were marred by omitting $\bar{x}^2$,not multiplying by 9 or 18 throughout and losing the final mark by using 5.83 instead of 5.833 or better.

Answers: **(i)** 5.83 **(ii)** 1.46

Question 6

(i) Those candidates familiar with the binomial distribution invariably scored maximum marks. The only problem was that some interpreted the request to find 'the probability… 5, 6 or 7 households have a printer' as three individual probabilities and not as a total.

(ii) Many centres were not aware of the normal approximation and those that were invariably their candidates did not use a continuity correction

(iii) The justifications for using the Normal Distribution as an approximation for the Binomial Distribution are that np and nq are both greater than 5. Please note NOT the mean (np) and variance (npq).

Answers: **(i)** 0.722 **(ii)** 0.595 **(iii)** np = 340, nq = 160, both > 5

Question 7

(a) (i) Many candidates recognised the necessity to accommodate the instances of replication and divided by 2! × 2! × 3! Unfortunately, a significant number of candidates used the wrong sequence on their calculator by pressing 9! ÷ 2! × 2! ×3!

(ii) Few candidates recognised that the even numbers ending in 2, 6 and 8 needed to be considered as 3 separate events; similarly in determining the odd numbers ending in 3 or 7 and subtracting from the total.

(b) A pleasing response from many candidates who realised that there were up to 6 ways of meeting the criteria set out in the question. Many used combinations and the majority recognised the need to find the sum of a series of triple products of combinations.

Answers: **(a)(i)** 15120 (15100) **(ii)** 10080 (10100) **(b)** 13839 (13800)

MATHEMATICS

Paper 9709/62
Paper 62

Key Messages

To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method. Candidates should label graphs and axes, show dotted lines for finding the median or quartiles from a graph, and choose sensible scales.

General Comments

This paper was well attempted by the majority of candidates who had worked and prepared for it. Most candidates appeared to have covered all the topics and were able to make a start on the questions.

Comments on Specific Questions

Question 1

The majority of those who knew that this was a binomial situation had no problems at all. Some used decimals instead of fractions which resulted in the answer not being correct to 3 significant figures, and thus gained only 2 marks out of 3. It is important for candidates to realise that they must work with at least 4 significant figures.

Answer: 0.222

Question 2

A surprising number of candidates did not realise that mid-points have to be used. Frequently class widths, upper class boundaries lower class boundaries were used. Once again, the final mark was lost by many candidates through not working with 4 figures. They used 45.8 when subtracting $\overline{x}^2$ whereas 45.83 would have given the correct answer to 3sf.

Answers: 45.8, 14.9

Question 3

When a question asks candidates to draw a graph, it is expected that it will be on graph paper especially as the instructions said that, and that rulers and pencils will be used and very importantly, any numerical axes should be clearly marked and labelled. Three quarters of the candidates lost a mark because they did not put the units, seconds, on the axis. A fair number were unable to find the quartiles, using 22, 44 and 66 instead of the time in seconds. In part **(ii)** it is not enough to calculate both extreme fences for outliers. We need to see a comment that there are no values outside these extreme fences.

Answers: **(i)** LQ 2.6, median 3.8, UQ 6.4. **(ii)** lower fence –3.1, upper fence 12.1 no values exist hence no outliers

Question 4

A high proportion of candidates gained full marks for this question which was well done and accessible to all who knew what conditional probability was.

Answers: **(i)** 0.81, **(ii)** 0.387

Question 5

This question proved a good discriminator between candidates with some candidates not appearing to understand what '3 discs at random without replacement' means, and the better candidates getting full marks. Some managed to get a probability P(EEO) to be 3/5×2/4×2/3 but did not appreciate that you can also have P(EOE) and P(OEE). They gained a method mark. Part **(ii)** was ably done by the grade A candidates. 10 different selections were expected but many candidates attempted to list all 60 permutations, in a completely haphazard fashion rather than the 10 selections.

