

MATHEMATICS

Paper 9709/11

Paper 11

Key messages

- In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- It is important for candidates to not spend too much time on any one question.

General comments

The paper allowed the majority of candidates to show what they had learned and understood. It was particularly the case in this paper that quite a number of questions contained a straightforward part, or parts, followed by a part that was more searching. This allowed even weaker candidates to score a reasonable number of marks and also gave stronger candidates the opportunity to demonstrate their understanding and ability on the more demanding parts of questions.

Comments on specific questions

Question 1

Most candidates were able to identify which term was independent of x . Care needs to be taken with the use of brackets, as some candidates had $2x^4$, instead of $(2x)^4$, leading to an incorrect answer of 30 instead of a correct answer of 240.

Answer: 240.

Question 2

Almost all candidates differentiated correctly, but few candidates realised that all that was needed was to transform the result to $(3x - 2)^2$ which is never negative.

Answer: $(3x - 2)^2 \geq 0$.

Question 3

Many candidates were able to sketch the graphs correctly. Others were not able to draw the correct cosine curve. Most candidates correctly sketched the graph of $y = 1/2$. Fewer candidates deduced that the answer to part (ii) was the number of intersections of the two graphs, or that the answer to part (iii) was 5 times the answer to part (ii).

Answers: (ii) 4; (iii) 20.

Question 4

Most candidates were successful in part (i). In part (ii), however, whilst the majority of candidates integrated correctly many candidates did not include an arbitrary constant and progressed no further. Other candidates thought that $f(0) = -4$ rather than $f(3) = -4$.

Answers: (i) 3; (ii) $x^2 - 6x + 5$.



Question 5

Candidates generally answered part (i) well. Part (ii) was also well done although a number of candidates lost accuracy in the final answer by not carrying sufficient significant figures in their answers for the areas of OAB and OCB . In general, if the final answer is required to be given correct to 3 significant figures, candidates should carry 4 significant figures in calculations leading to the answer.

Answers: (i) $r + r\theta + r\cos\theta + r\sin\theta$; (ii) 55.2.

Question 6

This question was answered well. There was considerable improvement in this topic compared to previous years. In both parts of this question candidates were able to obtain the majority of the marks available.

Answers: (a) 29; (b) $r = 4/5$, $S = 5$.

Question 7

Most candidates were able to complete part (i) successfully and to use their answer in part (ii). In part (ii) a number of candidates made errors, such as $A = 4xy + 3xy$, in expressing the area in terms of x and y and so did not reach the given answer. In part (iii) it was very common to see the derivative equated to zero giving, correctly $x = 3$, without taking the necessary further step of finding the corresponding value of the area. Many candidates found the second derivative to be negative and correctly stated that the area was a maximum. Candidates who used other methods for determining the nature of the stationary point, such as considering the sign of the gradient either side of the turning point, did not always give sufficient detail in their working – particularly since the answer ‘maximum’ was given in the question paper. In future, candidates should be advised to find the *actual* values of the gradient at two particular points rather than just to indicate, for example, ‘positive’ and ‘negative’.

Answers: (i) $y = \frac{1}{6}(48 - 8x)$; (ii) $A = 48x - 8x^2$; (iii) maximum area = 72m².

Question 8

In part (i) many candidates gave the answer to the scalar product as a vector quantity involving $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. Candidates making a mistake in the scalar product often may little progress with the rest of the question. Relatively few candidates were successful with part (ii). Some method marks were often awarded in part (iii) but, again, relatively few candidates reached the correct final answer. Some candidates omitted the ‘ $\pm$ ’ at the final stage.

Answers: (i) $25 + p^2$; (ii) $25 + p^2 = 0$ has no real solutions; (iii) $p = \pm\sqrt{15}$.

Question 9

Part (i) was done particularly well with large numbers of candidates scoring 4 or 5 marks. Part (ii) required, first of all, simplification into a 3-term quadratic equation followed by application of the condition for equal roots. Many candidates found this section challenging, and many did not attempt this section at all.

Answers: (i) $\sqrt{45}$, $(-1/2, 5)$; (ii) 3, 11.

Question 10

Most candidates obtained the correct answers for part (i). Part (ii) was also done well with most of the candidates who were able to differentiate accurately progressing to the correct final answer. A number of candidates, on differentiating, did not multiply by the derivative of the bracket $(1+2x)$. In part (iii) a large numbers of candidates, when finding the volume, rotated about the x -axis instead of the y -axis.

Answers: (i) $(0, 1), (4, 3)$; (ii) $y = -3x + 15$; (iii) $\frac{2}{15}\pi$.

Question 11

Part (i) was reasonably well done. Some candidates still do not appear to be confident in applying this process. Obtaining correct answers for parts (ii) and (iii) was a challenge for many candidates. In part (iv) most candidates obtained the correct sketch graph of $y = g(x)$, but few candidates were able to sketch correctly the half-parabola of $y = f(x)$ or the reflection of this in $y = g(x)$ for $y = f^{-1}(x)$. In part (v) the correct method was employed by many candidates. Candidates need to make sure the ' $\pm$ ' is not forgotten when taking the square root, as this omission can lead to the incorrect answer.

Answers: (i) $2(x-2)^2 + 2$; (ii) $2 \leq f(x) \leq 10$; (iii) $2 \leq x \leq 10$; (v) $2 - \sqrt{\frac{x-2}{2}}$.

MATHEMATICS

Paper 9709/12

Paper 12

Key messages

- Candidates need to read the questions carefully, and only draw accurate graphs when required.
- Working needs to be presented in a clear and readable form.
- Candidates need to take care to avoid arithmetic and algebraic errors.

General comments

The paper enabled candidates to show what they had learnt and what they had been taught. There were few poor scripts and there were sufficient testing parts of questions to differentiate between the more able candidates. The standard of numeracy and algebra was generally good though misuse of brackets, as in **Question 8**, presented problems. It was still common to see candidates spending time in drawing accurate graphs when only a sketch was needed. The presentation of answers was generally good. In questions requiring a proof (as in **Questions 5 and 6**), candidates should take extra care in the showing of all necessary working.

Comments on specific questions

Question 1

- (i) This was an extremely well answered question, with the use of binomial coefficients being particularly accurate.
- (ii) Less than a half of all candidates realised the link between parts (i) and (ii) and there were a large number of candidates who attempted to expand $(2 - 2x + x^2)^5$ or even $(2 - 2x - x^2)^5$. Those substituting $y = 2x - x^2$ into the answer to part (i) were usually correct, though a significant number still failed to realise the need to consider the sum of two terms.

Answers: (i) $32 - 80y + 80y^2$; (ii) 400

Question 2

This was a well answered question.

- (i) Most candidates realised the need to set “ x ” as “ $3x+a$ ” to find $ff(x)$, but the answer $3(3x + a)$ instead of $3(3x + a) + a$ was common. Candidates were more accurate in finding b , either by finding an expression for the inverse or solving the equation $g(3)=0$ directly.
- (ii) This part was well answered, though occasionally candidates failed to substitute their values of a and b to obtain a linear expression. Only rarely was gf used instead of fg .

Answers: (i) $a = -2, b = 8$; (ii) $22 - 6x$



Question 3

- (i) Most candidates used the scalar product accurately and set this to 0, though -1 and 90 were common errors. Unfortunately many candidates produced a mass of working through not appreciating that $\frac{a}{b} = 0$ implies that $a = 0$.
- (ii) Few attempts to this part were correct. The idea of a unit vector still presents difficulty and the concept of multiplying the unit vector by 28 posed a challenge to many candidates.

Answers: (i) $-8\frac{1}{2}$; (ii) $-12\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}$

Question 4

- (i) This was a source of high marks with candidates accurately obtaining, and then solving, a quadratic equation in x or in y .
- (ii) Several candidates failed to realise the need to obtain a quadratic equation in x or in y and then to set " $b^2 - 4ac$ " to 0 . Algebraic errors in the coefficients of a , b and c were common, particularly with those candidates taking the longer route of obtaining a quadratic equation in x . A small minority attempted the alternative method of equating gradients, but these were rarely correct.

Answers: (i) $(2, 3), (6, 1)$; (ii) $8\frac{1}{2}$

Question 5

- (i) Despite many curves being too straight or $y = \cos 2x$ being parabolic, most candidates obtained some marks for this part and most obtained the correct end-points for both curves. Many candidates unnecessarily drew accurate graphs and wasted a considerable amount of time.
- (ii) Stating that $x = 30^\circ$ because it looks approximately correct on a sketch graph gained no credit. Candidates should realise that "verify" implies showing that $\sin 30^\circ$ and $\cos 60^\circ$ have the same value. Very few candidates realised that the other root (by symmetry or otherwise) was 150° .
- (iii) Few solutions were able to use the sketch graphs to obtain the correct range.

Answers: (i) Sketch; (ii) 150° ; (iii) $0^\circ \leq x < 30^\circ, 150^\circ < x \leq 180^\circ$

Question 6

- (i) Many candidate lost a mark through not explaining why the height was $3\sqrt{3}$. In a question requesting "show", it is insufficient to state that $6\sin 60 = 3\sqrt{3}$.
- (ii) Although a few candidates mistakenly used "arc length" for "sector area", the majority used the formula $s = r\theta$ correctly to find the arcs DX and EX . Finding the length of DE proved difficult with the majority failing to realise that $DE = 16 - 6\cos 60 - 10\cos \theta$. The final accuracy mark was also often lost through not stating the answer to the accuracy requested.

Answers: (i) Proof; (ii) 16.20

Question 7

- (i) Whilst the majority of candidates realised the need to find the gradient of the line and to equate the product of this and $\frac{dy}{dx}$ to -1 , there were many errors. It was particularly common to see the gradient of $3y + x = 17$ quoted as -1 or as $\frac{1}{3}$ and many incorrectly assumed that the gradient of the normal was $\frac{dy}{dx}$. Setting $\frac{dy}{dx}$ to 0 or to -3 instead of 3 was common.
- (ii) This was well answered with many correct answers. A significant minority however still omitted the constant of integration.

Answers: (i) (2, 5); (ii) $y = 5x + \frac{8}{x} - 9$

Question 8

- (i) Although most candidates realised that the function was composite, many multiplied $\frac{1}{2}(8x - x^2)^{-\frac{1}{2}}$ by $8 \times 2x$ instead of $(8 - 2x)$. Misuse of the bracket around $8 - 2x$ was a common error.
- (ii) A large number of candidates integrated either πy or y instead of πy^2 . For those integrating y^2 , the integration was generally accurate. Few candidates were able to deduce that the limits for integration were the values for which $8x - x^2$ was 0. Using the incorrect limits of 0 to 4 was common.

Answers: (i) (4, 4); (ii) $\frac{256\pi}{3}$

Question 9

- (i) Most candidates found the equations of AC and BD correctly and proceeded to find the coordinates of M and D . A significant number however assumed that M was the mid-point of AC . A small minority took D to have coordinates (x, y) and used $AB = AD$ and $BC = CD$ to obtain 2 quadratic expressions in x and y . This led to D from which M could then be deduced. Finding D from M or M from D was accurately done.
- (ii) The majority of candidates obtained AM and MC as $\sqrt{45}$ and $\sqrt{20}$ respectively, but many of these stated the ratio as $45 : 20$, instead of $3 : 2$. Very few used "vector steps" to deduce the ratio $3 : 2$.

Answers: (i) $M(5, 2)$, $D(7, -2)$; (ii) $3 : 2$

Question 10

- (a) Parts (i) and (ii) were well answered, though a small percentage of candidates interchanged the formulae for the n th term and the sum of n terms. Part (iii) caused many problems. Many candidates realised that 0 was one of the terms of the progression and that the first positive term was 3. However, using $a = 0$ with $n = 19$ (instead of 20) or $a = 3$ with $n = 20$ (instead of 19), were very common errors. Most candidates obtained $n = 19$ or 20, but then incorrectly used $a = -15$. Finding the sum of the negative terms $(-15, -12, -9, -6, -3 = -45)$ was common but many of these candidates then used the answer $525 - 45$, instead of $525 - (-45)$.
- (b) Several candidates incorrectly assumed that the question required the use of the arithmetic progression. Others calculated r as 0.05 or as 1.5 instead of 1.05. The most common error, however, was to use ar^{11} instead of ar^{10} in part (i) and to give the answer to part (ii) as $\frac{4000(1.05^{10} - 1)}{1.05 - 1}$ instead of $\frac{4000(1.05^{11} - 1)}{1.05 - 1}$.

Answers: (a) (i) 3; (ii) 57; (iii) 570; (b) (i) \$6520; (ii) \$56,820



MATHEMATICS

Paper 9709/13

Paper 13

Key messages

- Candidates need to understand that only non-exact numerical answers should be given to 3 significant figures, or 1 decimal place in the case of angles, so 36 need not be followed up with ≈ 36.0 .
- Candidates need to ensure that sufficient working is seen for 'show that' answers. For example candidates who use their calculators to factorise quadratics need to realise that full credit will not be awarded when the answer is given in the question.
- Any errors which occur in using a calculator to integrate will mean that no credit can be awarded if no working is shown.

General comments

Most candidates made a reasonable attempt at the paper and on the whole the standard of presentation was good. Weaker candidates were able to gain significant marks in **Question 1**. Yet, many lost marks by not reading the questions in detail, or taking time to calculate unnecessary terms. In **Questions 7** and **10**, diagrams could have benefited many candidates by clarifying their answers.

Comments on specific questions

Question 1

Candidates generally gained full marks on this question. Occasionally they forgot to square the $\frac{1}{3}$ even though the term was usually expressed correctly in their initial expansion. Their nCr notation where used was usually interpreted correctly. Some candidates occupied unnecessary time in writing out the expansion in full.

