This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO  Correct Working Only – often written by a ‘fortuitous’ answer
ISW  Ignore Subsequent Working
MR   Misread
PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.
1 Rearrange as \( e^{2x} - e^x - 6 = 0 \), or \( u^2 - u - 6 = 0 \), or equivalent 
B1
Solve a 3-term quadratic for \( e^x \) or for \( u \) 
M1
Obtain simplified solution \( e^x = 3 \) or \( u = 3 \) 
A1
Obtain final answer \( x = 1.10 \) and no other 

2 EITHER: Use chain rule 
M1
obtain \( \frac{dx}{dt} = 6 \sin t \cos t \), or equivalent 
A1
obtain \( \frac{dy}{dt} = -6 \cos^2 t \sin t \), or equivalent 
A1
Use \( \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \) 
M1
Obtain final answer \( \frac{dy}{dx} = -\cos t \) 
A1
OR: Express \( y \) in terms of \( x \) and use chain rule 
M1
Obtain \( \frac{dy}{dx} = k(2 - \frac{x}{3})^2 \), or equivalent 
A1
Obtain \( \frac{dy}{dx} = -(2 - \frac{x}{3})^2 \), or equivalent 
A1
Express derivative in terms of \( t \) 
M1
Obtain final answer \( \frac{dy}{dx} = -\cos t \) 

3 (i) EITHER: Attempt division by \( x^2 - x + 1 \) reaching a partial quotient of \( x^2 + kx \) 
M1
Obtain quotient \( x^2 + 4x + 3 \) 
A1
Equate remainder of form \( lx \) to zero and solve for \( a \), or equivalent 
M1
Obtain answer \( a = 1 \) 
A1
OR: Substitute a complex zero of \( x^2 - x + 1 \) in \( p(x) \) and equate to zero 
M1
Obtain a correct equation in \( a \) in any unsimplified form 
A1
Expand terms, use \( i^2 = -1 \) and solve for \( a \) 
M1
Obtain answer \( a = 1 \) 
[SR: The first M1 is earned if inspection reaches an unknown factor \( x^2 + Bx + C \) and an equation in \( B \) and/or \( C \), or an unknown factor \( Ax^2 + Bx + 3 \) and an equation in \( A \) and/or \( B \). The second M1 is only earned if use of the equation \( a = B - C \) is seen or implied.]

(ii) State answer, e.g. \( x = -3 \) 
B1
State answer, e.g. \( x = -1 \) and no others 
B1 [2]

4 Separate variables and attempt integration of at least one side 
M1
Obtain term \( \ln(x + 1) \) 
A1
Obtain term \( k \ln \sin 2\theta \), where \( k = \pm 1, \pm 2, \text{ or } \pm \frac{1}{2} \) 
M1
Obtain correct term \( \frac{1}{2} \ln \sin 2\theta \) 
A1
Evaluate a constant, or use limits \( \theta = \frac{1}{12} \pi \), \( x = 0 \) in a solution containing terms \( a \ln(x + 1) \) and \( b \ln \sin 2\theta \) 
M1
Obtain solution in any form, e.g. \( \ln(x + 1) = \frac{1}{7} \ln \sin 2\theta - \frac{1}{7} \ln \frac{1}{x} \) (f.t. on \( k = \pm 1, \pm 2, \text{ or } \pm \frac{1}{2} \)) 
A1 √
Rearrange and obtain \( x = \sqrt{(2 \sin 2\theta)} - 1 \), or simple equivalent 
A1 [7]
5 (i) Make recognisable sketch of a relevant graph over the given interval \[ B1 \]
Sketch the other relevant graph and justify the given statement \[ B1 \] \[ 2 \]

(ii) Consider the sign of \( \sec x - (3 - \frac{1}{2} x^2) \) at \( x = 1 \) and \( x = 1.4 \), or equivalent \[ M1 \]
Complete the argument with correct calculated values \[ A1 \] \[ 2 \]

(iii) Convert the given equation to \( \sec x = 3 - \frac{1}{2} x^2 \) or work vice versa \[ B1 \] \[ 1 \]

(iv) Use a correct iterative formula correctly at least once \[ M1 \]
Obtain final answer 1.13 \[ A1 \]
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135) \[ A1 \] \[ 3 \]

[SR: Successive evaluation of the iterative function with \( x = 1, 2, \ldots \) scores M0.]

6 (i) State or imply \( R = \sqrt{10} \) \[ B1 \]
Use trig formulae to find \( \alpha \) \[ M1 \]
Obtain \( \alpha = 71.57^\circ \) with no errors seen \[ A1 \] \[ 3 \]
[Do not allow radians in this part. If the only trig error is a sign error in \( \cos(x - \alpha) \) give M1A0]

(ii) Evaluate \( \cos^{-1} \left( \frac{2}{\sqrt{10}} \right) \) correctly to at least 1 d.p. (50.7684\(\ldots^\circ \)) (Allow 50.7\(^\circ \) here) \[ B1 \sqrt{ } \]
Carry out an appropriate method to find a value of \( 2\theta \) in \( 0^\circ < 2\theta < 180^\circ \) \[ M1 \]
Obtain an answer for \( \theta \) in the given range, e.g. \( \theta = 61.2^\circ \) \[ A1 \]
Use an appropriate method to find another value of \( 2\theta \) in the above range \[ M1 \]
Obtain second angle, e.g. \( \theta = 10.4^\circ \), and no others in the given range \[ A1 \] \[ 5 \]
[Ignore answers outside the given range.]
[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]
[SR: The use of correct trig formulae to obtain a 3-term quadratic in \( \tan \theta \), \( \sin 2\theta \), \( \cos 2\theta \), or \( \tan 2\theta \) earns M1; then A1 for a correct quadratic, M1 for obtaining a value of \( \theta \) in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]
7 (i) Use a correct method to express $\overrightarrow{OP}$ in terms of $\lambda$ M1
Obtain the given answer A1 [2]