Answers: **(i)** 0.6, **(ii)** P(1) = 0.6, P(2) = 0.3, P(4) = 0.1

Question 6

Many students were aware they needed to use 2! and 3! in this question, preferably by dividing, but were not sure exactly where to use them. There were many good answers to parts **(i)** and **(ii)**. In part **(b)** there were many different ways of approaching this problem, and 3 ways were common. Overall this question was one of the better attempted ones.

Answers:**(a)(i)** 20, **(ii)** 60, **(b)** 1210

Question 7

An entire question on the normal distribution. Very good for those candidates who knew their work. The first part was a straightforward 3 marks for standardising and using tables to arrive at an answer of 0.1345. This answer then had to be multiplied by 52 to get 6.99 to 3 significant figures. Those candidate who rounded up to 0.135 and then multiplied by 52 got an answer of 7.02 to 3 significant figures and lost the final accuracy mark. Part **(b)** was very well done by the majority of candidates, even those who were unable to do parts **(i)** and **(ii)**.

Answers: **(a)(i)** 6.99, **(ii)** 37.0, **(b)** μ = 8.94, σ = 2.42

MATHEMATICS

Paper 9709/63
Paper 63

Key Messages

Candidates should be encouraged to show all necessary workings. A significant number of candidates did not show sufficient working to make their approach clear, and were unable to gain full credit.

Candidates should be aware that they need to work to a greater degree of accuracy than the 3 significant figures required in the answer to ensure that final answers are correct.

When drawing graphs, candidates should be encouraged to use scales that enable accurate readings to be achieved.

General Comments

Answers to **Questions 1**, **3** and **4** were generally stronger than answers to other questions.

The majority of candidates used the answer booklets provided effectively, however a number failed to utilise the available space appropriately either by answering the entire paper on a single page, or dividing the page into 2 columns, both making their reasoning difficult to follow. This also resulted in some candidates making corrections that were very unclear and resulted in the inaccurate transfer of answers. It was concerning the number of candidates who failed to secure the graph paper to their main answer booklet.

A number of candidates made more than a single attempt at a question and then did not indicate which their submitted solution was.

Comments on Specific Questions

Question 1

Most candidates used the normal distribution formula correctly. A few candidates incorrectly used a continuity correction. A common error was to use 3 significant figures throughout, resulting in premature approximation and an inaccurate final answer. Good solutions included a sketch of the normal distribution to assist understanding.

Answer: 170

Question 2

Many candidates did not appreciate that the table in part **(i)** was an aid to interpret the information in the question to assist in the performance of part **(ii)**.

(i) Good candidates recognised that the probability table was completed by ensuring that the totals were correct, and simply found the missing value in a line of 3 terms. The majority of candidates were able to complete the 'Meal not served on time' row, but then used the rules of independent probabilities for the remainder of the table. There were a concerning number of candidates who submitted solutions with negative values or probabilities greater than 1.

(ii) Almost all candidates completed the conditional probability process correctly, using the appropriate values from their table. Weaker candidates attempted to calculate P('Meal served on time' ∩ 'Kitchen left in a mess').

Answer: **(ii)** $\frac{5}{6}$

Question 3

The majority of candidates successfully attempted this question, using the normal approximation. Good candidates confirmed that this was an appropriate method by stating that $np > 5$ and $nq > 5$. Most candidates identified that there was a need for a continuity correction. Many solutions included a sketch to assist in identifying the required probability value. However, a significant number of solutions either failed to use calculations to 4 significant figures or had arithmetical errors. Candidates should be encouraged to use their calculators efficiently even if they approximate values within their written workings. The weakest candidates attempted to use the binomial distribution but were not sufficiently accurate to gain full credit.

Answer: 0.180

Question 4

Candidates of all abilities attempted this question with some success.