Answer: 3

Question 2

This was one of the better answered questions. Even weaker candidates obtained considerable marks on this question. Only occasionally candidates worked out the 10th term rather than the sum. Usually a and d had their correct values in part (i) and a and r in part (ii). Some knew the formula for the geometric progression but used $r=4$.

Answers: (i) 220, (ii) 4092

Question 3

In part (i) most candidates correctly equated the equations but then usually divided by x before or after rearranging to zero instead of calculating factors, and concluding that the quadratic factor therefore equalled zero, being the required result. In part (ii) most candidates recognised that they were dealing with a quadratic in x^2 and factorised it using a substitution for x^2 . Many then forgot to substitute back and thought that $+\frac{1}{2}$ and -2 were the required solutions. Of those who did substitute back, some did not do $\pm$ or thought $\sqrt{-2}$ was a solution, this then becoming $-\sqrt{2}$. Others find y from either their one or two values of x . The question states the need for exact solutions; this means that use of calculators and corresponding decimal solutions are not appropriate.

Answers: (ii) $(1/\sqrt{2}, 2/\sqrt{2})$, $(-1/\sqrt{2}, -2/\sqrt{2})$



Question 4

Many completely correct solutions were seen and the need to use $\frac{1}{2}r^2\theta$ in part (ii) and $r\theta$ in part (iii) was generally realised. Those candidates who did not to score full marks had most problems with part (i). Usually BC (or AD) was found using trigonometry and then incorrectly used as the height of the parallelogram. Some candidates found the area of one triangle but then omitted to double it. A small number of candidates converted the radians to degrees before calculating the sine, but most coped well with radian measure, and remembered the formulae accurately.

Answers: (i) 71.7, (ii) 80, (iii) 36

Question 5

In part (i) a large majority of candidates used the Pythagorean identity correctly, with very few errors. Candidates with equation solvers on their calculators did not justify their answers by giving the breakdown into brackets or an equivalent method. Most however showed the working clearly, and explained why the second solution was not needed. A very small number attempted verification but were usually unsuccessful. Occasionally carelessness was seen with $\cos x$ reduced to just $\cos$ or for candidates who solved the quadratic in $\cos x$ by replacing $\cos x$ with y ended up with $y = -2/3$ and did not complete the proof by stating that $\cos x = -2/3$. In part (ii), often the link between x and $\theta + 70$ was not recognised with candidates attempting to manipulate the new equation rather than simply state that $\cos(\theta + 70) = -2/3$. Those who saw the link usually obtained the first solution – though other candidates at this stage added 70 instead of subtracting it. The more able candidates recognised the need to look for a further solution.

Answers: (ii) 61.8, 158.2

Question 6

In part (i) the method of using the scalar product to find the angle between two vectors was familiar to virtually all candidates and most picked up full marks although a few approximated prematurely.

Part (ii) proved more testing. Though there were a good number of fully correct responses, most candidates thought OC was $\frac{1}{2}AB$ and hence used $\frac{1}{2}(b-a)$ or $\frac{1}{2}(a-b)$ instead of $\frac{1}{2}(a+b)$. They did however usually correctly put their expression for OC along with $2OD$ to find an expression for DC (or CD). Diagrams were rarely seen and a simple triangle OAB with C on AB and D on OB produced would have been helpful.

Answers: (i) 71.4, (ii) $-6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$

Question 7

This question proved very testing for many candidates and it was rare to see fully correct solutions. The equation of the line in part (i) was usually correct; weaker candidates were confused by having the non-numeric value of m for the gradient or interchanged the values of x and y . Most candidates started part (ii) correctly by equating the equations of their line and the curve and rearranged to form a quadratic in x though errors in the signs of one or more terms were often made. The need to use $b^2 - 4ac$ on the quadratic was generally recognised but for those with an incorrect rearrangement this then resulted in non-exact values for m and generally stopped at this point. Many of those who did end up with the correct pair of values for m did not then use their values to find the point where each line touched the curve. Some candidates arrived at a correct point(s) but from invalid methods. Using $y = dy/dx$ for the curve was a common misconception and although it gave one of the required points could not be credited. Another misunderstanding was to take the value for m from $2x - 4$ and assume m was equal to 2. Completing the square in part (iii) was done incorrectly by a number of candidates with $(x-2)^2 - 4 + 5$ sometimes becoming $(x-2)^2 - 1$.

Answers: (i) $y=m(x-2)$, (ii) (3, 2), (1, 2), (iii) $(x-2)^2 + 1$, giving minimum at (2, 1)

Question 8

Many candidates were confident in interpreting the information given in the question and gained full marks. A number of candidates did not establish the initial requirement that k was equal to -2 because they did not realise that the value 3 needed to be substituted into the first derivative, but they often then went on to gain the rest of the marks. Occasionally, there was a little confusion over which of the two values of x from the quadratic referred to point P and which to point Q . In part (iii), most candidates knew to integrate $f'(x)$ to get back to $f(x)$ and the integration was invariably done correctly. Many candidates forgot to include a constant of integration in part (iii). Those who did include it invariably knew how to find its value.

Answers: (i) -2 , (ii) min when $x = 3$ and max when $x = -2$, (iii) $f(x) = \frac{2}{3}x^3 - x^2 - 12x + 17$

Question 9

Most candidates were familiar with functions and picked up most of the marks available on this question. The notation for an inverse function was usually recognised in part (i); a few candidates misunderstood this as $1/f(x)$ and a few differentiated. Most candidates had no problem in finding the inverse of the linear function or solving it with the original equation. In part (ii) most candidates confidently sketched the two linear graphs, but not necessarily over the correct domains. Some then did not state the coordinates of the point of intersection as specified in the question or omitted to show the relationship between the graphs (by either not showing the mirror line or not stating its equation) or both. In part (iii) the concept of a compound function was generally known and the majority of candidates correctly tried to find $gf(x)$ as required though many only applied $f(x)$ to the x^2 term in $g(x)$ and forgot to apply it to the $-6x$ term. The resulting quadratic was usually set to ≤ 16 and the quadratic solved. Whilst the majority of candidates gave two solutions (some only stated the positive root) many thought that the required solutions were outside the pair of values obtained rather than between them. The need to also consider the given domains for f and g was generally not realised and it was rare to see a correct final solution. A large number of candidates had attached graph paper and had therefore plotted an accurate graph for part (ii). A few candidates who did this did not relate their answer to part (i) to this graph and quoted the point of intersection as -3.2 as their graph was not very accurate. Candidates who sketched the graph on lined or plain paper were more likely to draw the line of reflection and label the point of intersection. Those who had plotted it often relied on having labelled -3 on the two axes to score this mark.

Answers: (i) $\frac{1}{2}(x-3), -3$, (iii) $-\frac{5}{2} \leq x \leq 0$

Question 10

Most candidates made a good attempt at part (i). The curve was usually integrated correctly with correct limits almost always used. The simplest method for the area of the triangle was sometimes seen but candidates often preferred to integrate the equation of the line. Many who elected to integrate $(y_2 - y_1)$, rather than to integrate y_2 then minus the integral of y_1 , made a sign error. There were a good number of fully correct solutions in part (ii). Many candidates either mis-read the question or did not understand that rotating about different axes would give different volumes and rotated the curve and line about the x axis. Of these, some correctly followed through by using the x limits. Others incorrectly used the y limits. There were no diagrams to indicate the relevant strips of area (in the x or y direction) which the candidate was using. The candidates should be encouraged to do so in order to clarify their thinking. A number of candidates tried to integrate $(y^2 - 1)^2$ without expanding it first.

Answers: (i) $\frac{1}{6}$, (ii) $\frac{1}{5}\pi$

MATHEMATICS

Paper 9709/21

Paper 21

Key messages

- The paper sets out to test candidates' knowledge of the rules and results of the calculus and also their grounding in algebra, logarithmic theory, solution of pairs of simultaneous linear equations and general manipulative skills.
- Candidates need to take care to avoid arithmetic and sign errors.
- It is important for candidates to not spend too much time on any one question.

General comments

Results covered the full mark range from 0 to 50. **Questions 1, 5 and 6** were answered well. **Questions 3, 7, 8** were poorly answered by candidates. There were a high proportion of good solutions to **Question 4**. Though **Question 8(i)** posed few problems for candidates, many did not use the result from part (i) to convert the integral in part (ii) to a simple form. Many candidates attempted a direct integration of $\cos^3 x$. The differentiation in **Question 7(i)** was a challenge for many candidates.

Comments on specific questions

Question 1

This question was well answered by candidates. Some sign and/or arithmetic errors caused problems in solutions. Some candidates showed that $25x^2 - 40x + 7 = 0$, produced the critical values, but then were challenged by factorisation the left-hand side. Other candidates set $(4 - 5x) = 3$, thus yielding only one critical value.

Answer: $\frac{1}{5} < x < \frac{7}{5}$

Question 2

Almost all candidates started this question correctly, getting a term in $k \ln(4x + 1)$, with $k = 1, 2$ or the correct result $k = \frac{1}{2}$. After substituting most candidates could simplify the contents of the bracket to the form $(\ln 25 - \ln 9) = \ln(5/3)^2$. Other candidates opted to work from the given result and to move to the latter form without any intermediate working. Using $(\ln a - \ln b) = \ln(a/b)$ and $\ln c^2 = 2 \ln c$ had to be explicitly shown as parts of the simplification process.

Question 3

Many candidates obtained a derivative of the form $k \sec^2 2x$, with $k = 1$, the correct form, or $k = \frac{1}{2}$. Solving the equation $\sec^2 2x = 4$, in the form $\cos^2 2x = \frac{1}{4}$, proved a challenge for many candidates, who found only the two values of $2x$ or a single value of x . Candidates ranged over the entire marks from 0 to 5.

Answer: $\pi/6, \pi/3$



Question 4

Almost all candidates saw that the equation could be re-written as $y^2 - 7y + 10 = 0$, where $y = 3^x$, and found that $y = 2, 5$ were the solutions. Some candidates did not take logarithms to obtain $x \ln 3 = \ln 2, \ln 5$ and hence the final result.

Answer: 0.631, 1.46(5)

Question 5

This was a well answered question. Almost all candidates set $p(\frac{1}{2}) = 10$, though calculating x^3 and x^2 at $x = \frac{1}{2}$ sometimes resulted in errors. Arithmetic and sign errors sometimes resulted in an incorrect value for a . This could have been spotted as part (i) required proof that $p(3) = 0$.

In part (ii), even those with an incorrect result in part (i) could gain method marks for attempting to divide the original cubic equation by $(x - 3)$, or to factorise it in the form $(x - 3)(4x^2 - \alpha x - 3)$, where α a constant.

Answer: (i) -16 (ii) $x = 3, -\frac{1}{2}, \frac{3}{2}$.

Question 6

- (i) Almost all candidates calculated $f(0.7)$ and $f(0.3)$, where $f(x) \equiv x^3 - 2x^2 + 5x - 3$, showing $f(0.7), f(0.8)$ had different signs, and hence the result.
- (ii) This part was a challenge for many candidates who either made no progress or were unable to re-write the cubic equation of part (i) into the required form in part (ii)
- (iii) Those candidates with incorrect values from part (ii) were unable to progress with part (iii), as they were using the wrong iterative formula. Those who succeeded in (ii) were able to answer part (iii). Many candidates lost accuracy in the iterations, did not perform sufficient iterations, or did not round their final answer to 2 decimal places.

Answer: (ii) $a = 2, b = 5$; (iii) 0.74

Question 7

- (i) A number of candidates did not see that $\frac{dx}{dt} = 3e^{3t}$ and/or $\frac{dy}{dt} = t^2 e^t + 2te^t$. Almost everyone correctly stated that $dy/dx = dy/dt \div dx/dt$.
- (ii) Few candidates fully explained that y' must be zero, and that this occurs when $t = 0$ ($x = 1, y = 3$) making the tangent parallel to the x -axis.
- (iii) $y' = 0$ also at $t = -2$, and hence the exact values of $x(-2)$ and $y(-2)$ followed.

Answers: (iii) $(e^{-6}, 4e^{-2} + 3)$

Question 8

- (i) Few problems were encountered with this question. It was crucial to use (i) to re-write the integrand in the form $(\frac{1}{2} \cos 3x + \frac{3}{2} \cos x - \cos x)$. Some candidates did not use the result from (ii) and found a variety of incorrect forms for $\int 2 \cos^3 x \, dx$, for example $\frac{1}{2} \cos^4 x$.

Answers: (ii) $5/12$



MATHEMATICS

Paper 9709/22

Paper 22

Key messages

- The paper is a test of candidates' knowledge of the rules and results of calculus, and their proficiency in both algebra and logarithmic theory and general manipulative skills.
- Care needs to be taken to avoid arithmetic and sign errors.
- Candidates need to be careful to read the question in detail and answer as indicated.
- Attention needs to be paid to making sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answer.

General comments

Many problems were encountered in **Questions 3, 4 and 6**, whereas **Question 7** was well answered by all strengths of candidates. **Questions 1, 5 and 8 (i)** were also generally well answered.

Comments on specific questions

Question 1

Candidates scored well on this question. Almost all candidates squared on each side of the equation and obtained $(3/4)x^2 + 6x = 0$, to give the solutions for x . Some candidates missed the solution $x = 0$ and/or were unable to correctly find the non-zero solution. Some candidates simply set $x + 2 = (1/2)x - 2$ to obtain a single solution $x = -8$ as a critical value.

Answer: $x < -8, x > 0$.

Question 2

Many candidates started this question well. The first two of the four marks were usually scored by correctly taking logarithms. The later two marks were often lost due to arithmetic or sign errors.

Answer: 3.39

Question 3

- (i) Many candidates were challenged by the differentiation of $y(x)$. The errors were mainly related to the differentiation of $\ln x$.
- (ii) Few correct answers to this question were seen. Many candidates, instead of using the stated trapezium rule, tried to integrate the expression directly. Another tendency was to use five ordinates, adding $x = 1$ to the four correct values many others used only two intervals, with $x = 2, 3.5$ and 5 .
- (iii) Candidates invariably referred to the concave or convex nature of the curve, not mentioning that the tops of the latter lie above the actual curve.

