Cambridge International AS & A Level

**Cambridge International Examinations**
Cambridge International Advanced Subsidiary and Advanced Level

**MATHEMATICS** **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2015**

**1 hour 45 minutes**

*3195737770*

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.




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1 Express $2x^2 - 12x + 7$ in the form $a(x+b)^2 + c$, where $a$, $b$ and $c$ are constants. [3]

2 A curve is such that $\frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$ and the point $(4, 7)$ lies on the curve. Find the equation of the curve. [4]

3 (i) Write down the first 4 terms, in ascending powers of $x$, of the expansion of $(a-x)^5$. [2]

(ii) The coefficient of $x^3$ in the expansion of $(1-ax)(a-x)^5$ is $-200$. Find the possible values of the constant $a$. [4]

4 (i) Express the equation $3\sin\theta = \cos\theta$ in the form $\tan\theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

(ii) Solve the equation $3\sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

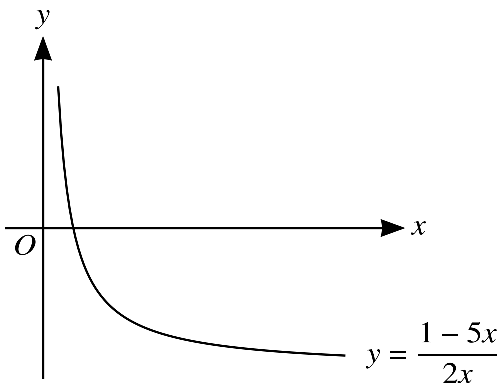
5 Relative to an origin $O$, the position vectors of the points $A$, $B$ and $C$ are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Show that angle $ABC$ is 90°. [4]

(ii) Find the area of triangle $ABC$, giving your answer correct to 1 decimal place. [3]

6




The diagram shows the graph of $y = f^{-1}(x)$, where $f^{-1}$ is defined by $f^{-1}(x) = \frac{1-5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $f(x)$ and state the domain of f. [5]

(ii) The function g is defined by $g(x) = \frac{1}{x}$ for $x \geqslant 1$. Find an expression for $f^{-1}g(x)$, giving your answer in the form $ax + b$, where $a$ and $b$ are constants to be found. [2]

**7** The point $A$ has coordinates $(p, 1)$ and the point $B$ has coordinates $(9, 3p + 1)$, where $p$ is a constant.

**(i)** For the case where the distance $AB$ is 13 units, find the possible values of $p$. [3]

**(ii)** For the case in which the line with equation $2x + 3y = 9$ is perpendicular to $AB$, find the value of $p$. [4]

**8** The function f is defined by $\mathrm{f}(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x > -1$.

**(i)** Find $\mathrm{f}'(x)$. [3]

**(ii)** State, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

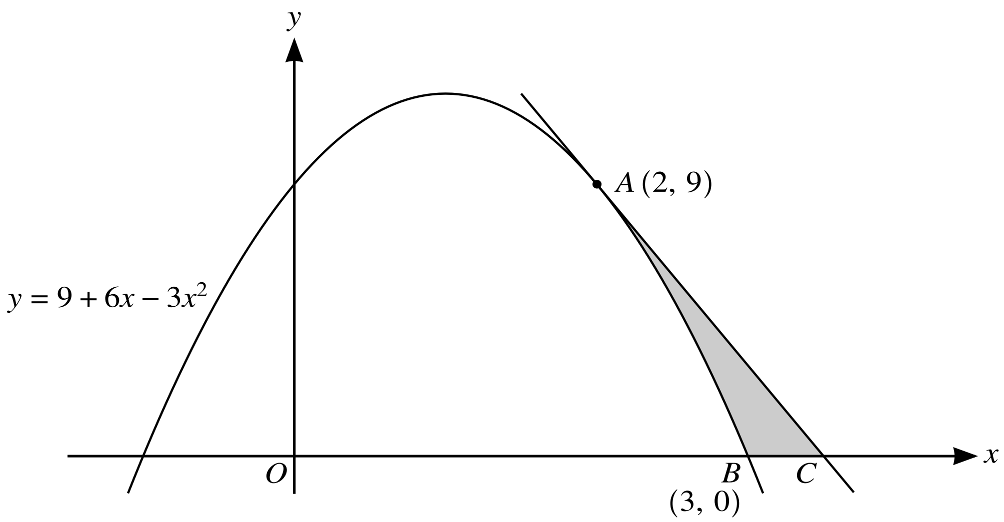
The function g is defined by $\mathrm{g}(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x < -1$.

**(iii)** Find the coordinates of the stationary point on the curve $y = \mathrm{g}(x)$. [4]

**9** **(a)** The first term of an arithmetic progression is $-2222$ and the common difference is 17. Find the value of the first positive term. [3]

**(b)** The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of $\theta$ for which the progression is convergent. [5]

**10**



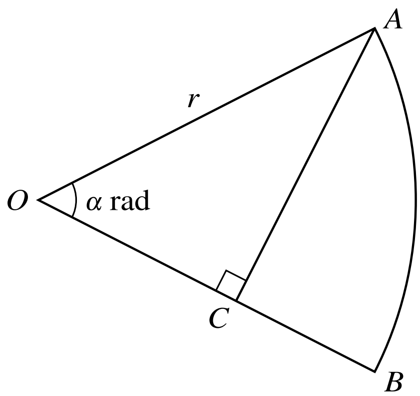

Points $A\,(2, 9)$ and $B\,(3, 0)$ lie on the curve $y = 9 + 6x - 3x^2$, as shown in the diagram. The tangent at $A$ intersects the $x$-axis at $C$. Showing all necessary working,

**(i)** find the equation of the tangent $AC$ and hence find the $x$-coordinate of $C$, [4]

**(ii)** find the area of the shaded region $ABC$. [5]

**[Question 11 is printed on the next page.]**

**11**




In the diagram, $OAB$ is a sector of a circle with centre $O$ and radius $r$. The point $C$ on $OB$ is such that angle $ACO$ is a right angle. Angle $AOB$ is $\alpha$ radians and is such that $AC$ divides the sector into two regions of equal area.

**(i)** Show that $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$. [4]

It is given that the solution of the equation in part **(i)** is $\alpha = 0.9477$, correct to 4 decimal places.

**(ii)** Find the ratio

perimeter of region $OAC$ : perimeter of region $ACB$,

giving your answer in the form $k : 1$, where $k$ is given correct to 1 decimal place. [5]

**(iii)** Find angle $AOB$ in degrees. [1]