(i) Many candidates appeared not to appreciate that where they are asked to show a statement is true, a clearly expressed rationale for their work is required. The best solutions used a tree diagram, and then calculations linked clearly to the appropriate outcomes. Good candidates using the alternative combination approach justified the formula and showed clear calculations. However many candidates simply stated a two-factor product which was then doubled.

(ii) Almost all candidates drew a correct probability distribution table. Only the weakest candidates calculated the probabilities incorrectly, often appearing to use a different approach to their solution to **(i)**.

(iii) The majority of candidates attempted to calculate the expected value. A common misunderstanding was that the context of the question required an integer result, with many candidates rounding their accurate answer. Unfortunately, a number of candidates did not transfer the values from **(ii)** accurately.

Answers: **(ii)** $P(W,W) = \frac{1}{12}$, $P(\overline{W}, \overline{W}) = \frac{5}{12}$ **(iii)** $\frac{4}{3}$

Question 5

Most candidates attempted to use the normal distribution correctly. However, the failure to include explanation as to the purpose of a calculation appears to have caused confusion to some candidates in identifying the appropriate values elsewhere in the question

(i) Most candidates were able to standardise correctly, with each section identified clearly. The best solutions often had a sketch of the normal distribution to aid the candidate. The application of the binomial distribution was good, with few candidates incorrectly including 2 large books within the required outcomes. A number of solutions incorrectly reversed the probability values. Many candidates showed workings to 3 significant figures, but used a more accurate value for their calculations to gain full credit, whilst weaker candidates introduced rounding errors resulting in incorrect final answers.

(ii) Most candidates constructed an inequality of the correct format. Poor solutions used the answer from **(i)** or other values that had been calculated previously. Good solutions provided clear logarithmic manipulations to solve the inequality. However, many simply stated a single value, which may have been achieved by trial and improvement, but had misinterpreted the inequality and therefore could gain little credit.

Answers: **(i)** 0.718 **(ii)** 28

Question 6

Few fully correct solutions to this question were seen. Many candidates had scales which resulted in inaccurate plotting and reading. Candidates should be encouraged to have axes which focus on the values to be plotted, and a scale which ensures that the small 'squares' on the graph paper can be used appropriately. Scales such as 2cm = 7 units should be avoided. A number of candidates used non-linear scales which could gain no credit.

(i) Most candidates attempted to draw a clear cumulative frequency graph. The best solutions had a smooth, increasing curve with a horizontal axis from 3.5 to 5.0. The use of line segments is acceptable at this level. Weaker candidates used a continuity correction for plotting the values which was not appropriate for the context. Many candidates failed to label the axes fully, often omitting either 'cumulative frequency' or 'nitrogen content'.

(ii) It was anticipated that candidates would use the graph that they had constructed to obtain the values to calculate the required percentage. This was indicated by the system of question numbering. Good solutions were obtained by candidates who had used a scale that could be read accurately in **(i)**. A number of candidates attempted to use linear interpolation, but did not use the data provided accurately. A number of solutions were for a nitrogen content of less than 4.4.

(iii) As in part **(ii)**, good solutions followed from clear graphs in **(i)**. Most candidates provided evidence that they understood the concept of the median, frequently identifying the median term. However, many students then simply identified that it was in the range $4.0 < m < 4.2$, or calculated the mean of the boundaries of this region. A small number successfully attempted linear interpolation.

(iv) Many candidates did not attempt this part, although this does not appear to be from a time constraint as most then completed **Question 7**. Good solutions had a clear frequency table which was used as a structure to generate the frequency densities required for the histogram. Many candidates simply calculated the frequency and then plotted a frequency graph. Good solutions used an appropriate scale, the most common being vertical scale of 2 cm for 10 units and a horizontal scale of 2 cm for 0.5 units. These enabled the bars to be plotted accurately, including indicating that the frequency density for $4.5 < x < 4.8$ was approximately 26.7. Most candidates recognised that the same horizontal values as **(i)** should be used, although a number of solutions introduced a continuity correction.