Answers: (i) 2; (ii) 3.23; (iii) over-estimate.



Question 4

Few candidates answered this question correctly. Some candidates substituted the limits into the integrands, rather than integrating. The candidates who integrated correctly obtained the equation $e^{4k} - 4e^{2k} + 3 = 0$. Few candidates recognised that it was a quadratic equation in $y = e^{2k}$. Many solutions were based on the false premise that taking logarithms of the lefthand side gave $4k - 4.2k + \ln 3 = 0$.

Answer: $1/2 \ln 3$

Question 5

- (i) Many candidates were unable to draw good quality graphs. Many candidates drew $y = 1/x$ as a straight line. An omitted horizontal scale made it impossible to judge candidates graphs of $y = \sin x$.
- (ii) Almost all candidates correctly evaluated $f(1.1)$ and $f(1.2)$, where $f(x) \equiv \pm (1/x - \sin x)$, noted the difference in their signs, and proved the proposition.
- (iii) Many good attempts to this question dropped marks by working the iterations to too few decimal places, by not iterating sufficiently often, or by not rounding answers to 2 decimal places.

Answers: (iii) 1.11

Question 6

- (i) Many candidates were challenged by the differentiation of $(1 + 2\sin 2\theta)$ to give correctly $4 \sin \theta \cos \theta$; $dx/d\theta = 2 \cos 2\theta$ was a common error. Other candidates struggled to correctly differentiate $y = 4 \tan \theta$. Almost all candidates correctly set dy/dx to $dy/d\theta \div dx/d\theta$.
- (ii) Many candidates did not obtain correct values for x , y and dy/dx when $\theta = \frac{1}{4}\pi$. The value of $\sin \pi/4$ and $\cos \pi/4$ were often incorrectly given.

Answers: (ii) $y = 4x - 4$

Question 7

- (i) Most candidates realised that $p(-2) = 0$ and that $p(-1) = 12$, though weaker candidates sometimes lost the 12. Sometimes sign or arithmetic errors resulted in calculations giving incorrect values of a and b .
- (ii) This part was well answered by most candidates. Even candidates who obtained incorrect values in (i) were able to score method marks in this part. Some candidates wrongly tried to divide $p(x)$ by $(x + 1)$ or $(x + 1)(x + 2)$.

Answers: (i) $a = 2$, $b = 6$; (ii) $(x + 2)(2x - 1)(x - 3)$.

Question 8

- (i) This part was well answered by candidates. A few candidates stated the approximate value 5.83. Some others had $\tan \alpha = 5/3$ instead of $3/5$.
- (ii) Many candidates found at least one of the two values of θ . Some incorrectly added 180° to this to get a second solution or took 15.7° from 360° .
- (iii) Few candidates noted that $-3R$ was needed.

Answers: (i) $R = \sqrt{34}$, $\alpha = 30.96^\circ$; (ii) 15.7° , 282.3° (or 4); (iii) $-3\sqrt{34}$



MATHEMATICS

Paper 9709/23

Paper 23

Key messages

- To do well on this paper, a sound knowledge of calculus (rules and results for differentiating and integrating) and of manipulative skills in arithmetic and algebra are required.
- It is important for candidates to not spend too much time on any one question or part of question.
- Answers should be tidy and legibly presented in the examination.
- Candidates should check working to ensure high accuracy and to avoid arithmetic errors.

General comments

Candidates scored well with **Questions 1, 2, 6, 7 (ii) and (iii) and 8 (i)**. More challenges were found in **Questions 3, 4(i) and (ii), 7 (i) and 8 (ii)**. Arithmetic and algebraic errors reduced scores in many solutions, which were initially well attempted. In questions, such as **2, 3, 5 and 6**, the answers were well started but sign and numerical errors reduced the overall marks gained. In the later questions candidates knew how to address the problem(s) posed, but some lacked confidence in their answers. In **Questions 6, 7 (ii) and (iii), and 8(i)** few problems were encountered, and generally very accurate solutions were seen.

Comments on specific questions

Question 1

This question was generally solved without problems by candidates. Some candidates did not use the chain rule, thus losing the factor 5 in the numerator of the gradient.

Answer: $5/21$

Question 2

Almost everyone squared each side and solved the resulting quadratic equation in x , namely $5x^2 + 12x - 9 = 0$. Some candidates made arithmetic or sign errors resulting in different equations to be solved. Some candidates were challenged by solving the correct equation. Weaker candidates simply set $2x - 3 = 3x$ to obtain $x = -3$ only.

Answer: $x \leq -3, x \geq \frac{3}{5}$

Question 3

Many candidates did not state that $2 \ln(x + 3) = \ln(x + 3)^2$ and/or use the rule $\ln a - \ln b = \ln(a/b)$. Candidates who obtained a correct quadratic equation almost always solved it correctly. Some retained the false solution $x = -1$

Answer: $x = 9$



Question 4

- (i) Many candidates were challenged by this part of the question. Around half of candidates answered correctly, using the formula $\cos 2x \equiv 2 \cos^2 x - 1$.
- (ii) This part was dependent on obtaining a correct answer to part (i). Some candidates struggled to obtain a reasonably correct form for the integral; often not scoring the later method marks.

Answers: (i) $\frac{1}{2}(1 + \cos 2x)$; (ii) $\left(\frac{\pi}{12} + \frac{1}{4} + \frac{\sqrt{3}}{8}\right)$ from $\left[\frac{\sin 2x}{4} + \frac{x}{2} - \frac{1}{2} \cos 2x\right]_0^{\pi/6}$

Question 5

Most candidates used $\sec^2 2\theta = 1 + \tan^2 2\theta$, and set up and solved the resultant quadratic equation in $\tan 2\theta$. Not all four solutions for θ were always found. Some candidates incorrectly set $\sec^2 2\theta = (1 - \tan^2 2\theta)$ or $(\tan^2 2\theta - 1)$.

Answer: $22\frac{1}{2}^\circ$, 70.7° , 112.5° , 160.7°

Question 6

- (i) This question was answered well by almost all candidates. Some candidates made sign errors in ax^3 and bx when $x = -2$. Occasionally, sign or arithmetic errors meant that the 2 correct simultaneous linear equations in a and b were not solved correctly.
- (ii) Candidates made few errors in this part. Most candidates used algebraic long division, or 'spotting' of terms in the quotient by inspection.

Answers: (i) $a = 3$, $b = -5$; (ii) $x^2 + 2x - 1$.

Question 7

- (i) Many candidates were challenged by this part. Some did not use the rule for differentiating a product, or lost the $\frac{1}{2}$ factor in the derivative of $e^{\frac{1}{2}x}$.
- (ii) Most candidates focused on finding $f(3.1)$ and $f(3.3)$ where $f(x) \equiv \pm(x - 2 - 6e^{-\frac{1}{2}x})$, and noted the difference in their signs. Some candidates struggled to make progress.
- (iii) Candidates answered this question well. Some solutions were not rounded to 2 significant figures, or insufficient iterations were performed.

Answers: (iii) 3.21

Question 8

- (i) This part was answered well by almost all candidates. A few candidates did not seem to know $d(y^2) = 2ydy$.
- (ii) This question was well answered by candidates apart from a few sign errors. Candidates were initially required to calculate $4x - 3 = 2y - 3$, and so note that $y = 2x$ should be set into the original equation in part (i). Not all candidates noted this. The resultant quadratic equation in x (or y) must then be solved.

Answers: (ii) $(-1/2, -1)$ and $(2, 4)$

MATHEMATICS

Paper 9709/31

Paper 31

Key messages

- Candidates should check working to ensure high accuracy and to avoid arithmetic errors.
- It is important for candidates to not spend too much time on any one question.
- It is important to read questions in detail and to be careful when copying expressions into answers.

General comments

The standard of work on this paper varied between candidates resulting in a spread of candidate marks from zero to full marks. The questions that were most successfully answered were **Question 5** parts (ii), (iii) and (iv) (iteration), **Question 8** (partial fractions) and **Question 9** (calculus). Those that were least well answered were **Question 4** (differential equation), **Question 7** (vector geometry) and **Question 10** (complex numbers).

Candidates presented their work well and appeared to allocate time appropriately between the questions. Some candidates need to be careful to avoid errors when copying expressions either from the question paper or from their own work. Miscopying of both kinds was most common in **Question 2**, **Question 7**, **Question 8** and **Question 10 (i)**. Candidates also need to be careful to use brackets correctly. Errors caused by incorrect use of brackets were found in **Question 4** and **Question 8**.

Comments on Specific Questions

Question 1

This question was well answered by candidates. Most candidates derived a correct quadratic equation in e^x or in u , rejected the negative root and rounded the final value of x correctly. Some candidates misinterpreted $6e^{-x}$ as $(6e)^{-1}$. Some candidates did not reject the negative root. A considerable number of candidates rounded the final answer to 3 decimal places instead of 3 significant figures.

Answer: 1.10

Question 2

Many candidates understood the chain rule and were successful in answering this question. Other candidates were challenged by the differentiation of x and/or y with respect to t . In finding the derivative of $\sin^2 t$ some candidates made no use of the chain rule and took the derivative to be $2\sin t$. Other candidates, realising that $\cos t$ should be involved in some way mistook the derivative to be $2\sin t \cos^2 t$. Nearly all

candidates seemed aware of the relation $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and used it correctly. A minority commenced by attempting to express x and y in terms of multiple angles. This was often done correctly, but this approach made the route to the final answer more demanding.

At the end of the solution a substantial number of candidates omitted the minus sign in the simplification and gave $\cos t$ as the final answer.

Answer: $-\cos t$

Question 3

- (i) The majority of candidates obtained the correct quotient in this question by using long division by $x^2 - x + 1$, or working with an identity. Those candidates who divided normally went on to equate the remainder to zero. They did not always realise that correct working led to a remainder that only involved terms in x , so that the presence of a constant in the remainder was an indication that an error had been made. An alternative method of finding a involved substituting one of the complex zeros of $x^2 - x + 1$ in $p(x)$, equating the result to zero and solving for x . Since such a zero is a zero of $x^3 + 1$ this method allowed for a rapid evaluation of a .
- (ii) This part asked for the real roots of $p(x) = 0$. While many candidates answered correctly, there were some who did not. Thus, some said $(x + 1)$ and $(x + 3)$ were the real roots. Some simply stopped after factorising $p(x)$, while others followed the statement $x = -1$ and $x = -3$ with the assertion that the real roots were $(x + 1)$ and $(x + 3)$. Future candidates would benefit from a clear understanding of the meaning of terms such as 'factor' and 'root', 'factorise' and 'solve'.

Answers: (i) $a = 1$; (ii) $x = -1, x = -3$

Question 4

Many candidates were challenged by this question. Most candidates tried to separate variables and the majority successfully completed that stage. Integration then usually produced the term $\ln(x + 1)$, but the integration of $\frac{\cos 2\theta}{\sin 2\theta}$ was found to be difficult. Those who recognised the type of integrand to be of the form

$\frac{f'(x)}{f(x)}$ obtained an integral of the form $k \ln(\sin 2\theta)$ and many had the correct value of k , i.e. $1/2$. Some

success was achieved by those candidates who first converted the integrand to $\frac{1}{2}(\cot \theta - \tan \theta)$. Those candidates with integrals of the right form were often successful in evaluating the arbitrary constant. To complete the solution it is necessary to remove the logarithms accurately. This final stage was a challenge to many of the candidates.

Answer: $x = \sqrt{2 \sin 2\theta} - 1$

Question 5

Parts (ii), (iii) and (iv) were answered well by the majority of candidates. Few candidates used degrees rather than radians, for the iteration in part (iv), and nearly all candidates took care to make clear the function whose sign they were considering in the calculations for part (ii). In part (iii) sufficient working was normally presented by candidates to support the proof.

The graphical work in part (i) was more of a challenge for candidates. Candidates could benefit from focusing on syllabus item graphs, such as $y = \sec x$. Adequate parabolic sketches of $y = 3 - \frac{1}{2}x^2$ were uncommon; many of the incorrect sketches were of straight lines or intersected the x -axis between 0 and $\frac{1}{2}\pi$. When correct sketches were made, candidates often omitted to say or indicate that the intersection meant that there was a root in the interval.

Answer: (iv) 1.13

Question 6

Most candidates answered part (i) accurately. The main errors were $R = 2$ and failure to give α correct to 2 decimal places. In part (ii) some candidates were challenged by the replacement of x by 2θ . Candidates with a correct method found the answer 61.2° , but the second solution eluded most of them. This answer is best found by using the value of $\cos^{-1}(2/\sqrt{10})$ lying in the interval $(-90^\circ, 0^\circ)$.

Answer: (i) $R = \sqrt{10}$, $\alpha = 71.57^\circ$; (ii) $10.4^\circ, 61.2^\circ$



Question 7

In part (i) a full explanation was required. It was not enough to form a general equation of the line AB using a parameter λ without showing that this parameter was the same as that used in the definition of P in the question.

In part (ii) many candidates made good progress in setting up expressions for the cosines of AOP and BOP in terms of λ , though a minority used the wrong sign of the scalar product, e.g. $\vec{AO} \cdot \vec{OP}$ rather than $\vec{OA} \cdot \vec{OP}$. The question advised candidates to equate these expressions. Those candidates that did so often went on to solve successfully for λ , especially if they saw that the factor of $|\vec{OP}|$ could be removed. Some otherwise correct solutions were ruined by incorrectly copying one of the numerators, $14 + 11\lambda$, as $4 + 11\lambda$. Some candidates introduced the cosine of AOB . They then often set up incorrect equations such as $\cos AOB = \cos AOP + \cos BOP$ or $\cos AOB = 2 \cos AOP$. A valid approach using AOB is to equate $\cos AOP$ or $\cos BOP$ to $\cos \frac{1}{2} AOB$. This leads to a quadratic with a spurious negative root that must be rejected.

