UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/23 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Obtain derivative of the form $\frac{k}{5x+1}$, where $k = 1$, 5 or $\frac{1}{5}$ M1

Obtain correct derivative $\frac{5}{5x+1}$ A1

Substitute $x = 4$ into expression for derivative and obtain $\frac{5}{21}$ A1√ [3]

2 **EITHER** State or imply non-modular inequality $(2x - 3)^2 \leqslant (3x)^2$, or corresponding equation or pair of linear equations M1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1

Obtain critical values –3 and $\frac{3}{5}$ A1

State correct answer $x \leqslant -3$ or $x \geqslant \frac{3}{5}$ A1

OR State one critical value, e.g. $x = -3$, by solving a linear equation (or inequality) or from a graphical method or by inspection B1

State the other critical value correctly B2

State correct answer $x \leqslant -3$ or $x \geqslant \frac{3}{5}$ B1 [4]

3 Use $2\ln(x + 3) = \ln(x + 3)^2$ M1

Use law for addition or subtraction of logarithms M1

Obtain correct quadratic expression in x A1

Make reasonable solution attempt at a 3-term quadratic M1

State $x = 9$ and no other solutions (condone $x = -1$ not deleted) A1 [5]

4 **(i)** State correct expression $\frac{1}{2} + \frac{1}{2}\cos 2x$, or equivalent B1 [1]

(ii) Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly M1

State correct integral $\frac{1}{2}x + \frac{1}{4}\sin 2x$, or equivalent A1

Obtain correct integral (for $\sin 2x$ term) of $-\frac{1}{2}\cos 2x$ B1

Attempt to substitute limits, using exact values M1

Obtain given answer correctly A1 [5]

5 Use trig identity correctly to obtain a quadratic in $\tan 2\theta$ M1

Solve the quadratic correctly M1

Obtain $\tan 2\theta = 1$ or $-\frac{4}{5}$ A1

Obtain one correct answer A1

Carry out correct method for second answer from either root M1

Obtain remaining 3 answers from 22.5°, 112.5°, 70.7°, 160.7° and no others in the range A1

[Ignore answers outside the given range] [6]

6 **(i)** Substitute $x = 1$ or $x = -2$ and equate to zero — M1
Obtain a correct equation in any form with powers of x values calculated — A1
Obtain a second correct equation in any form — A1
Solve a relevant pair of equations for a or for b — M1
Obtain $a = 3$ and $b = -5$ — A1 [5]

(ii) Attempt division by $x^2 + x - 2$, or equivalent, and reach a partial quotient of $x^2 + kx$ — M1
Obtain partial quotient $x^2 + 2x$ — A1
Obtain $x^2 + 2x - 1$ with no errors seen — A1
S.C. M1A1√ if 'a' and/or 'b' incorrect — [3]

7 **(i)** At any stage, state the correct derivative of $e^{\frac{1}{2}x}$ — B1
Use product rule — M1
Obtain correct derivative in any form — A1
Equate derivative to 3 and obtain given equation correctly — A1 [4]

(ii) Consider sign of $2 + 6e^{-\frac{1}{2}x} - x$, or equivalent — M1
Complete the argument correctly with appropriate calculations — A1 [2]

(iii) Use the iterative formula correctly at least once — M1
Obtain final answer 3.21 — A1
Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (3.205, 3.215) — B1 [3]

8 **(i)** State $2y\frac{dy}{dx}$ as derivative of y^2, or equivalent — B1
Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ — M1
Obtain given answer correctly — A1 [3]

(ii) Equate gradient expression to –1 and rearrange — M1
Obtain $y = 2x$ — A1
Substitute into original equation to obtain an equation in x^2 (or y^2) — M1
Obtain $2x^2 - 3x - 2 = 0$ (or $y^2 - 3y - 4 = 0$) — A1
Correct method to solve their quadratic equation — M1
State answers $(-\frac{1}{2}, -1)$ and $(2, 4)$ — A1 [6]