Answers: **(ii)** 21.4% **(iii)** 4.15

Question 7

Many candidates scored well on this question. The best solutions had clear workings provided at all stages. However a number of candidates provided calculations with no statement of what was being calculated, which made it difficult to follow their logical working where incorrect solutions were obtained.

(i) The best solutions clearly stated the possible options prior to making any attempt at calculating. Each option was evaluated before a total was stated. Although many candidates made good progress, many omitted an option, often where there was no use of pattern in the numbers being totalled to 5. A few candidates mistakenly identified the number of possible ways of choosing 1 of each object and then considering options for the remaining 2.

(ii) Good solutions identified the possible arrangements of having a duck at each end and then considered the remaining 8 ornaments required from the 10 non-duck ornaments. A number of candidates achieved success by stating they were considering DNNNNNNNND and identifying the number of ornaments available for each position prior to finding the final product. Weaker candidates often either stated that there were 2 options for the ducks or ignored the importance of the order and evaluated combinations.

(iii) Few candidates gained full credit for this part. Many good candidates only evaluated DSWSWSWSWD and not the alternative DWSWSWSWSD. Many candidates recognised that this was a subset of solutions from **(ii)** and successfully used their approach again.

Answers: **(i)** 894 **(ii)** 10 886 400 **(iii)** 103 680

MATHEMATICS

Paper 9709/71
Paper 71

Key Messages

Candidates should consider carefully which distribution is appropriate in a given situation.

Clear hypotheses should be stated when significance testing and a clear comparison made between a calculated value and the critical value, which need to be expressed in the same mathematical form

General Comments

Many candidates found the two questions involving significance testing challenging, with many solutions simply repeating the information given in the question. **Question 6** caused difficulties as many candidates used the binomial which would indicate that they thought there could only be a maximum of one error on a page. The best answered questions were **Questions 3** and **7(i), 7(ii),** which tested combinations of variables, and **Question 5** testing confidence intervals. **Question 7(iii)** was the most demanding on the paper.

Comments on Specific Questions

Question 1

Many candidates were quickly able to find the value of a as they recognised that the area of the triangle was 1 and that therefore $a^2 = 2$. They then went on to demonstrate, as required, that E(X) was equal to 0.943. Some candidates used the given result to find the value of a rather than use the area of the triangle, and partial credit was given for this.

Question 2

(i) This part of the question tested the formulation of hypotheses. The correct response that H_0 p=0.2 and H_1 p>.0.2 or equivalent were stated by many candidates. There were many candidates who tried to state their hypotheses in words taken from the question. Candidates need to identify clearly that the significance test is undertaken on the basis that H_0 is true.

(ii) In this part candidates needed to calculate P(X≥13) and compare with 10%. It was expected that candidates would use the normal approximation to the binomial. A number of fully correct solutions were seen, either doing a comparison of probabilities or z values. A number of candidates who calculated the correct test statistic did not show any comparison thereby invalidating any conclusions drawn.

Answers: **(i)** H_0 p=0.2, H_1 p>0.2 **(ii)** Not significant, insufficient evidence that Sami can read minds

Question 3

(i) The question asked for the mean and variance of the combined activities of Yu Ming. Many candidates found the correct values. Some candidates then incorrectly divided these values by 3.

(ii) Candidates were then required to perform a standardisation using these values to find the probability that the mean of these activities took between 33 and 35 minutes over a 70 day period. Many candidates performed this correctly and used tables appropriately to find the required probability. The omission of 70 in the standardisation was a common error, although marks could still be gained for a method to use normal tables to find the correct probability. If the correct mean was obtained, it was clear to many candidates that the required area was symmetrical.