Those candidates with a correct value of λ , still found part (iii) a challenge. Rather than observe that $AP:BP = \lambda:(1-\lambda)$ and thus 3:5, most attempts at the solution involved the calculation of the position vector of P and then the vectors $\vec{AP}$ and $\vec{BP}$.

Answer: (ii) $\lambda = \frac{3}{8}$

Question 8

Part (i) of this question was well answered by almost all candidates. Almost all candidates carried out the evaluation of the constants clearly and accurately. A common error was the omission to place $Bx + C$ in brackets in the basic identity $12 + 8x - x^2 \equiv A(4 + x^2) + (Bx + C)(2 - x)$.

Part (ii) was also well answered. However, before attempting to integrate each partial fraction, candidates needed to check that the partial fractions correctly recorded the outcome of part (i). As well as errors in transcribing the values of A , B , and C , the miscopying of $(4 + x^2)$ as $(4 - x^2)$ was also seen. Candidates with $C = 0$ often saw that their fraction $\frac{Bx}{4 + x^2}$ was of the form $\frac{f'(x)}{f(x)}$. Those candidates with correct indefinite integrals almost always gave enough working after the substitution of limits to justify the given answer.

Answer: (i) $\frac{3}{2-x} + \frac{4x}{4+x^2}$

Question 9

Part (i) of this question was generally well answered by candidates. Most found the exact value of x correctly, but many lost the mark for the y -coordinate either because they calculated an approximation or made a mistake in finding an exact value. In part (ii) integration by parts was needed to find the correct indefinite integral. The lower of the two limits was $x = 1$, but the use of $x = 0$ was sometime seen.

Answer: (i) $x = \frac{1}{\sqrt{e}}$, $y = -\frac{1}{2e}$; (ii) $\frac{1}{9}(2e^3 + 1)$

Question 10

Most candidates had a sound method in part **(a)** involving setting up and solving simultaneous equations in x and y , or a and b . Some candidates had errors of sign, usually arising when equating real and imaginary parts or from a miscopying of the complex number in the question. Those who reached a correct final pair of values for x (or y) often lost the final mark by making a mistake in stating the complex roots.

In part **(b)** there were many correct sketches. There were also many incorrect sketches with the centre of the circle often at $-3i$, or less frequently at $2 + 3i$, $2 - 3i$ or the origin. Very few candidates had a completely correct method for finding the maximum argument. There were many incorrect attempts involving inexact methods such as working with angles or distances measured from a diagram.

Answer. **(i)** $\pm (\sqrt{3} - i\sqrt{2})$ **(ii)** 131.8° (or 2.30 radians)



MATHEMATICS

Paper 9709/32

Paper 32

Key messages

- Candidates should check working to ensure high accuracy and to avoid arithmetic errors.
- It is important for candidates to not spend too much time on any one question.
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General comments

The standard of work on this paper varied between candidates resulting in a spread of candidate marks from zero to full marks. The questions that were most successfully answered were **Question 5** parts (ii), (iii) and (iv) (iteration), **Question 8** (partial fractions) and **Question 9** (calculus). Those that were least well answered were **Question 4** (differential equation), **Question 7** (vector geometry) and **Question 10** (complex numbers).

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Answer: (i) $R = \sqrt{10}$, $\alpha = 71.57^\circ$; (ii) $10.4^\circ, 61.2^\circ$



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Part (ii) was also well answered. However, before attempting to integrate each partial fraction, candidates needed to check that the partial fractions correctly recorded the outcome of part (i). As well as errors in transcribing the values of A , B , and C , the miscopying of $(4 + x^2)$ as $(4 - x^2)$ was also seen. Candidates with $C = 0$ often saw that their fraction $\frac{Bx}{4 + x^2}$ was of the form $\frac{f'(x)}{f(x)}$. Those candidates with correct indefinite integrals almost always gave enough working after the substitution of limits to justify the given answer.

Answer: (i) $\frac{3}{2-x} + \frac{4x}{4+x^2}$

Question 9

Part (i) of this question was generally well answered by candidates. Most found the exact value of x correctly, but many lost the mark for the y -coordinate either because they calculated an approximation or made a mistake in finding an exact value. In part (ii) integration by parts was needed to find the correct indefinite integral. The lower of the two limits was $x = 1$, but the use of $x = 0$ was sometime seen.

Answer: (i) $x = \frac{1}{\sqrt{e}}$, $y = -\frac{1}{2e}$; (ii) $\frac{1}{9}(2e^3 + 1)$

Question 10

Most candidates had a sound method in part **(a)** involving setting up and solving simultaneous equations in x and y , or a and b . Some candidates had errors of sign, usually arising when equating real and imaginary parts or from a miscopying of the complex number in the question. Those who reached a correct final pair of values for x (or y) often lost the final mark by making a mistake in stating the complex roots.

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Answer. **(i)** $\pm (\sqrt{3} - i\sqrt{2})$ **(ii)** 131.8° (or 2.30 radians)



MATHEMATICS

Paper 9709/33

Paper 33

Key messages

- It is important to read questions in detail and to be careful when copying expressions into answers.
- Candidates need to show working to obtain the full marks available for each question.

General comments

The majority of candidates offered responses to all ten questions on this paper, and worked the questions in the order set. A full range of achievement was seen, with a number of candidates scoring full marks. All questions were accessible to all candidates. Later questions proved challenging, usually due to the algebraic and manipulation skills required to obtain answers. Many candidates were proficient in working the basic processes. Questions asking for interpretation, such as the ends of **Questions 4, 6 and 9** proved to be more difficult.

All candidates should be advised to double check that their responses meet the demands of the questions. In this paper **Question 6** was a particular problem, with many candidates finding ω^2 and ω^3 correctly, but making no mention of the modulus or argument of either number. Candidates also need to be reminded that when an answer is given in the question they are expected to show full working in their solution.

Question 1

Many fully correct responses were seen. Some candidates gave a correct un-simplified statement of the binomial expansion of $(1 + \frac{1}{2}x)^{-2}$ but then made slips. Those candidates who attempted the expansion of $(2 + x)^{-2}$ usually made mistakes in dealing with the powers of 2. The most common mistake was to take the factor of 2 out of the bracket but not gain a factor of 2^{-2} . Some candidates did not attempt to deal with the negative power.

Answer: $4 - 4x + 3x^2$

Question 2

The majority of candidates used the correct quotient rule or product rule correctly to find $\frac{dy}{dx}$. Some candidates chose to rearrange the given equation to the form $y(1 + e^{2x}) = e^{2x}$ and use implicit differentiation. In order to derive the given result the candidates needed to substitute $x = \ln 3$. In many instances full working was not presented.

Answer: $\frac{9}{50}$



Question 3

- (i) Many correct answers were seen to this question. Some candidates confused $\tan^{-1} \frac{15}{8}$ and $\tan^{-1} \frac{8}{15}$. Some candidates lost marks as a result of the invalid statements $\sin \alpha = 15$ and $\cos \alpha = 8$ in the course of their working.
- (ii) Most candidates obtained a correct value for $\cos^{-1} \frac{12}{17}$ and went on to obtain $\theta = 107.0^\circ$. The second value for θ was often omitted. Many candidates obtained 376.8° as a possible solution but rejected this as inadmissible rather than consider subtracting 360° . A minority considered the possibility of $\cos^{-1} \frac{12}{17} = -45.1^\circ$ as part of their working.

Answer: (i) $17 \cos(\theta - 61.93^\circ)$; (ii) 107.0° , 16.8°

Question 4

- (i) Most candidates started this question by separating the variables and attempting to integrate. There were mistakes in dealing with the constants 0.5 and -0.02. Other common mistakes included $\int N^{-\frac{1}{2}} dN = \frac{1}{2} \ln N$, $\int e^{-0.02t} dt = \frac{1}{0.98} e^{0.98t}$ and $\int e^{-0.02t} dt = te^{-0.02t}$. Many candidates did go on to find the constant of integration correctly. Relatively few were able to find a correct expression for N from a correct expression for $2N^{\frac{1}{2}}$. Some omitted this step, and others thought that they needed to find the square root of $N^{\frac{1}{2}}$ to obtain N .
- (ii) The question expected candidates to give the limiting value of N for large values of t . Some simply said "it increases" or "it decreases", and some selected a "large" value of t to substitute into their expression for N .

Answer: (i) $N = (40 - 30e^{-0.02t})^2$; (ii) 1600

Question 5

- (i) The integration by parts was usually completed correctly. A small number of candidates attempted the alternative route, integrating $\ln x$, often successfully. Many went on to use the limits correctly. Some candidates used limits 0 and a . Sign errors in the substitution and mistakes in the algebra limited some candidates' answers.
- (ii) Fully correct answers were common to this question. Some candidates did not use the required level of accuracy. There were errors in the application of the iterative formula – some of these were due to misquoting the formula.

Answer: (i) $\sqrt{\frac{87}{2 \ln a - 1}}$ (given); (ii) 5.86

Question 6

- (i) Some candidates started by finding the modulus and argument of ω and using the results for modulus and argument of a product to obtain their answers. Others preferred to find ω^2 and ω^3 first and then find the moduli and arguments. The moduli were often correct, and most candidates had a correct method for finding arguments. Some candidates consider which quadrant they should be looking in for their answer. Candidates working from $\omega^2 = -2i$ often claimed that the argument was not defined.

- (ii) Only a minority of candidates were able to identify the correct circle and find its equation. Candidates working with the correct points did not all appreciate the significance of the information that ω^2 and ω^3 were the ends of a diameter of the required circle, and were not able to make much progress. Some candidates worked with ω^2 and ω^3 rather than with ω and ω^2 .

Answers: (i) $2, 2\sqrt{2}, -\frac{1}{2}\pi, \frac{1}{4}\pi$; (ii) $\left|z + \frac{1}{2} + \frac{1}{2}i\right| = \frac{1}{2}\sqrt{10}$

Question 7

- (i) Many candidates used the remainder theorem to find the value of a and then went on to divide in order to find the quadratic factor of $p(x)$. Candidates using synthetic division frequently obtained the incorrect factorisation $p(x) = (2x - 1)(2x^2 + 4)$. Those candidates who started with $p(x) = (2x - 1)(Ax^2 + Bx + C)$ found the value of a and the factors in the same working. The question asked candidates to factorise $p(x)$, and they were expected to express $p(x)$ as a product of factors in their answer to part (i) in order to gain full marks. Some candidates thought that they were expected to go on and express $(x^2 + 2)$ as a product of complex factors.
- (ii) Most candidates identified the correct form for their partial fractions, and many evaluated all three constants correctly without algebraic or numerical slips.

Answers: (i) $2, (2x - 1)(x^2 + 2)$; (ii) $\frac{-4}{2x - 1} + \frac{2x + 5}{x^2 + 2}$

Question 8

- (i) Most candidates attempted to find $\frac{dy}{dt}$ and $\frac{dx}{dt}$, and went on to use these to find $\frac{dy}{dx}$. There were mistakes due to incorrect differentiation of y , but the most common problem was that candidates did not spot the factorisation $3\sin^2 t \cos t - 3\cos^2 t \sin t = 3\sin t \cos t(\sin t - \cos t)$. Because the answer was given, some candidates claimed to have reached it from totally incorrect working, and some candidates who did not factorise went to considerable effort to try to find an alternative route to the answer.
- (ii) Many candidates were challenged by this part of the question. Some assumed that $t = 0$ at the origin. Other candidates solved the equation $\tan t = -1$, but concluded that $t = -\frac{\pi}{4}$ (outside the set of values for t), rather than $t = \frac{3\pi}{4}$.
- (iii) The most common approach to this question was to express the gradient in terms of $\sin 2t$. Having done that, both the 2 and the negative value of $\sin 2t$ presented difficulties in solving for values of t . Several candidates opted to square the initial equation and form a quadratic in $\sin^2 \theta$ or $\cos^2 \theta$. This method introduces additional invalid solutions which were not always rejected. A number of candidates offered solutions in degree mode, although the question is clearly set in radian mode.

Answers: (i) $-3\sin t \cos t$ (given); (ii) $\frac{3}{2}$; (iii) 1.9, 2.8

Question 9

- (i) The majority of candidates attempted the scalar product of the direction of l with the normal to p . Many claimed an answer of zero without any supporting working. Working is needed to demonstrate a given answer. Some candidates gave very clear explanations of how their result demonstrated that l and p were parallel.

- (ii) Many candidates substituted the coordinates of a general point on the line into the equation of the plane, although it was only necessary to use $(a, 1, 4)$. The most common mistake was for 10 to become 0 when moving between lines of working.
- (iii) Many scripts offered no solution to this part of the question. Some candidates, attempted to apply a vector product, with no success. Those candidates who had a sound method frequently considered only one of the two options for the position of the line.

Answers: (ii) 4; (iii) -5, 13

Question 10

- (i) Many candidates had difficulty with the integration by substitution. They usually started correctly, with a statement equivalent to $\frac{du}{dx} = \sec^2 x$, but made little further progress because they could not simplify the resulting integral, often because they did not notice the factorisation $\tan^{n+2} x + \tan^n x = \tan^n x(\tan^2 x + 1)$.
- (ii) The question asked candidates to use their work from part (i) to find two further integrals. Candidates needed to show working in this question.
- (a) Candidates who attempted to use the substitution $1 + \tan^2 x = \sec^2 x$ were often successful, although there were some solutions weakened by algebraic or arithmetic errors. Common mistakes were the assumption that $1 + \tan^4 x = \sec^4 x$, or the assumption that $\int \sec^4 x dx = \tan^2 x$.
- (b) Few candidates spotted that if they split 5 as $(1 + 4)$ then they could apply the result from part (i). Several took out the term $5(\tan^7 x + \tan^5 x)$, but could not then deal with $\tan^9 x + \tan^3 x$. Another common, but mistaken approach, was to assume that the result from (i) still applied if one of the two terms was multiplied by 5.

