CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- **Note:** B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR – 1 A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA – 1 This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1	State or imply ordinates 0, 0.405465…, 0.623810…, 0.693147…	B1	
	Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
	Obtain answer 0.72	A1	**[3]**
2	Use laws of indices correctly and solve for u	M1	
	Obtain u in any correct form, e.g. $u = \frac{16}{16-1}$	A1	
	Use correct method for solving an equation of the form $4^x = a$, where $a > 0$	M1	
	Obtain answer $x = 0.0466$	A1	**[4]**
3	EITHER: Use correct product rule	M1	
	Obtain correct derivative in any form, e.g. $-\sin x\cos 2x - 2\cos x\sin 2x$	A1	
	Use the correct double angle formulae to express derivative in cos x and sin x, or cos $2x$ and sin x	M1	
	*OR*1: Use correct double angle formula to express y in terms of cos x and attempt differentiation	M1	
	Use chain rule correctly	M1	
	Obtain correct derivative in any form, e.g. $-6\cos^2 x\sin x + \sin x$	A1	
	*OR*2: Use correct factor formula and attempt differentiation	M1	
	Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
	Use correct trig formulae to express derivative in terms of cos x and sin x, or sin x	M1	
	Equate derivative to zero and obtain an equation in one trig function	M1	
	Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$	A1	
	Obtain answer x = 1.15 (or 65.9°) and no other in the given interval	A1	**[6]**
	[Ignore answers outside the given interval.]		
	[SR: Solution attempts following the *EITHER* scheme for the first two marks can earn the second and third method marks as follows:		
	Equate derivative to zero and obtain an equation in tan $2x$ and tan x	M1	
	Use correct double angle formula to obtain an equation in tan x	M1]	
4	**(i)** State $R = \sqrt{13}$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 33.69°$ with no errors seen	A1	**[3]**
	(ii) Evaluate $\sin^{-1}(1/\sqrt{13})$ to at least 1 d.p. (16.10° to 2 d.p)	B1$\sqrt{}$	
	Carry out an appropriate method to find a value of θ in the interval $0° < \theta < 180°$	M1	
	Obtain answer θ = 130.2° and no other in the given interval	A1	**[3]**
	[Ignore answers outside the given interval.]		
	[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]		
5	**(i)** State or imply $AT = r\tan x$ or $BT = r\tan x$	B1	
	Use correct arc formula and form an equation in r and x	M1	
	Rearrange in the given form	A1	**[3]**
	(ii) Calculate values of a relevant expression or expressions at x = 1 and x = 1.3	M1	
	Complete the argument correctly with correct calculated values	A1	**[2]**

	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.11	A1	
		Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115)	A1	**[3]**
6	**(i)**	State or imply $\mathrm{d}u = -\frac{1}{2\sqrt{x}}\mathrm{d}x$, or equivalent	B1	
		Substitute for x and dx throughout	M1	
		Obtain integrand $\frac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
		Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
	(ii)	Integrate and obtain at least two terms of the form $a\ln u$, bu, and cu^2	M1*	
		Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
		Substitute limits correctly	M1(dep*)	
		Obtain the given answer correctly having shown sufficient working	A1	**[4]**
7	**(i)**	Square $x + \mathrm{i}y$ and equate real and imaginary parts to -1 and $4\sqrt{3}$	M1	
		Obtain $x^2 - y^2 = -1$ and $2xy = 4\sqrt{3}$	A1	
		Eliminate one unknown and find an equation in the other	M1	
		Obtain $x^4 + x^2 - 12 = 0$ or $y^4 - y^2 - 12 = 0$, or three term equivalent	A1	
		Obtain answers $\pm(\sqrt{3} + 2\mathrm{i})$	A1	**[5]**
		[If the equations are solved by inspection, give B2 for the answers and B1 for justifying them]		
	(ii)	Show a circle with centre $-1 + 4\sqrt{3}$ in a relatively correct position	B1	
		Show a circle with radius 1 and centre not at the origin	B1	
		Carry out a complete method for calculating the greatest value of arg z	M1	
		Obtain answer 1.86 or 106.4°	A1	**[4]**
8	**(i)**	State or imply the form $\frac{A}{3-2x} + \frac{Bx+C}{x^2+4}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A = 3$, $B = -1$, $C = -2$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	**[5]**
	(ii)	Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√+A1√	
		Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	M1	
		Obtain final answer $\frac{1}{2} + \frac{5}{12}x + \frac{41}{72}x^2$, or equivalent	A1	**[5]**

[Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the first M1.The f.t. is on A, B, C.]

[In the case of an attempt to expand $(5x^2+x+6)(3-2x)^{-1}(x^2+4)^{-1}$, give M1A1A1for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

9	(i)	Separate variables correctly and attempt integration of one side	B1	
		Obtain term $\ln x$	B1	
		Obtain term of the form $a\ln(k+e^{-t})$	M1	
		Obtain term $-\ln(k+e^{-t})$	A1	
		Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a\ln(k+e^{-t})$ and $b\ln x$	M1*	
		Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k+e^{-t}) + \ln(k+1)$	A1	[6]
	(ii)	Substitute $x = 20$, $t = 1$ and solve for k	M1(dep*)	
		Obtain the given answer	A1	[2]
	(iii)	Using $e^{-t} \to 0$ and the given value of k, find the limiting value of x	M1	
		Justify the given answer	A1	[2]

10	**(i)**	Carry out a correct method for finding a vector equation for *AB*	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		Equate at least two pairs of components of general points on *AB* and *l* and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$; or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$	A1	
		Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1	**[5]**
	(ii)	*EITHER*: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Use scalar product to obtain an equation in *a*, *b* and *c*, e.g. $3a + b - c = 0$	B1	
		Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for one ratio, e.g. $a : b$	M1	
		Obtain final answer $a : b : c = 1 : 4 : 7$ A1		
		Use coordinates of a relevant point and values of *a*, *b* and *c* in general equation and find *d*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*1: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1	
		Obtain two correct components	A1	
		Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $x + 4y + 7z = d$, or equivalent, and find *d*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*2: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a 2-parameter equation for the plane	M1	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(3\mathbf{i} + \mathbf{j} - \mathbf{k})$	A1	
		State 3 correct equations in *x*, *y*, *z*, *s* and *t*	A1	
		Eliminate *s* and *t*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*3: Using the coordinates of *A* and two points on *l*, state three simultaneous equations in *a*, *b*, *c* and *d*, e.g. $a + b + 2c = d$, $2a - b + 3c = d$ and $4a + 2b + c = d$	B1	
		Solve and find one ratio, e.g. $a : b$	M1	
		State one correct ratio	A1	
		Obtain a correct ratio of three of the unknowns, e.g. $a : b : c = 1 : 4 : 7$, or equivalent	A1	
		Either use coordinates of a relevant point and the found ratio to find the fourth unknown, e.g. *d*, or find the ratio $a : b : c : d$	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*4: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-2 & y+1 & z-3 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0$	A1	
		Attempt to expand the determinant	M1	
		Obtain or imply two correct cofactors	A1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	**[6]**