Answers: **(i)** 34, 13.29 **(ii)** 0.978

Question 4

(i) The question tested significance testing of a normal mean. Candidates were required to state the hypotheses; calculate the test statistic comparing two means and perform a comparison, drawing the correct conclusion. Some candidates performed this correctly, showing a clear comparison. Hypotheses were sometimes omitted, and many candidates did not show the comparison of the test statistic with 1.96 or a comparison of the two probabilities.

(ii) This part required candidates to interpret what a Type II error was in context. Some correctly deduced that it meant concluding that the population mean had not increased when it in reality had. A significant number of candidates described a Type I error, or gave no response.

(iii) Candidates were required to indicate that the new mean would need to be known. Those candidates who responded and had correctly responded to part **(ii)** generally identified this.

Answers: **(i)** Not significant, no evidence of an increase in mean. **(ii)** Concluding mean time is still 12.4 when it has increased. **(iii)** The value of the new mean.

Question 5

(i) The majority of candidates found the correct values for the unbiased estimators of the population mean and variance. Those candidates who tried to find these figures by using formulas for E(M) were rarely successful.

(ii) Many candidates were aware of the correct form for the equation of a confidence interval and found the required interval with no errors. A common error was to use the wrong z value, most often 2.054.

(iii) Candidates were required to recognise that 98% of the 50 intervals would be likely to contain the true value of μ. This part was often omitted. Those candidates who attempted this part usually calculated the correct value.

Answers: **(i)** 52.5, 108 **(ii)** 49.8 to 55.2 **(iii)** 49

Question 6

This question tested a context in which the poisson distribution applied. The question indicated that errors occurred at a rate of 1 per 3 pages. Many errors made by the candidates were made by assuming the probability that only a single error could occur on each page with a probability of $\frac{1}{3}$, and thus the binomial distribution could be applied. This was the incorrect approach made by a significant number of the candidates who attempted the question.

(i) Candidates who recognised that the distribution was $\text{Po}\left(\frac{10}{3}\right)$ normally calculated $P(X=2)$ correctly

(ii) The majority of the candidates who obtained the correct answer in the first part then correctly used Po(2) to correctly find $P(X\geq 3)$ by finding $1 - P(X\leq 2)$. A few candidates incorrectly found $1 - P(X\leq 3)$

(iii) The majority of candidates identified that a normal approximation was required, including those using the Binomial. A common error was to make the wrong continuity correction, or to make none at all. A significant number of candidates who used the correct distribution throughout the question produced fully correct solutions for this part.

Answers: **(i)** 0.198 **(ii)** 0.323 **(iii)** 0.0178

Question 7

(i) Candidates were required to combine the binomial and poisson means and subtract 2. Many correct solutions were seen, although a significant number of candidates used n rather than np as the binomial mean.

(ii) Candidates were required to recognise that the variance of the binomial is found by calculating npq and the variance of the poisson is the value of λ. That when combining multiples of variables that the multiplier must be squared, that variances need to be added, and that a constant has no effect on the total variance. Many fully correct solutions were seen, but many other candidates did not recognise the need to follow all of these rules, the most common error was adding 3, or by not squaring the multiplier. It was particularly important here that candidates working was fully shown as marks were available for valid steps that were made in solutions presented.

(iii) Candidates were required to identify that $P(2X-Y=18)$ was possible in just two ways; $P(X=10) \times P(Y=2)$, or $P(X=9) \times P(Y=0)$. This was not identified by the majority of candidates who attempted this part of the question. The most common incorrect approach was to attempt to treat the situation as one that could be solved by combining the two distributions as a single normal variable. As this is not possible there is no merit in this approach. Those who identified the correct process generally correctly combined the probabilities.

Answers: **(i)** 69 **(ii)** 52.6 **(iii)** 0.374 or 0.375

MATHEMATICS

Paper 9709/72
Paper 72

General comments

In general, candidates found the start of the paper quite demanding and questions towards the end more accessible. Questions particularly well attempted were **Questions 4(ii), 5(i), 6 and 7(ii)** whilst **Questions 1, 2 and 3** proved particularly demanding.