Answers: (i) $\frac{1}{n+1}$ (given); (ii) (a) $\frac{1}{3}$, (b) $\frac{25}{24}$

MATHEMATICS

Paper 9709/41

Paper 41

Key messages

- It is important for candidates to read the questions carefully, and answer as required.
- Candidates need to take care to avoid arithmetic and sign errors.

General Comments

This paper requires some answers derived from just the use of basic ideas and some answers derived from the use of basic ideas in the context of problem solving. Even questions focusing on basic ideas proved challenging for some candidates.

Comments on Specific Questions

Question 1

Many candidates did not find the relevant distance of 16 m. Many candidates did not use the idea that the work done by a force of magnitude F N, acting on a block at an angle α to the direction of motion of the block, for a distance of d m is given by 'work done = $F d \cos \alpha$ J'.

Answer: 376 J

Question 2

Few candidates made progress with this question.

Answers: Tension = 4.55 N, Magnitude = 9.1 N

Question 3

Most candidates resolved forces in a given direction to find the component of the resultant in that direction. Fully correct answers for parts (i), (a) and (b) were uncommon.

Part (ii) is dependent on the answers in part (i). Many candidates omitted to note that $R^2 = X^2 + Y^2$ and $\tan \theta = Y \div X$.

Answers: (i) (a) 8.74 N; (b) 11.5 N; (ii) 14.4 N, 52.7° to the i direction

Question 4

Most candidates were aware of the formulae for motion in a straight line with constant acceleration. Many did not develop a strategy for using these formulae effectively.

Answers: (i) $u = 1.4$, $a = 2$; (ii) $\theta = 11.5^\circ$

Question 5

Most candidates were aware of the need to resolve forces vertically and horizontally. Many did not find $R = 27.2$ in (i) or $R = 12.8$ in (ii), or $F = 9.6$ in (i) and (ii), where R N and F N are the normal and frictional forces acting on the block.

Answer: (ii) $\mu < 3/4$



Question 6

In part (i) there is no change in speed and hence no change in kinetic energy. The car moves uphill throughout, giving a change in potential energy. Most candidates mistakenly invoked the principle of conservation of energy. This part requires a solution to a work/energy balance question.

Parts (ii) and (iii) are more difficult exercises on problem solving.

Answers: (i) Work done is 900 000 J or 900 kJ; (iii) Ratio is 0.75

Question 7

Some candidates were able to answer part (i). Part (ii) was more of a challenge. Part (iii) was the best answered part of this question. Most candidates were challenged to develop a strategy for solving the problem in part (iv).

Answers: (ii) 16.9 ms^{-1} (or $16 \frac{7}{8} \text{ ms}^{-1}$); (iii) 1070 m; (iv) $t = 144$ seconds



MATHEMATICS

Paper 9709/42

Paper 42

Key messages

- It is important for candidates to read the questions carefully, and answer as required.
- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.

General Comments

A requirement of the examination rubric is that an answer should be correct to 3 significant figures, unless it is exact to 1 or 2 significant figures, as in **Question 1(i)**, **Question 5(i)**, **Question 6(i)** and **(ii)** and **Question 7(i)**, **(ii)** and **(iii)**. In other questions and parts of questions, candidates' answers were often not in accordance with the rubric. Such answers arise from premature approximation, or by giving an answer correct to only 2 significant figures. The way errors of this type arise is well illustrated in **Question 2**.

The question paper and mark scheme always ensure that some marks are available for straightforward application of knowledge of the syllabus. Other marks are available that can be scored by using principles in circumstances that are not standard. Many candidates did not score highly in questions in non-standard circumstances.

Care needs to be taken in reading questions. Omitting this can lead to mistakes as seen in **Question 5(ii)** where $a = 2$ and $t = 0.3$ appear in the same kinematic equation, and in **Question 6(iii)** where the bracketed term $(1240 + 1860)$ appears in an equation.

Comments on Specific Questions

Question 1

Part **(i)** of this question was well answered. In part **(ii)** few candidates adopted the reasoning that $a > 0 \Rightarrow P/v - R > 0 \Rightarrow 720/v - 48 > 0 \Rightarrow 720/v > 48 \Rightarrow v < 720/48 = 15$. Many candidates simply showed that $a > 0$ when v is equal to some particular number between 0 and 15.

Answers: **(i)** $R = 48 \text{ N}$; **(ii)** $v < 15 \text{ ms}^{-1}$

Question 2

In part **(i)** most candidates used Newton's second law and found that $6g \sin 8^\circ - 0.2 \times 6g \cos 8^\circ = 6a$. Many candidates lost numerical accuracy by rounding answering during their working.

These answers in part **(i)** were used in part **(ii)**. Numerical mistakes in part **(i)** were compounded at this point.

Answers: **(i)** 0.589 ms^{-2} ; **(ii)** 7.64 m

Question 3

This question was well attempted by all candidates. Many candidates obtained the exact answer of 110.72 m , which is of course an acceptable alternative to 111 m , which is correct to 3 significant figures.

A significant number of candidates applied formulae relevant to motion with constant acceleration.

Answers: 111 m



Question 4

The four main approaches to part (i) of this question were:

- resolving forces horizontally and vertically
- using a triangle of forces
- using the formula $R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \Phi$ where R is the resultant of forces of magnitudes F_1 and F_2 that act at a point with angle Φ between their directions
- Lami's theorem

The main mistake in the first and fourth approaches were to assume falsely that the angle between the forces of magnitudes 4 N and C N is 90° , and thus the force of magnitude C N acts at 60° to the horizontal. Candidates should be aware that diagrams are not exact unless stated.

Many candidates incorrectly assumed in part (ii), that the normal force has magnitude equal to the weight of P .

Answers: (i) $C = 8.72$; (ii) 0.433

Question 5

Part (i) was generally well attempted by candidates, although a minority used the formula $a = ((M - m)g) \div (M + m)$ to find the acceleration, but did not find the tension in the string. Such candidates had no reason to apply Newton's second law.

In part (ii) a large proportion of candidates did not realise the relevance of the motion of B while the string is slack, and wherever a constant acceleration formula was used the acceleration was taken to be 2 ms^{-2} .

Answers: (i) 2 ms^{-2} , 7.2 N; (ii) 2.25 m

Question 6

The wording of the question suggests that in part (i) it is necessary to set up an equation based on work/energy balance. Such an equation has four terms, representing kinetic energy change, potential energy change, work done against resistance and work done by the driving force. Many candidates omitted the potential energy change component.

In calculating potential energy change in (i) and in calculating the distance BD in part (ii), very many candidates used $\sin \theta = \sin 2.87^\circ$ or $\sin 2.9^\circ$ instead of the given $\sin \theta = 1/20$. In the case of 2.87 the answers found in parts (i) and (ii) are 199.7 m and 299.58 m which, if rounded to 3 significant figures, would be the correct answers. However in the 2.9 case the answers found in parts (i) and (ii) are 198 m and 296.5 m which do not round to correct answers.

Part (iii) is more difficult than the earlier parts and few candidates scored all four of the available marks. In most cases where an attempt was made candidates assumed that the resistance to motion is equal to $(1240 + 1860) \text{ N}$ at some part of the journey from B to D .

Answers: (i) 200m; (ii) 300m; (iii) 61.3 m

Question 7

The requirements in parts (i) (a) and (b) and part (ii) (a) were a challenge for candidates. In part (i) (a) common incorrect answers were 3,200 m and 3,600 m. Parts (ii) (b) and (c) are more difficult than the first three parts of the question.

Answers: (i) (a) 4000 m; (b) 0.02 ms^{-2} , -0.02 ms^{-2} ; (ii) (a) 200, 600



MATHEMATICS

Paper 9709/43

Paper 43

Key Messages

- Candidates need to recognise the direction of the frictional force and the normal reaction in a given situation. Accurate force diagrams could have helped some candidates in answering questions.
- For motion questions, candidates need to assess whether there is constant speed, constant acceleration or neither. This may help in selecting an appropriate method of solution.

General comments

The majority of candidates attempted all questions and much work was of a high standard. Most candidates could successfully attempt at least part of each question. Some candidates could not access the full level of mathematics examined in this paper. **Question 1** and **Question 2** were the best answered questions. **Question 3**, **Question 6** and **Question 7 part (iii)** were found to be more challenging.

Comments on specific questions

Question 1

- (i) Most candidates calculated the accelerations either by using the gradient property or $v=u+at$. Sometimes the acceleration for $30 < t < 40$ was stated as 0.21 instead of -0.21 and occasionally for $0 < t < 30$ the gradient was given as $1.5/30 = 0.05$ or $2.1/30 = 0.07$.
- (ii) and (iii) Many candidates were unsure of the difference between the distance AB and the total distance walked, giving the same answer, e.g. 86.5 m, for both parts of the question or giving the two answers the wrong way round. In part (ii) some calculated only the area of the triangle for $40 < t < 60$.

Answers: (i) 0.02 ms^{-2} , -0.21 ms^{-2} ; (ii) 42.5 m; (iii) 86.5 m

Question 2

Many candidates gave fully correct solutions. Others found difficulty in describing the direction of the resultant. Those who calculated 41.1° did not always make it clear that this was 41.1° anticlockwise from the 31 N force. Many calculated $\tan^{-1}(5/12)$ or found $\sin \alpha = 5/13$ and $\cos \alpha = 12/13$. Incorrect expressions such as $31+26 \cos(12/13)$ were also seen. It was usual for candidates to resolve in the directions of the 31 N and 58 N forces but some omitted at least one component and others made sign errors.

Answers: 73 N, 41.1° to i direction



Question 3

Candidates found this question one of the most challenging. Many were able to state an equation of motion for particle P and thus to find the acceleration before P reached the ground. There was some confusion about the distance required, as well as how to obtain it. 0.4 m (not doubled) and 2.8 m (including the initial 2 m) rather than 0.8 m were common incorrect answers. Candidates often calculated unnecessary information such as the mass of particle Q or the time taken for P to reach the ground. Some used incorrect values such as $a = 2$ rather than $a = g$ for the acceleration of Q after the string became slack.

Answer: 0.8 m

Question 4

- (i) The most concise solutions equated energy at A and at C . Some found the velocity at B and then recognised that the velocity at C was the same. Others mistakenly used a constant acceleration formula for the motion along the horizontal smooth surface with e.g. $a = g$. A few candidates incorrectly attempted $v^2 = u^2 + 2$ as for the motion along the curved surface.
- (ii) Those who used Newton's Second law to find the acceleration from B to C sometimes obtained $a = 3$ rather than $a = -3$ leading to a value of 10.5 ms^{-1} for the speed at C . Others who used a work/energy approach were often successful. Mistakes included work done by friction = 0.3×8 or $0.3 \times 8 \times 4$.

Answers: (i) 8.94 ms^{-2} ; (ii) 7.07 ms^{-2}

Question 5

This question was often well done with candidates recognising the need for integration to obtain displacement and differentiation to find acceleration. Some candidates attempted to use constant acceleration formulae although the acceleration was variable.

- (i) Some answers contained a constant of integration which should have been found to be zero.
- (ii) Many candidates found $t = 6/k$, but some solutions were incomplete giving k in terms of t . Others were incorrect from solving $s = 0$ instead of $v = 0$.
- (iii) Whilst a correct equation in k was often found, algebraic errors were sometimes made in attempting to solve the equation.
- (iv) Candidates needed to find t when $a = 0$ for maximum velocity but did not always differentiate to find the acceleration.

Answers: (i) $2t^3 - kt^4/4$; (ii) $6/k$; (iii) 1; (iv) 32

Question 6

This was a more challenging question with many candidates experiencing difficulty with the direction of forces and in particular the direction of the normal reaction R . Those who set up two equations for the frictional force F and for R were usually able to apply $F = \mu R$ and solve the equations to find a value for T . Frequently candidates assumed $R = mg$. The best solutions considered both the case with the frictional force preventing upwards motion and the case with the frictional force preventing downwards motion, but sometimes only one case was considered.

Answers: 28.3, 68.5

Question 7

Candidates were expected to recognise three different situations: constant power and constant speed; no power and deceleration; constant power and varying acceleration. This proved increasingly challenging.

- (i) Many correct solutions were seen, often with the speed found correctly as 40 ms^{-1} . Some candidates used $s = \frac{1}{2} ((u + v) \times t)$ which was correct with $u = 40$ and $v = 40$ but was regularly seen with $u = 0$ suggesting constant acceleration instead of constant speed.
- (ii) Many candidates used correct methods. Some mistakes occurred with acceleration calculated as 1.25 ms^{-2} rather than -1.25 ms^{-2} . Sign errors were also evident in some work/energy solutions.
- (iii) A work/energy equation was expected for solving this problem. Whilst the work done by the engine or the change in kinetic energy was sometimes found, these both needed to be included to form a suitable equation for the distance CD . Candidates often assumed that the acceleration was constant writing e.g. $v = u + at$, $30 = 20 + 14a$ and continuing with $v^2 = u^2 + 2$ as to obtain a distance of 350 m. Other candidates assumed that the acceleration was $750/600$ and obtained a distance of 200 m. Some who attempted to use the constant power of 30 kW calculated $P/20$ or $P/30$.

Answers: (i) 4000 m; (ii) 480 m; (iii) 360 m

MATHEMATICS

Paper 9709/51

Paper 51

Key messages

- It is important for candidates to not spend too much time on any one question.
- It is important to read questions in detail and to be careful when copying expressions into answers.
- Candidates need to make full use of the supplied list of formulae.