(ii) EITHER: Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$ in terms of $\lambda$ M1
Using the correct method for the moduli, divide scalar products by products of moduli and express $\cos AOP = \cos BOP$ in terms of $\lambda$, or in terms of $\lambda$ and $OP$ M1*

OR1: Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of $\lambda$. M1
Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express $\cos AOP = \cos BOP$ in terms of $\lambda$, or $\lambda$ and $OP$ M1*

Obtain a correct equation in any form, e.g. \[
\frac{9 + 2\lambda}{3\sqrt{(9 + 4\lambda + 12\lambda^2)}} = \frac{11 + 14\lambda}{5\sqrt{(9 + 4\lambda + 12\lambda^2)}}
\] A1
Solve for $\lambda$ M1(dep*)
Obtain $\lambda = \frac{3}{8}$ A1 [5]

[SR: The M1* can also be earned by equating $\cos AOP$ or $\cos BOP$ to a sound attempt at $\cos \frac{1}{2} AOB$ and obtaining an equation in $\lambda$. The exact value of the cosine is $\sqrt{13/15}$, but accept non-exact working giving a value of $\lambda$ which rounds to 0.375, provided the spurious negative root of the quadratic in $\lambda$ is rejected.]

[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for $OP$ to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]

(iii) Verify the given statement correctly B1 [1]

8 (i) Use any relevant method to determine a constant M1
Obtain one of the values $A = 3$, $B = 4$, $C = 0$ A1
Obtain a second value A1
Obtain the third value A1 [4]

(ii) Integrate and obtain term $-3 \ln(2 - x)$ B1√
Integrate and obtain term $k \ln(4 + x^2)$ M1
Obtain term $2 \ln(4 + x^2)$ A1√
Substitute correct limits correctly in a complete integral of the form \[a \ln(2 - x) + b \ln(4 + x^2), \; ab \neq 0\] M1
Obtain given answer following full and correct working A1 [5]
9 (i) Use product rule

Obtain correct derivative in any form

Equate derivative to zero and solve for \( x \)

Obtain answer \( x = e ^ { - \frac{1}{2}} \), or equivalent

Obtain answer \( y = - \frac{1}{2} e ^ { - 1} \), or equivalent

(ii) Attempt integration by parts reaching \( kx^3 \ln x \pm k \int x^3 \cdot \frac{1}{x} \, dx \)

Obtain \( \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^3 \, dx \), or equivalent

Integrate again and obtain \( \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \), or equivalent

Use limits \( x = 1 \) and \( x = e \), having integrated twice

Obtain answer \( \frac{\ln e}{3} (2e^3 + 1) \), or exact equivalent

[SR: An attempt reaching \( ax^2 \left( x \ln x - x \right) + b \int 2x(x \ln x - x) \, dx \) scores M1. Then give the first A1 for \( I = x^2 \left( x \ln x - x \right) - 2I + \int 2x^2 \, dx \), or equivalent.]

10 (a) EITHER: Square \( x + iy \) and equate real and imaginary parts to 1 and \(-2\sqrt{6}\) respectively

Obtain \( x^2 - y^2 = 1 \) and \( 2xy = -2\sqrt{6} \)

Eliminate one variable and find an equation in the other

Obtain \( x^4 - x^2 - 6 = 0 \) or \( y^4 + y^2 - 6 = 0 \), or 3-term equivalent

Obtain answers \( \pm \sqrt{3} - i \sqrt{2} \), or equivalent

OR: Denoting \( 1 - 2\sqrt{6}i \) by \( Re^{i\theta} \), state, or imply, square roots are \( \pm \sqrt{Re^{i\left( \frac{\pi}{4} \right)}} \)

and find values of \( R \) and either \( \cos \theta \) or \( \sin \theta \) or \( \tan \theta \)

Obtain \( \pm \sqrt{5}(\cos \frac{1}{2}\theta + i \sin \frac{1}{2}\theta) \), and \( \cos \theta = \frac{1}{\sqrt{5}} \) or \( \sin \theta = -\frac{2\sqrt{6}}{\sqrt{5}} \) or \( \tan \theta = -2\sqrt{6} \)

Use correct method to find an exact value of \( \cos \frac{1}{2}\theta \) or \( \sin \frac{1}{2}\theta \)

Obtain \( \cos \frac{1}{2}\theta = \pm \frac{\sqrt{5}}{\sqrt{5}} \) and \( \sin \frac{1}{2}\theta = \pm \frac{i}{\sqrt{5}} \), or equivalent

Obtain answers \( \pm (\sqrt{3} - i \sqrt{2}) \), or equivalent

[Condone omission of \pm except in the final answers.]

(b) Show point representing \( 3i \) on a sketch of an Argand diagram

Show a circle with centre at the point representing \( 3i \) and radius 2

Shade the interior of the circle

Carry out a complete method for finding the greatest value of \( \arg z \)

Obtain answer 131.8° or 2.30 (or 2.3) radians

[The f.t. is on solutions where the centre is at the point representing \(-3i\).]