Presentation was generally good, though it is important that candidates make sure that all numerical digits are clearly written; it was very difficult to decipher the handwriting for some candidates, with 4's and 9's looking very similar in some cases. A few candidates did not number their questions clearly.

On the whole full working was shown; it is important that candidates show their full method of solution, as on occasions marks can be withheld for lack of all essential working being shown.

The following comments, referring to particular questions, highlight common errors made. However, it should be noted that many fully correct solutions were also seen.

Comments on specific questions

Question 1

To find the variance of $4X$-$5Y$, the calculation required was 16×9+25×36. Many candidates used 4 and 5 rather than 16 and 25, some used 3 and 6 instead of 9 and 36 and some subtracted instead of adding. A few candidates left their answer as a variance, not making the final step of square rooting to get the standard deviation as required in the question.

Answer: 32.3

Question 2

In part **(i)**, candidates were usually successful in stating the null and alternative hypotheses, but part **(ii)** was not well attempted. Some candidates only found a single value, P(X=2), and a valid comparison was often not made. It is important that the comparison is clearly made so that the conclusion drawn is justified. Approximating distributions were not valid; a Poisson distribution (as stated in the question) was required. It should also be noted that final conclusions should, preferably, be in context and not definite.

Answer: **(i)** H_0: λ=0.5
H_1: λ>0.5
(ii) There is evidence to support the claim

Question 3

Many candidates did not fully understand what was required here. Most candidates realised they needed to calculate 5 x 0.15 to find the mean for the 5 day period. However, the calculations to find the expectation (200 x 0.75) and the variance (200^2 x 0.75) were performed successfully by only a small number of candidates.

Answer: 150
30,000

Question 4

Candidates continue to struggle to interpret situations "in the context of the question". In part **(i)** some candidates merely used a text book definition of a Type I error, though it was pleasing to note that there was not as much evidence of this as has been the case in the past. The candidates that tried to answer using the context given, did not always clearly state what had been concluded (that flight times had been affected) and what in actual fact was the case (flight times, in reality, had not been affected); the distinction between conclusion and reality needed to be clear to gain the marks, as we are describing a potential error in the conclusion of a test.

Many candidates answered part **(ii)** with confidence. The steps required in performing the test were generally well followed, though some candidates omitted to define their null and alternative hypotheses, and again a clear and valid comparison was needed in order to support the conclusion drawn. Again, the conclusion drawn should, preferably, be in context and not definite.

Part **(iii)** required a reason; this was not always supplied.

Answers: **(i)** Conclude that flight times affected, when in fact they have not been.
(ii) No evidence that flight times affected
(iii) H_0 was not rejected. Type II.

Question 5

Parts of this question were well attempted.

Most candidates successfully found unbiased estimates of the population mean and variance; there were very few candidates who only found a biased estimate of the variance. Candidates were better at substituting correctly into a correct formula for the unbiased estimate of the variance than has been noted previously.

Many candidates made a good attempt at finding α but some were unable to make the correct final step of using the z value they had found (1.406) to find the value of α. Many thought α was $\Phi(1.406)$ and gave α as 92% not appreciating that we were calculating α for a symmetric confidence interval.

Some candidates successfully found the probability in part **(iii)**, but incorrect working out was often seen.

Answer: **(i)** 296 188
(ii) 84%
(iii) 0.849

Question 6

Questions testing knowledge of probability density functions are usually well attempted, and this question was no exception. Candidates answered part **(i)** very well, and there were only a small number of cases where marks were withheld for lack of essential working. When candidates are asked to 'show that' a given value is true, it is important that candidates do show all stages of their working in order to reach the given answer convincingly.

Part **(ii)** was also well attempted; the main error seen was integrating from 0 to 10 and failing to subtract from 1. Candidates who integrated from 10 to 15 generally reached the correct probability.

