READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
1 Solve the inequality $|4 - 5x| < 3$. [3]

2 Show that $\int_2^6 \frac{2}{4x + 1} \, dx = \ln \frac{2}{5}$. [5]

3 The diagram shows the part of the curve $y = \frac{1}{2} \tan 2x$ for $0 \leq x \leq \frac{\pi}{2}$. Find the $x$-coordinates of the points on this part of the curve at which the gradient is 4. [5]

4 Solve the equation $3^{2x} - 7(3^x) + 10 = 0$, giving your answers correct to 3 significant figures. [5]

5 The polynomial $4x^3 + ax^2 + 9x + 9$, where $a$ is a constant, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(2x - 1)$ the remainder is 10.

(i) Find the value of $a$ and hence verify that $(x - 3)$ is a factor of $p(x)$. [3]

(ii) When $a$ has this value, solve the equation $p(x) = 0$. [4]
6  (i) Verify by calculation that the cubic equation
\[ x^3 - 2x^2 + 5x - 3 = 0 \]
has a root that lies between \( x = 0.7 \) and \( x = 0.8 \). [2]

(ii) Show that this root also satisfies an equation of the form
\[ x = \frac{ax^2 + 3}{x^2 + b}, \]
where the values of \( a \) and \( b \) are to be found. [2]

(iii) With these values of \( a \) and \( b \), use the iterative formula
\[ x_{n+1} = \frac{ax_n^2 + 3}{x_n^2 + b} \]
to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7  The parametric equations of a curve are
\[ x = e^{3t}, \quad y = t^2 e^t + 3. \]

(i) Show that \( \frac{dy}{dx} = \frac{t(t + 2)}{3e^{3t}} \). [4]

(ii) Show that the tangent to the curve at the point \((1, 3)\) is parallel to the \(x\)-axis. [2]

(iii) Find the exact coordinates of the other point on the curve at which the tangent is parallel to the \(x\)-axis. [2]

8  (i) By first expanding \( \cos(2x + x) \), show that
\[ \cos 3x \equiv 4 \cos^3 x - 3 \cos x. \] [5]

(ii) Hence show that
\[ \int_0^{\frac{\pi}{8}} (2 \cos^3 x - \cos x) \, dx = \frac{5}{12}. \] [5]