General Comments

The work by most candidates was generally neat and well presented. Few candidates used premature approximations and most candidates gave their answers to 3 significant figures. $g = 9.8$ or 9.81 was very rarely seen. The paper clearly states that $g = 10$ should be used. **Questions 1** and **3** proved to be the most accessible questions on the paper. **Questions 4** and **6** proved to be the most challenging.

Comment on Specific Questions

Question 1

- (i) Most candidates obtained full marks on this part of the question. Some candidates did not have a moment equation or the correct moment equation.
- (ii) Good marks were normally scored here. Candidates were required to find the friction force F and the normal reaction R and then to use $F = \mu R$.

Answers: (i) $T = 12\text{N}$; (ii) $\mu = 0.289$

Question 2

- (i) Some candidates used $\tan 45 = (\text{vertical velocity})/(\text{horizontal velocity})$ instead of $\tan 45 = (\text{vertical distance})/(\text{horizontal distance})$.
- (ii) This part of the question was generally well done with many candidates obtaining full marks.

Answers: (i) $v = 8.2(0) \text{ ms}^{-1}$; (ii) $t = 0.3(00) \text{ s}$.

Question 3

Many candidates scored all marks on this question.

- (i) By using $T = \lambda x/L$ many candidates went on to score both the available marks.
- (ii) To solve this part of the question it is necessary to set up an energy equation. This equation should be $20 \times (0.05)^2 / (2 \times 0.4) + 0.25v^2 / 2 = 0.25g \times 0.45$, leading to $v = 2.92 \text{ ms}^{-1}$.
- (iii) If d is the greatest distance then $20(d - 0.4)^2 / (2 \times 0.4) = 0.25gd$. This sets up a quadratic equation and when solved results in the greatest distance.

Answers: (i) Distance $OP = 0.45 \text{ m}$; (ii) Speed $= 2.92 \text{ ms}^{-1}$; (iii) Greatest distance $OP = 0.656 \text{ m}$.



Question 4

This question challenged many candidates.

- (i) This part of the question was solved by using $\tan \theta = 0.7/(2.4/4)$, $\theta = 49.4^\circ$.
- (ii) Here the expression needed was $h/2 = 2.4/4$, $h = 1.2$.
- (iii) This part is solved by setting up an equation by taking moments about some point. If moments are taken about the vertex V then the equation becomes $4w \times VG = w \times 2.4 \times \frac{3}{4} + 3w(2.4 + h/2)$ with G as the centre of mass of the whole solid. If α is the semi-vertical angle of the cone then $\tan \alpha = 0.7/2.4$ and so $\cos \alpha = 2.4/2.5$. $VG = 2.5/\cos \alpha = 2.5/(2.4/2.5) = 125/48$ and so $4 \times 125/48 = 0.6 \times 3 + 7.2 + 1.5h$ giving $h = 0.994$ m.

Answers: (i) $\theta = 49.4^\circ$; (ii) Greatest possible value of $h = 1.2$ m; (iii) Least possible value of $h = 0.994$ m

Question 5

This question was a good source of marks for many candidates.

- (i) Few candidates did not set up the given equation. Most candidates integrated correctly to set up a logarithmic function. A number of candidates did not manipulate the algebra to get the correct function for v in terms of t .
- (ii) Full marks were often scored on this part of the question. Those candidates who had mistaken expressions v could often still obtain marks.

Answers: (i) $v = 50 - 50e^{-0.2t}$; (ii) $h = 17.6$ m

Question 6

This question caused problems. Few candidates gained full marks.

- (i) Many candidates did not realise that the length of AB was 0.7 m and not 0.9 m. If θ is the angle that AB makes with the vertical then $\sin \theta = 0.2/0.7$. By resolving vertically the tension can now be found. Having found the tension then by using radial acceleration the value of ω can be calculated. It is then possible to find the kinetic energy.
- (ii) Here it is necessary to first find the distance AB . This can be done by using the right angled triangle ABC with $BC = 0.9 - AB$ and then stating that $\tan BAC = (0.9 - AB)/AB = 1/2$. This gives $AB = 0.6$. The radius of the circle is then equal to $0.6 \sin BAC$. T can now be calculated by resolving vertically giving $T \cos BAC - T \sin BAC = 0.3g$ and so $T = 6.71$ N. Using radial acceleration gives $T \cos BAC + T \sin BAC = 0.3\omega^2 \times 0.6 \sin BAC$ and so $\omega = 10.6$.

Answers: (i) $\omega = 5$, 0.402 J; (ii) $T = 6.71$, $\omega = 10.6$

MATHEMATICS

Paper 9709/52

Paper 52

Key messages

- It is important for candidates to not spend too much time on any one question.
- It is important to read questions in detail and to be careful when copying expressions into answers.
- Candidates need to make full use of the supplied list of formulae.

General Comments

The work by most candidates was generally neat and well presented. Few candidates used premature approximations and most candidates gave their answers to 3 significant figures. $g = 9.8$ or 9.81 was very rarely seen. The paper clearly states that $g = 10$ should be used. **Questions 1** and **3** proved to be the most accessible questions on the paper. **Questions 4** and **6** proved to be the most challenging.

Comment on Specific Questions

Question 1

- (i) Most candidates obtained full marks on this part of the question. Some candidates did not have a moment equation or the correct moment equation.
- (ii) Good marks were normally scored here. Candidates were required to find the friction force F and the normal reaction R and then to use $F = \mu R$.

Answers: (i) $T = 12\text{N}$; (ii) $\mu = 0.289$

Question 2

- (i) Some candidates used $\tan 45 = (\text{vertical velocity})/(\text{horizontal velocity})$ instead of $\tan 45 = (\text{vertical distance})/(\text{horizontal distance})$.
- (ii) This part of the question was generally well done with many candidates obtaining full marks.

Answers: (i) $v = 8.2(0) \text{ ms}^{-1}$; (ii) $t = 0.3(00) \text{ s}$.

Question 3

Many candidates scored all marks on this question.

- (i) By using $T = \lambda x/L$ many candidates went on to score both the available marks.
- (ii) To solve this part of the question it is necessary to set up an energy equation. This equation should be $20 \times (0.05)^2 / (2 \times 0.4) + 0.25v^2 / 2 = 0.25g \times 0.45$, leading to $v = 2.92 \text{ ms}^{-1}$.
- (iii) If d is the greatest distance then $20(d - 0.4)^2 / (2 \times 0.4) = 0.25gd$. This sets up a quadratic equation and when solved results in the greatest distance.

Answers: (i) Distance $OP = 0.45 \text{ m}$; (ii) Speed $= 2.92 \text{ ms}^{-1}$; (iii) Greatest distance $OP = 0.656 \text{ m}$.



Question 4

This question challenged many candidates.

- (i) This part of the question was solved by using $\tan \theta = 0.7/(2.4/4)$, $\theta = 49.4^\circ$.
- (ii) Here the expression needed was $h/2 = 2.4/4$, $h = 1.2$.
- (iii) This part is solved by setting up an equation by taking moments about some point. If moments are taken about the vertex V then the equation becomes $4w \times VG = w \times 2.4 \times \frac{3}{4} + 3w(2.4 + h/2)$ with G as the centre of mass of the whole solid. If α is the semi-vertical angle of the cone then $\tan \alpha = 0.7/2.4$ and so $\cos \alpha = 2.4/2.5$. $VG = 2.5/\cos \alpha = 2.5/(2.4/2.5) = 125/48$ and so $4 \times 125/48 = 0.6 \times 3 + 7.2 + 1.5h$ giving $h = 0.994$ m.

Answers: (i) $\theta = 49.4^\circ$; (ii) Greatest possible value of $h = 1.2$ m; (iii) Least possible value of $h = 0.994$ m

Question 5

This question was a good source of marks for many candidates.

- (i) Few candidates did not set up the given equation. Most candidates integrated correctly to set up a logarithmic function. A number of candidates did not manipulate the algebra to get the correct function for v in terms of t .
- (ii) Full marks were often scored on this part of the question. Those candidates who had mistaken expressions v could often still obtain marks.

Answers: (i) $v = 50 - 50e^{-0.2t}$; (ii) $h = 17.6$ m

Question 6

This question caused problems. Few candidates gained full marks.

- (i) Many candidates did not realise that the length of AB was 0.7 m and not 0.9 m. If θ is the angle that AB makes with the vertical then $\sin \theta = 0.2/0.7$. By resolving vertically the tension can now be found. Having found the tension then by using radial acceleration the value of ω can be calculated. It is then possible to find the kinetic energy.
- (ii) Here it is necessary to first find the distance AB . This can be done by using the right angled triangle ABC with $BC = 0.9 - AB$ and then stating that $\tan BAC = (0.9 - AB)/AB = 1/2$. This gives $AB = 0.6$. The radius of the circle is then equal to $0.6 \sin BAC$. T can now be calculated by resolving vertically giving $T \cos BAC - T \sin BAC = 0.3g$ and so $T = 6.71$ N. Using radial acceleration gives $T \cos BAC + T \sin BAC = 0.3\omega^2 \times 0.6 \sin BAC$ and so $\omega = 10.6$.

Answers: (i) $\omega = 5$, 0.402 J; (ii) $T = 6.71$, $\omega = 10.6$

MATHEMATICS

Paper 9709/53

Paper 53

Key messages

- It is important for candidates to not spend too much time on any one question.
- It is important to read questions in detail and to be careful when copying expressions into answers.
- Candidates need to make full use of the supplied list of formulae.

General Comments

Few candidates used premature approximations and most candidates gave their answers to 2 significant figures. The use of $g = 9.8$ or 9.81 was rarely seen. The paper clearly states that $g = 10$ should be used. **Questions 1, 3 and 4** proved the most accessible questions on the paper. **Question 6** was the most challenging question.

Comments on Specific Questions

Question 1

This question was generally well done by the candidates. Some candidates only found the vertical component of the velocity. The full solution required finding both the horizontal and the vertical components and then calculating the resultant to give the required speed.

Answer: Speed is $13(.0) \text{ ms}^{-1}$

Question 2

This question required the candidate to be able to take moments about certain points. This was a challenge for some candidates.

- (i) This section of the question was generally well done.
- (ii) To solve this it was necessary to take moments about A . Some candidates attempted to take moments about B not realising that there would be forces acting at A . Occasionally only the weight of one rod was used in the moment equation. The correct moment equation should be $0.3 \cos 45^\circ \times 2 \times 7 = (2 \times 0.6 \sin 45^\circ) \times F$.
- (iii) The same approach as in (ii) was taken by some candidates. The correct moment equation should be $0.3 \cos 45^\circ \times 2 \times 7 = 0.6 F$.

Answers: (i) 0.212; (ii) (a) $F = 3.5 \text{ N}$, (b) $F = 4.95 \text{ N}$



Question 3

- (i) Most candidates were able to express x and y in terms of t and then go on to eliminate t to get the required equation. A few candidates having found x and y incorrectly restarted the question by quoting the trajectory equation and used it to get the required result. The instruction “hence” meant no credit could be given.
- (ii) This part of the question was well done.

Answers: (i) $x = (25 \cos 45^\circ) t$, $y = (25 \sin 45^\circ) t - gt^2/2$; (ii) Distance = 57.5 m

Question 4

- (i) This part of the question was usually well done. Some candidates omitted the minus sign.
- (ii) When integrating $1/(1 + 4t)$ some candidates arrived at just $\ln(1 + 4t)$ instead of $\frac{1}{4} \ln(1 + 4t)$. Otherwise, this part of the question was generally well answered.

Answers: (i) $v = 8/(1 + 4t)$; (ii) $OP = 3.89$ m

Question 5

- (i) Candidates used radial acceleration on R to get $0.2 \omega^2 \times 1.2 = 6$. Occasionally 0.4 m was seen instead of 1.2 m.
- (ii) This part of the question was more challenging. In setting up the equation required quite often only the 10 N force was seen and not the 6 N force as well. The correct equation should have been $m \times \omega^2 \times 2 \times 0.4 = 10 - 6$.
- (iii) Few candidates successfully completed this section. The correct solution should have been $(0.2 \times (5 \times 1.2)^2)/2 = (M(5 \times 0.4)^2)/2$ where M is the mass of P . $M = 1.8$ kg, therefore, $(T - 10) = (1.8 \times 5^2 \times 0.4)$, hence $T = 28$ N.

Answers: (i) $\omega = 5$; (ii) Mass of Q is 0.2 kg; (iii) Tension in OP is 28 N.

Question 6

This was the most challenging question in the paper.

- (i) The candidate needed to take moments about O . The resulting equation should have contained 3 elements namely the hemi-sphere, the cylinder and the whole body. It was possible to take moments about other points and then to make the final adjustment. When moments are taken about O the required equation is
 $(\pi \times 0.6^2 \times 0.6 \times 0.3) - (\frac{2}{3} \pi \times 0.6^3 \times \frac{3}{8} \times 0.6) = (\pi \times 0.6^3 + \frac{2}{3} \pi \times 0.6^3)d$, hence $d = 0.09$ m.
- (ii) Few candidates were able to set up a correct moment equation. When moments are taken about O the equation is
 $(\frac{2}{3} \pi \times 0.6^3 \times \frac{3}{8} \times 0.6) - (\pi \times 0.6^3 \times 0.3) + (0.48A \times 0.36) = 0$, $A = 3\pi/16$.
- (iii) Here it is necessary to extend the hole into the hemi-sphere so that the new height of the cylinder is 7.2 m. The increase in length will be 0.24 m.

Answers: (i) Distance of centre of mass from O is 0.09 m; (iii) Increase in length of hole is 0.24 m.