In part **(iii)** the main error was to confuse mean with median.

Answer: **(i)** $\frac{4}{27}$

(ii) $\frac{45}{8}$

Question 7

Questions involving the Poisson distribution are usually well attempted, as was part **(ii)** of this question. Other parts were more challenging.

In part **(i)**, not all candidates were able to justify their answer. The given binomial distribution could be approximated to a Poisson distribution because $n>50$ and $np<5$. Some candidates incorrectly thought a normal distribution was a valid approximation or stated binomial as the approximation; many correctly stated Poisson but not all could not justify this fully.

In part **(ii)** many candidates (including those who had not previously stated 'Poisson') correctly used Po(2.1) and reached the required probability.

Calculating the conditional probability was poorly attempted, though most candidates managed to score some marks for finding $P(X \geq 1)$ or for $P(X=1, 2, 3)$. A lack of understanding of conditional probability was evident as few candidates then went on to use the correct formula for finding the conditional probability.

Answer: **(i)** Poisson $n>50$ $np<5$
(ii) 0.839
(iii) 0.816

MATHEMATICS

Paper 9709/73
Paper 73

Key Messages

Candidates continue to perform well on questions testing continuous random variables, as well as the application of the Poisson distribution.

Candidates must ensure that they use the context when describing weaknesses in a given sampling method.

There continues to be an improvement in the performance of significance tests.

General Comments

The majority of solutions were well presented, and most candidates were able to demonstrate their understanding of the topics covered by this specification. Candidates in general performed well on **Questions 4, 6 and 7**. **Questions 1 and 2** proved the most demanding, with many candidates giving answers which ignored the context given. Nearly all candidates gave numerical solutions to at least 3 significant figures.

Comments on Specific Questions

Question 1

Many candidates were able to give short but precise answers in both parts of this question. Part **(i)** required two reasons in context why sampling a single table of 5 in a canteen was unsatisfactory. Examiners were looking for answers that indicated weaknesses for example the method excluded those not in the canteen, or who sat on tables not consisting of 5 people. A number of responses indicated only that the sample was not random, or was unrepresentative without saying why. A number of candidates indicated as a weakness that the sample size was small or just from one School year, thus ignoring the fact that the sample was to be 5 members of her year. Part **(ii)** required candidates to identify the three key aspects of using the random numbers given to select a sample. The three key points were that the digits should be split into two digit numbers; that numbers outside the range 01 to 82 should be discarded, and that any repeating pairs of digits should be ignored. Many candidates identified one of these aspects, without identifying all of them. A minority of candidates gave a general explanation of sampling methods without using the numbers given in the question.

Answers: **(i)** Not all candidates in canteen, not all sat on tables of 5 **(ii)** Split into two digit blocks; ignore if outside range; ignore repeats.

Question 2

Some candidates identified the hypotheses correctly in part **(i)**, while many incorrect answers were seen, the most common wrong answer being to use p as 0.3. A number also used wrong parameters for example μ=3.

In part **(ii)** candidates were required to work out P(X≥3) using B(10,0.125). The majority of candidates identified that the correct distribution to calculate the required probability, although a number of candidates calculated with incorrect values of p, normally 0.3. A significant number of candidates worked out P(X≤2) indicating that possibly they were unsure what a Type I error was in this context.. In part **(iii)** candidates were expected to state the significance level of the test, which should have been their answer to part **(ii)** expressed as a percentage. Many candidates stated this correctly, although a number calculated 1 – their previous answer.

