If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
1 The independent random variables $X$ and $Y$ have standard deviations 3 and 6 respectively. Calculate the standard deviation of $4X - 5Y$. [3]

2 Cloth made at a certain factory has been found to have an average of 0.1 faults per square metre. Suki claims that the cloth made by her machine contains, on average, more than 0.1 faults per square metre. In a random sample of 5 m$^2$ of cloth from Suki’s machine, it was found that there were 2 faults. Assuming that the number of faults per square metre has a Poisson distribution,

(i) state null and alternative hypotheses for a test of Suki’s claim, [1]
(ii) test at the 10% significance level whether Suki’s claim is justified. [4]

3 In a golf tournament, the number of times in a day that a ‘hole-in-one’ is scored is denoted by the variable $X$, which has a Poisson distribution with mean 0.15. Mr Crump offers to pay $200 each time that a hole-in-one is scored during 5 days of play. Find the expectation and variance of the amount that Mr Crump pays. [5]

4 In the past, the flight time, in hours, for a particular flight has had mean 6.20 and standard deviation 0.80. Some new regulations are introduced. In order to test whether these new regulations have had any effect upon flight times, the mean flight time for a random sample of 40 of these flights is found.

(i) State what is meant by a Type I error in this context. [2]
(ii) The mean time for the sample of 40 flights is found to be 5.98 hours. Assuming that the standard deviation of flight times is still 0.80 hours, test at the 5% significance level whether the population mean flight time has changed. [4]
(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in your answer to part (ii). [2]

5 The volumes, $v$ millilitres, of juice in a random sample of 50 bottles of Cooljoos are measured and summarised as follows.

$$n = 50 \quad \Sigma v = 14,800 \quad \Sigma v^2 = 4,390,000$$

(i) Find unbiased estimates of the population mean and variance. [3]
(ii) An $\alpha\%$ confidence interval for the population mean, based on this sample, is found to have a width of 5.45 millilitres. Find $\alpha$. [4]

Four random samples of size 10 are taken and a 96% confidence interval for the population mean is found from each sample.

(iii) Find the probability that these 4 confidence intervals all include the true value of the population mean. [2]
The waiting time, $T$ minutes, for patients at a doctor’s surgery has probability density function given by

$$f(t) = \begin{cases} \frac{k(225 - t^2)}{2250} & 0 \leq t \leq 15, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{1}{2250}$. [3]

(ii) Find the probability that a patient has to wait for more than 10 minutes. [3]

(iii) Find the mean waiting time. [4]

In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable $X$ denotes the number of prize-winning tickets that have been sold.

(i) State, with a justification, an approximating distribution for $X$. [3]

(ii) Use your approximating distribution to find $P(X < 4)$. [3]

(iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geq 1$. [4]