Question 7

- (i) Most candidates scored full marks on this part of the question.
- (ii) An energy equation is required in this part. This equation should be (Initial Elastic Energy) + (Initial Kinetic Energy) = (Final Potential Energy).
- (iii) An energy equation is required in this answer. This equation should be (Initial Potential Energy) = (Final Kinetic Energy). If d is the length of the plane then the equation would be $0.8gd \sin 30 = (20(d - 0.4)^2)/(2 \times 0.4)$

Answers: (i) The extension is 0.08 m; (ii) Speed of projection is $2.1(0) \text{ ms}^{-1}$; (iii) Length of the slope is 0.745 m.

MATHEMATICS

Paper 9709/61

Paper 61

Key messages

- It is important for candidates to read the questions carefully, and answer as required.

General comments

Many candidates scored well on this paper, based on their syllabus knowledge. Other candidates were challenged by this paper.

Comments on specific questions

Question 1

This was a normal approximation to a binomial type question. Those candidates who recognised this found the mean and variance, standardised correctly and obtained full marks.

Answer: 0.794

Question 2

This question was well attempted by candidates and many correct solutions were obtained. Most candidates realised that the coded mean + 25 was equal to the true mean. They were then able to solve for n , the number of data points, which turned out to be 40. One subsequent mistake was for candidates to mix coded data with uncoded data in determining the variance. Candidates should have used mean of coded squares minus (coded mean) squared to get 82.99 for the variance and hence find the standard deviation. In part (ii) since Σx^2 was asked for, candidates could use raw data and use $\Sigma x^2/40 - (\Sigma x/40)^2 = 82.99$ to solve for Σx^2 .

Answers: (i) 9.11; (ii) 35,412

Question 3

Many candidates took this situation to mean that individuals were chosen according to a binomial distribution, with a constant probability of success. This is equivalent to using sampling with replacement. In part (ii) candidates who used the binomial model were able to gain marks for determining the variance.

Answers: (i) $P(1) = 1/14$, $P(2) = 3/7$, $P(3) = 3/7$, $P(4) = 1/14$; (ii) $15/28$ (0.536)

Question 4

Almost everyone found the quartiles for the History marks correctly. Some candidates were challenged by box-and-whisker plots, and others lost marks by not labelling their axes or drawing two box-and-whisker plots superimposed on top of each other.



Question 5

This question was well answered by many candidates. One common mistake was to find the z-value for a probability of 0.94 rather than 0.97. Part (ii) relied on finding the answer to part (i). Those candidates with a mistake in part (i) could still score in part (ii). This also applied in part (iii).

Answers: (i) 6.38; (ii) 0.864; (iii) 0.0171

Question 6

This question was well answered with many candidates scoring high marks. Part (b) (i) found a variety of methods used, including writing that $nC4 = 3876$, finding n by trial and error to be 19, and then finding $19P4$. In part (b) (ii) many candidates arranged the personal possessions in $3!$ ways, and arranged the astronauts in $4!$ ways, but struggled to progress. Candidates were given credit for identifying either of these groupings.

Answers: (a) (i) 19,958,400, (ii) 362,880; (b) (i) 93,024, (ii) 31,104

Question 7

In part (i) many candidates scored well. Most candidates recognised that they had to find all the different options of two balls with the same number and then sum the options. Some candidates added the initial probabilities instead of multiplying them, and then multiplied the options instead of adding them. Some candidates multiplied the probabilities together for two 8s but did not multiply by the probability of being not-8 for Bag C. Part (ii) was recognised by some candidates to be a conditional probability type of question. The word "Given" should have alerted candidates to this fact. It was a question of getting the sentence the correct way round $P(\text{both 2 given the same number})$, and substituting in the conditional probability formula. Part (iii) required the strict definition of independence either as $P(X \mid Y) = P(X)$ or $P(X \cap Y) = P(X) \times P(Y)$. All probabilities needed to be stated, and/or found, and then substituted in the formula to show whether the events X and Y were independent or not.

Answers: (i) 47/140; (ii) 5/47; (iii) not independent

MATHEMATICS

Paper 9709/62

Paper 62

Key messages

- It is important for candidates to read the question carefully, and answer as required.

General comments

This paper was a challenge for many candidates. Some questions asked for answers that involved candidates having to think carefully what was happening, and not rushing into the first option that presented itself. In general, candidates expressed their working clearly and were competent with topics. They were especially familiar with the normal distribution. Some candidates were challenged by this paper.

Comments on specific questions

Question 1

In this question, it was anticipated that candidates would use the statistical mode on their calculators and write down the answers. Few did this, preferring to do everything longhand and consequently many lost accuracy as they did not work with enough figures to ensure a final accuracy of 3 significant figures. Candidates who only want to type the numbers in once could have shown how they obtained the standard deviation by finding Σx^2 from their calculator and using this in the formula. They would then have obtained some marks. The correct answers for this question, with no working, would have gained full marks.

Answers: (i) 59.4; (ii) 7.68

Question 2

In part (i) candidates needed to recognise that there were two choices each time (head or tail) and so the total number of choices was 2^{12} . Part (ii) was similar to finding the number of different ways of arranging the letters HHHHHHTTTTTT which candidates need to recognise as being similar to, say, the number of ways of arranging the letters of the word STATISTICS. Candidates need to recognise these different situations which all have a common idea.

Answers: (i) 4,096; (ii) 792

Question 3

Part (a) was well done by most candidates. Few were able to obtain full marks for (b), although most picked up one mark out of three for grouping the Gs together.

Answers: (i) 1,941,912; (ii) 5040

Question 4

There was nothing intrinsically difficult in this question but candidates lost marks through not reading the question carefully and not understanding exactly what was required. When asked to state which interval the median mark was in, it was not sufficient to say " <50 ". The lower bound was needed too. In part (ii) candidates need to be aware that the lower quartile (LQ) can occur anywhere in the interval 40 – 45 and the upper quartile (UQ) can occur anywhere in the interval 50 – 60. Thus the maximum Inter Quartile Range (IQR) would be when the LQ is 40 and the UQ is 60, giving the maximum IQR as 20. Similarly the smallest IQR would be when the LQ is 45 and the UQ is 50 giving the minimum IQR to be 5. Part (iii) involved finding a frequency from a cumulative frequency table, and then in part (iv) these had to be used to find the frequency density and hence draw the histogram. Many candidates drew a histogram of cumulative



frequencies. Others used every combination of frequency/class width, class width/frequency, frequency/mid interval, mid interval/frequency and then draw all these with cumulative frequencies used instead of frequencies. Marks were gained for axes which were correctly labelled. Most candidates did use a ruler.

Answers: (i) 45-50g; (ii) smallest IQR 5 largest 10; (iii) 50

Question 5

This question was the best attempted on the paper with a large majority of prepared candidates gaining full marks. Candidates multiplied the three probabilities together and then added the 6 different options. A few candidates took the probability of landing on a red side to be the same as the probability of landing on the blue side, and the probability of landing on the green side that is $\frac{1}{3}$, despite being told in part (i) that the probability of landing on the blue side is $\frac{1}{8}$. These candidates were awarded method marks for multiplying their probabilities and for summing 6 options even with the wrong probabilities, providing the sum of the three probabilities was 1. Part (iii) was a normal approximation to the binomial and many candidates scored full marks.

Answers: (ii) $\frac{9}{64}$ (0.141); (iii) 0.742

Question 6

This question tested candidates' ability to "think through" a question and analyse it into smaller steps. It proved a good differentiator between candidates. Many candidates drew a tree diagram, recognised a binomial distribution and scored full marks in part (i). Part (ii) proved more taxing, as candidates had to recognise that this was a conditional probability question without being told that it was. Those who did so gained full marks. Those who did not, managed to gain a method mark for using the binomial distribution even though their probabilities were wrong.

Answers: (i) 0.953; (ii) 0.701

Question 7

Part (i) involved using a normal distribution to find the probability that the random variable was between two particular values so was marginally more difficult than finding the probability that the random variable was say, greater than a particular value. In part (ii), candidates had to standardise and then cancel a fraction, which proved too difficult for many. Similarly in part (iii), candidates had to find the correct probability and then multiply it by 70. Many saw the number 70 but did not appreciate what they had to do with it. Part (iv) was well done apart from the fact that many candidates were unable to solve the equation $1.45 = \frac{6 - \mu}{2\mu}$.

Answers: (i) 0.373; (ii) 0.309; (iii) 11.1; (iv) 1.54

MATHEMATICS

Paper 9709/63

Paper 63

Key messages

- A considerable proportion of the total marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- It is important for candidates to not spend too much time on any one question.

General comments

Nearly every candidate was able to attempt all six questions. Answers to **Questions 1** and **6** part (ii), both of which concerned the normal distribution, were usually correct. **Question 5**, involving a box-and-whisker plot, was also answered well. Attempts at **Questions 2** (probability) and **6** part (i) (conditional probability and the binomial distribution) were generally less successful. Some candidates were able to produce satisfactory answers for only a few questions.

The majority of candidates completed the paper in the time allowed. A few candidates did not complete the paper.

Comments on specific questions

Question 1

- (i) This question was usually answered well. Some candidates used $(\mu - 25)$ instead of $(25 - \mu)$ in their numerator; and $\sigma = 3\mu$ instead of $\mu = 3\sigma$. A few candidates used their tables to find $\Phi(0.648)$ and others wrongly included a continuity correction. Occasionally a response stopped after solving an equation to find the value of either μ or σ , without continuing to find the value of the other.
- (ii) Most candidates realised that the binomial distribution was appropriate. Several calculated ${}^6C_4(0.648)^4(0.352)^2$ instead of ${}^6C_4(0.352)^4(0.648)^2$. Some candidates gave their answer as 0.097, rather than 0.0967. In the future candidates need to be careful to answer to 3 significant figures. A few rounded their probabilities and calculated ${}^6C_4(0.35)^4(0.65)^2$. Some used their values of μ and σ from part (i) to calculate a new value for $P(X < 25)$.

Answers: (i) $\mu = 22.2$, $\sigma = 7.40$; (ii) 0.0967.

Question 2

Let F , M and W represent the events the person chosen is female, the person chosen is male and the person chosen watches 'Kops are Kids' respectively. In both parts conditional probabilities were often used instead of probabilities, thus $P(F \text{ and } W) = \frac{3}{12}$ and $P(M) = \frac{13}{18}$ appeared instead of $\frac{3}{30}$ and $\frac{13}{30}$ in parts (i) and (ii) respectively.

- (i) The majority of candidates were challenged by this part of the paper. There was confusion between $P(F \text{ and } W)$ and $P(F \text{ or } W)$, together with their use. This often resulted in an answer of $\frac{28}{30}$ or $\frac{31}{30}$. Several answers involved the product of $\frac{12}{30}$, $\frac{16}{30}$, and $\frac{3}{30}$. In some instances, a correct tree diagram was generated, but the wrong branches were selected. Other candidates

incorrectly assumed that $P(F) = P(M) = \frac{1}{2}$. Alternatively Venn diagrams and 2-way tables often led to good answers. Some candidates produced several different equations.

- (ii) Generally answers to this question were good. Some candidates incorrectly used $P(M) \times P(W)$ to calculate $P(M \text{ and } W)$. Candidates need to understand the conditions for two or more events being 'independent', 'mutually exclusive' or 'exhaustive'.

Answers: (i) $\frac{5}{6}$ or 0.833; (ii) Not independent.

Question 3

Many candidates gained full marks for this question. Others were unable to produce a correct probability distribution table. Tree diagrams usually helped to obtain the correct answer and were particularly successful in showing the different number of arrangements in parts (ii) and (iii). Some answers were based on a multiple of five ropes and used probabilities without replacement.

- (i) Many candidates answered this question correctly. Some candidates' answers included the incorrect probabilities $P(3 \text{ m rope}) = \frac{1}{5}$ and $P(5 \text{ m rope}) = \frac{4}{5}$. Some candidates omitted the " $- \{E(X)\}^2$ " term. Some probabilities were left as e.g. $\frac{4x}{5x}$, whilst $\frac{4}{5x}$ and $\frac{1}{5x}$ featured in other attempts. A few candidates calculated the expectation and variance for each rope separately.
- (ii) Several candidates overlooked the possibility that the two ropes could be chosen in either order. Those using combinations made little progress, both here and in part (iii).
- (iii) The need for two ropes of length 3 m and one rope of length 5 m was understood by most candidates. The different arrangements were considered more often than in part (ii). A few candidates listed all eight possible arrangements and arrived at the answer $\frac{3}{8}$.

Answers: (i) expectation $\frac{2}{35}$ or 3.4 and variance $\frac{16}{25}$ or 0.64; (ii) $\frac{8}{25}$ or 0.32; (iii) $\frac{48}{125}$ or 0.384.

Question 4

- (i) This question was usually answered well. The most common errors were answers of $3! + 4! + 8!$, $15!$ or $\frac{15!}{3! 4! 8!}$.
- (ii) This question was usually answered well. The main errors were using permutations instead of combinations and summing the combinations instead of multiplying them.
- (iii) The majority of the candidates realised that there were three different ways in which 6 of the images could be chosen and proceeded to calculate the correct answer. Some attempts involved 8C_x and ${}^7C_{6-x}$. The incorrect answer 2400, from ${}^6C_3 \times {}^4C_1 \times {}^3C_1 \times {}^{10}C_1$, was given by several candidates.

Answers: (i) 34 836 480 or 34 800 000; (ii) 504; (iii) 2520.

Question 5

- (i) Candidates often gained all four marks. Nearly all scales were sensible, the most frequent being 2 cm to represent 10 thousand euros. Sometimes the box formed from the quartiles and median was symmetrical, often involving 20, 40 and 60 thousand euros. A few candidates omitted the units from their plot and others had a lower whisker which stopped at 0 instead of 5000 euros. Occasionally the whiskers ended at –1500 and 42 500 euros.
- (ii) Most candidates were able to make a relevant comment about the salaries. These comments usually concerned the shape of the distribution (e.g. positively skewed) or were factual (e.g. the range is 75 thousand euros). A frequent incorrect statement was “Most of the salaries are between 15 and 26 thousand euros.”
- (iii) (a) The majority of candidates answered this question well. Other candidates appeared unfamiliar with ‘the interquartile range’, often multiplying their upper and lower quartiles by 1.5.
- (b) This question was usually answered well. A few candidates calculated $1.5 \times$ their interquartile range – their lower quartile.