Answers: **(i)** $H_0\ p=\frac{1}{8}$ $H_1\ p>\frac{1}{8}$ **(ii)** 0.119 or 0.120 **(iii)** 12%

Question 3

Many candidates were able to correctly calculate the confidence interval for the proportion accurately in part **(i)**. Some candidates used a process more suited to the confidence interval for a mean, using 22 rather than 0.22 for p. Some candidates also used npq rather than $\frac{np}{q}$ for the variance. The majority of candidates correctly identified that the correct z value for a 97% interval was 2.17. A small number of candidates used $\frac{1}{6}$ rather than 0.22 for p. In part **(ii)** candidates were required to find the probability that in two 97% confidence intervals one would contain p. Many candidates correctly multiplied 0.97 by 0.03 and doubled the result. A common error was to try to calculate a second confidence interval, an approach that did not gain any marks.

Answers: **(i)** 0.13 to 0.31 **(ii)** 0.0582

Question 4

Many candidates in part **(i)** were able to correctly calculate unbiased estimators of the mean and variance using the summary statistics given. Some candidates calculated the biased variance, and other candidates incorrectly substituted into the formula given. In part **(ii)** the correct approach of multiplying the mean by 1.5 and adding 10, and the variance by 1.5^2 was followed by many candidates. A common error was to add 10 to the variance. Some candidates tried to adjust the summary statistics and then substitute into the standard formula for the mean and variance. Although this is possible, very few candidates taking this approach were successful.

Answers: **(i)** 30.16, 129 **(ii)** 55.24, 291

Question 5

The accepted responses for part **(i)** included the destructive nature of the test, or the cost and time saving of sampling. Most candidates indicated at least one of these features. In part **(ii)** candidates were required to perform a significance test. Most candidates wrote down the correct hypotheses based on μ=5 and calculated a test statistic for the mean. The correct comparison of z value with z value or probability with probability was in general correctly performed. A minority of candidates did not show a clear comparison. Part **(iii)** proved most demanding. The required response was that as the distribution of the population was unknown then yes the central limit theory was needed. Responses had to make reference to population to score full marks. A number of candidates indicated no, as the sample size was large indicating that they were not clear about when the CLT is applied.

Answers: **(i)** Testing destructive or too time consuming **(ii)** Test significant so evidence of drop in mean **(iii)** Population distribution unknown so yes.

Question 6

Many candidates were able to answer all three parts correctly. The question tested the application of the Poisson distribution and this continues to be a topic in which candidates show a good understanding. Part **(i)** required the adjustment of the rate from 1 minute to 5. Part **(ii)** tested the normal approximation to the Poisson. A common error was to omit the continuity correction or to apply an incorrect one. Most candidates performed the standardisation and were able to identify the correct area. Part **(iii)** proved to be the most challenging part. Candidates were required to find the mean rate for two checkouts in a two minute period. A significant number of candidates found the mean only for the second checkout (λ=1) rather than both (λ=2.4). These candidates scored the method marks only provided they calculated P(X>3).

Answers: **(i)** 0.216 **(ii)** 0.973 **(iii)** 0.221

Question 7

This question tested continuous random variables and was answered well by most candidates. Part **(i)** required that candidates show that c=2 and nearly all candidates knew that integration between 0 and c was required equated to 1. Very few made errors in obtaining the given result. Part **(ii)** required a sketch of the function. Most candidates recognised the function as an inverted parabola. To be fully correct candidates were required to illustrate 0 outside the range of 0 to 2, and this was not shown by a majority of candidates. With the correct symmetrical shape it can be seen that the median is 1, although many candidates calculated this, rather than used the symmetry of the function. A number of candidates plotted rather than sketched the function. Part **(iii)** was very well done, most candidates finding the correct probability by integrating the pdf between 0 and 1.5. In part **(iv)** it was expected that the symmetry of the graph would be used. Many candidates recognised this and wrote down the answer. Many other candidates integrated between 0.5 and 1. These candidates were not penalised if they obtained the correct answer. A minority of candidates attempted to standardise in both parts **(iii)** and **(iv)**. This is not correct so did not result in any credit.

Answers: **(ii)** Inverted parabola between (0,0) and (2,0) and 0 otherwise, median =1
(iii) 0.844 **(iv)** 0.344