Answers: (i) box-and-whisker plot; (ii) comment, e.g. $Q_2 - Q_1 < Q_3 - Q_2$; (iii) (a) 42 500 euros; (b) outliers < -1500 euros.

Question 6

Overall, this question was well answered by the majority of candidates. Use of the normal distribution in part (i) was not appropriate. Similarly, using the binomial distribution in part (ii) would take a considerable time, even if their calculator was able to carry out the required calculations.

- (i) Many candidates produced correct solutions to this question. Several candidates did not work to a sufficient degree of accuracy, usually resulting in a final answer of 0.155. An appreciable number of candidates did not realise that a conditional probability was required since the random sample was restricted to those with a Rhesus+ blood group. Consequently many answers involved 0.37 rather than $\frac{37}{83}$. Calculation of the binomial terms was very well performed by candidates. Nearly all candidates attempted to evaluate the correct three probabilities. Only on rare occasions was $P(0)$ omitted or $P(3)$ included. A few candidates’ attempts to use the binomial distribution included two probabilities which did not sum to 1 (e.g. 0.37 and 0.46).
- (ii) This question was generally well answered. Sometimes the continuity correction was omitted and, less often, standardising involved the variance instead of the standard deviation. Future candidates would benefit from working with a sufficient degree of accuracy resulted in accurate final answers.

Answers: (i) 0.156; (ii) 0.0854 or 0.0855.

MATHEMATICS

Paper 9709/71

Paper 71

Key messages

- It is important for candidates to read the questions carefully, and answer as required.
- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of script scores from high to low. In general, candidates scored well on **Questions 3 and 6**, whilst **Questions 2 and 5** proved particularly demanding.

Accuracy was not a great issue, though it should still be noted by candidates that for a final answer to be accurate to 3 significant figures, all working out up to the final answer needs to be accurate to at least 4 significant figures. Also, some candidates only gave final answers to two significant figures, this may have been due to a misunderstanding regarding zeros, or a confusion between significant figures and decimal places. In **Question 3(i)** the final answer was 0.0306 to 3 significant figures. Some candidates gave a final answer of 0.031, perhaps believing that this was 3 significant figures.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Many candidates correctly found the mean to be 2.6, but many stated that the variance was 2×1.3 , rather than 4×1.3 . The standard response (though other reasons were acceptable) for part (ii) was that the mean was not equal to the variance, thus causing confusion for candidates who made the error noted above.

Answer: (i) Mean=2.6, Variance=5.2; (ii) Variance $\neq$ Mean

Question 2

It is important on hypothesis testing questions that the Null and Alternative Hypotheses are stated. Many candidates omitted to do this. Most candidates who were able to make a reasonable attempt at the question realised that the approximating distribution was Normal, though errors with parameters were seen. It is important that the final conclusion made is fully justified by clearly stating the comparison (either z values or area) that is being made. This should be a clear inequality, or a clear diagram showing the relevant values. An un-qualified statement will not score even if correct.

Answer: Claim not justified.

Question 3

This was a particularly well attempted question, with many candidates scoring full marks. Some candidates worked with an incorrect variance, but other than this error, the general method required was well understood and applied.

Answer: 0.0306 or 0.0307



Question 4

There were many correct answers for part (i), though some candidates confused the two formulae for the unbiased estimate of the population variance. It is also important when applying either of these formulae that candidates appreciate the difference between Σx^2 and $(\Sigma x)^2$.

In part (ii) had many correct answers, though some candidates used the variance of 4.2 rather than the standard deviation ($\sqrt{4.2}$) when calculating the confidence interval.

In part (iii) many candidates realised that the claim was not supported, but it was important that a reason was clearly stated (i.e. saying that 333 was not in the interval). Merely repeating the confidence interval (330 to 332) from part (ii) without any real interpretation was not sufficient to gain the mark. It is important to make clear statements rather than just repeat previous findings.

Answer: (i) 331, 4.125; (ii) 330 to 332; (iii) No because 333 is not within the CI

Question 5

In part (i), many candidates used the answer given and showed that it was in the rejection region, whereas the question was asking the candidate to show that the rejection region was greater than 23.7. This was a common misunderstanding and illustrates the importance of reading the question carefully.

Some candidates successfully found the probability of a Type II error, but in general candidates find this topic challenging.

Answer: (ii) 0.0172

Question 6

It was important in part (i) that candidates gave their answers in the context of the question. Just to say 'independent' or 'at random' was not sufficient. It needed to be stated that 'customers arrived independently' or 'customers arrived randomly'.

Parts (ii), (iii) and (iv) were all well attempted. Some candidates forgot the continuity correction, or made mistakes in its calculation, in part (iv) but the method required was usually understood.

Answer: (i) Customers arrive independently; (ii) 0.161; (iii) 0.570; (iv) 0.869

Question 7

Questions on probability density functions are usually well attempted. However, this question proved challenging. Some candidates correctly identified the variables in part (i) but few were able to give a convincing argument. Part (ii) was not well attempted with few candidates correctly identified the equation of the line in fig 2. Candidates attempting to find the area of the trapezium were more successful though few used this method.

Part (iii) caused some confusion, particularly in part (b) when it was very often stated that $x \times x^n$ was x^{2n} rather than x^{n+1} .

Answer: (i) (a) X or 5; (b) V or 3, Higher and lower values more likely; (ii) 0.75; (iii) (b) $a=5$, $n=4$

MATHEMATICS

Paper 9709/72

Paper 72

Key messages

- It is important for candidates to read the questions carefully, and answer as required.
- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of script scores from high to low. In general, candidates scored well on **Questions 3 and 6**, whilst **Questions 2 and 5** proved particularly demanding.

Accuracy was not a great issue, though it should still be noted by candidates that for a final answer to be accurate to 3 significant figures, all working out up to the final answer needs to be accurate to at least 4 significant figures. Also, some candidates only gave final answers to two significant figures, this may have been due to a misunderstanding regarding zeros, or a confusion between significant figures and decimal places. In **Question 3(i)** the final answer was 0.0306 to 3 significant figures. Some candidates gave a final answer of 0.031, perhaps believing that this was 3 significant figures.

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Comments on specific questions

Question 1

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Answer: (i) Mean=2.6, Variance=5.2; (ii) Variance $\neq$ Mean

Question 2

It is important on hypothesis testing questions that the Null and Alternative Hypotheses are stated. Many candidates omitted to do this. Most candidates who were able to make a reasonable attempt at the question realised that the approximating distribution was Normal, though errors with parameters were seen. It is important that the final conclusion made is fully justified by clearly stating the comparison (either z values or area) that is being made. This should be a clear inequality, or a clear diagram showing the relevant values. An un-qualified statement will not score even if correct.

Answer: Claim not justified.

Question 3

This was a particularly well attempted question, with many candidates scoring full marks. Some candidates worked with an incorrect variance, but other than this error, the general method required was well understood and applied.

Answer: 0.0306 or 0.0307



Question 4

There were many correct answers for part (i), though some candidates confused the two formulae for the unbiased estimate of the population variance. It is also important when applying either of these formulae that candidates appreciate the difference between Σx^2 and $(\Sigma x)^2$.

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Part (iii) caused some confusion, particularly in part (b) when it was very often stated that $x \times x^n$ was x^{2n} rather than x^{n+1} .

Answer: (i) (a) X or 5; (b) V or 3, Higher and lower values more likely; (ii) 0.75; (iii) (b) $a=5$, $n=4$

MATHEMATICS

Paper 9709/73

Paper 73

Key messages

- It is important for candidates to read the questions carefully, and answer as required.

General comments

Many candidates answered this paper in an efficient manner, showing a good understanding of the contents of the syllabus. Work was generally well presented and clear to follow. Many candidates scored well on **Questions 3(ii), 4, 6 and 7** whilst some candidates found **Questions 1, 2, 3(i), (iii) and 5(ii)** more demanding.

The significance testing in **Question 5** was well handled by many candidates, with comparisons being fully shown.

Comments on specific questions

Question 1

Many correct solutions to this question were seen. Many candidates obtained the first equation correctly ($50 = a + 54b$).

Some candidates made mistakes in forming the variance equation, including the error $\text{Var}(a) = a$ (giving $100 = a + 144b^2$) or using b instead of b^2 (giving $100 = 144b$).

Answers: $b = 5/6$, $a = 5$

Question 2

Many correct solutions to this question were seen.

Some candidates did not establish the variance for the sampling distribution of proportions ($0.35 \times 0.65/n$). Some candidates included extra terms in n and other candidates omitted the $\sqrt{\quad}$ when setting up the equation for the width.

Some candidates incorrectly included 0.35 in the class interval width. Other candidates used the wrong value for z , but they were still able to obtain the method marks.

Answer: $n = 200$

Question 3

- (i) Candidates needed to refer to the numbering of the members, and not assume that this had already happened. Many candidates did state this initial step.

Then candidates needed to refer to sets of random numbers and the process of finding 3 digit numbers from them (to match to the previous member numbering).

Also candidates needed to explain what to do when numbers that were too large or repeated, were encountered.

The suggestion of drawing 8 out of 750 numbered papers from a container was not acceptable for this question. Apart from this being impractical for 750 members, the question required the use of random numbers (for example listed in a random number table) and how to use these random numbers.

- (ii) Many correct answers to this part were seen.

Some candidates mixed the two formula methods incorrectly.

A few candidates found only the sample variance, not the unbiased estimate of the population variance.

- (iii) Many candidates thought that population referred to the members of the club (or to the sample of 8 members). Candidates needed to realise that the variable being considered was the amount in dollars spent by the members last week in the cafe, so the population referred to these amounts spent individually by all of the members.

Answer: (ii) 20, 62.3; (iii) Amounts spent last week in the cafe by all club members.

Question 4

- (i) Many correct solutions showing sufficient steps of working for finding k were seen. Some candidates did make errors in the integration or in the substitution of the limits and made further adjustments to attempt to obtain the given answer. These candidates did not gain full marks.
- (ii) Many correct solutions were also seen for this part. Most candidates stated the correct function to be integrated. Some candidates did not perform the necessary integration by parts correctly.

The answer was required by the question to be in terms of e .

Answer: (ii) $(e-2)/(e-1)$

Question 5

- (i) The question asked for the relevant assumption to be stated. The appropriate assumption was that the population standard deviation remained the same (105). Some candidates suggested independent events or use of the same route. Many candidates simply omitted this statement.

Many correct testing methods were seen, showing the hypotheses ($H_0: \mu = 1150$ and $H_1: \mu < 1150$ or expressed in terms of “the population mean ...”) and the standardisation (using $105^2/20$), and a correct comparison and conclusion given in context. This work was well presented by many candidates.

Some candidates did not use the correct variance (sometimes omitting 20) or used the incorrect critical value (e.g. $z = -2.576$ which would have been appropriate for a 2-tail test).

The test could be completed using totals (21,800 and 23,000 and variance = 220,500). Candidates who attempted this procedure often mixed up the total distances with the variance of the means.

Most candidates did give their conclusion in the context of the question, as is appropriate.

- (ii) Sensibly many candidates wrote down the basic meaning of a Type I error and then attempted to convert this to the context of the question. Phrases such as “concluding that” or “saying that” were needed as part of the statement, so that two contradictory statements were not presented.

The probability was 0.01 and not for example 0.0053.

Answers: (i) There is evidence that the mean distance has decreased; (ii) 0.01, Concluding that the mean driving distance has decreased when in fact it has not.

Question 6

Many correct solutions were seen for at least one of the two parts of this question.

- (i) This part required the mean and the variance for the times for two mathematics lectures and one physics lecture to be found (127 and $2 \times 3.5^2 + 5.2^2 = 51.54$ for $X_1 + X_2 + Y$). Many candidates found these values. Other candidates incorrectly used 2^2 instead of 2 .
- (ii) This part required the mean and the variance for the times for one mathematics lecture and $1\frac{1}{2}$ x one physics lecture to be found (118.5 and $3.5^2 + 1\frac{1}{2} \times 5.2^2 = 73.09$ for $X + 1\frac{1}{2} \times Y$). Many candidates found these values. Other candidates incorrectly used $1\frac{1}{2}$ instead of $1\frac{1}{2}^2$.

A few candidates tried to use $N(118.5, 118.5)$.

Answer: (i) 0.965; (ii) 0.985

Question 7

Many fully correct solutions were seen for this question. Candidates handled the different time spans for the Poisson distributions very efficiently.

- (i) (a) Most candidates did change to the half-hour period and found $1 - P(0 \text{ or } 1 \text{ men})$ and $1 - P(0 \text{ women})$. Other candidates found $1 - P(0,1,2 \text{ men})$ and $1 - P(0,1 \text{ women})$, or omitted the “1 - “.

Most candidates multiplied their two probabilities. Other candidates added (even though the resulting answer was greater than 1), and some candidates left their two probabilities as separate answers.

- (b) Most candidates combined the Poisson distributions and found $1 - P(0, 1, 2)$ with $\lambda = 2.6$.

Other candidates used $\lambda = 5.2$ or found $1 - P(0, 1, 2, 3)$ or found $P(3)$.

- (ii) Most candidates realised the need to change from the Poisson distribution with large $\lambda = 52$ for the 10-hour period ($\lambda > 15$ as some candidates pointed out) to a Normal distribution $N(52, 52)$. A few candidates had $\lambda = 26$.

For this normal approximation a continuity correction was required (to 60.5). Many candidates did apply this correction. Other candidates omitted it or changed to 59.5.

Answer: (i) 0.254; (ii) 0.482; (iii) 0.119