CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2012 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		State or imply $\ln e = 1$	B1	
		Apply at least one logarithm law for product or quotient correctly (or exponential equivalent)	M1	
		Obtain $x + 5 = ex$ or equivalent and hence $\dfrac{5}{e-1}$	A1	[3]
2	**(i)**	State or imply $R = 25$	B1	
		Use correct trigonometric formula to find α	M1	
		Obtain $16.26°$ **with no errors seen**	A1	[3]
	(ii)	Evaluate of $\sin^{-1}\dfrac{17}{R}$ $(= 42.84\ldots°)$	M1	
		Obtain answer $59.1°$	A1	[2]
3	**(i)**	Either Use correct quotient rule or equivalent to obtain		
		$\dfrac{dx}{dt} = \dfrac{4(2t+3) - 8t}{(2t+3)^2}$ or equivalent	B1	
		Obtain $\dfrac{dy}{dt} = \dfrac{4}{2t+3}$ or equivalent	B1	
		Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \bigg/ \dfrac{dx}{dt}$ or equivalent	M1	
		Obtain $\dfrac{1}{3}(2t+3)$ or similarly simplified equivalent	A1	
		Or Express t in terms of x or y e.g. $t = \dfrac{3x}{4-2x}$	B1	
		Obtain Cartesian equation e.g. $y = 2\ln\left(\dfrac{6}{2-x}\right)$	B1	
		Differentiate and obtain $\dfrac{dy}{dx} = \dfrac{2}{2-x}$	M1	
		Obtain $\dfrac{1}{3}(2t+3)$ or similarly simplified equivalent	A1	[4]
	(ii)	Obtain $2t = 3$ or $t = \dfrac{3}{2}$	B1	
		Substitute in expression for $\dfrac{dy}{dx}$ and obtain 2	B1	[2]

4			Separate variables correctly and integrate one side	M1	
			Obtain $\ln y = \ldots$ or equivalent	A1	
			Obtain $= 3\ln(x^2+4)$ or equivalent	A1	
			Evaluate a constant or use $x = 0$, $y = 32$ as limits in a solution containing terms $a \ln y$ and $b\ln\left(x^2+4\right)$	M1	
			Obtain $\ln y = 3\ln(x^2+4) + \ln 32 - 3\ln 4$ or equivalent	A1	
			Obtain $y = \frac{1}{2}\left(x^2+4\right)$ or equivalent	A1	[6]
5	(i)	Either	Use correct product rule	M1	
			Obtain $3e^{-2x} - 6xe^{-2x}$ or equivalent	A1	
			Substitute $-\frac{1}{2}$ and obtain 6e	A1	
		Or	Take ln of both sides and use implicit differentiation correctly	M1	
			Obtain $\frac{dy}{dx} = y\left(\frac{1}{x} - 2\right)$ or equivalent	A1	
			Substitute $-\frac{1}{2}$ and obtain 6e	A1	[3]
	(ii)		Use integration by parts to reach $kxe^{-2x} \pm \int ke^{-2x}\,dx$	M1	
			Obtain $-\frac{3}{2}xe^{-2x} + \int \frac{3}{2}e^{-2x}\,dx$ or equivalent	A1	
			Obtain $-\frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x}$ or equivalent	A1	
			Substitute correct limits correctly	DM1	
			Obtain $-\frac{3}{4}$ with no errors or inexact work seen	A1	[5]
6	(i)		Find y for $x = -2$	M1	
			Obtain 0 and conclude that $\alpha = -2$	A1	[2]
	(ii)	Either	Find cubic factor by division or inspection or equivalent	M1	
			Obtain $x^3 + 2x - 8$	A1	
			Rearrange to confirm given equation $x = \sqrt[3]{8-2x}$	A1	
		Or	Derive cubic factor from given equation and form product with $(x - \alpha)$	M1	
			$(x+2)\left(x^3 + 2x - 8\right)$	A1	
			Obtain quartic $x^4 + 2x^3 + 2x^2 - 4x - 16\ (= 0)$	A1	
		Or	Derive cubic factor from given equation and divide the quartic by the cubic	M1	
			$\left(x^4 + 2x^3 + 2x^2 - 4x - 16\right) \div \left(x^3 + 2x - 8\right)$	A1	
			Obtain correct quotient and zero remainder	A1	[3]
	(iii)		Use the given iterative formula correctly at least once	M1	
			Obtain final answer 1.67	A1	
			Show sufficient iterations to at least 4 d.p. to justify answer 1.67 to 2 d.p. or show there is a change of sign in interval (1.665, 1.675)	A1	[3]

7	(i)	State or imply $du = 2\cos 2x\,dx$ or equivalent		B1	
		Express integrand in terms of u and du		M1	
		Obtain $\int \frac{1}{2}u^3(1-u^2)\,du$ or equivalent		A1	
		Integration to obtain an integral of the form $k_1u^4 + k_2u^6, k_1, k_2 \neq 0$		M1	
		Use limits 0 and 1 or (if reverting to x) 0 and $\frac{1}{4}\pi$ correctly		DM1	
		Obtain $\frac{1}{24}$, or equivalent		A1	[6]
	(ii)	Use 40 and upper limit from part **(i)** in appropriate calculation		M1	
		Obtain $k = 10$ with no errors seen		A1	[2]
8	(i)	State or imply general point of either line has coordinates $(5 + s, 1 - s, -4 + 3s)$ or $(p + 2t, 4 + 5t, -2 - 4t)$		B1	
		Solve simultaneous equations and find s and t		M1	
		Obtain $s = 2$ and $t = -1$ or equivalent in terms of p		A1	
		Substitute in third equation to find $p = 9$		A1	
		State point of intersection is $(7, -1, 2)$		A1	[5]
	(ii)	Either	Use scalar product to obtain a relevant equation in a, b, c e.g. $a - b + 3c = 0$ or $2a + 5b - 4c = 0$	M1	
			State two correct equations in a, b, c	A1	
			Solve simultaneous equations to obtain at least one ratio	DM1	
			Obtain $a : b : c = -11 : 10 : 7$ or equivalent	A1	
			Obtain equation $-11x + 10y + 7z = -73$ or equivalent with integer coefficients	A1	
		Or 1	Calculate vector product of $\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and $\begin{pmatrix}2\\5\\-4\end{pmatrix}$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct $\begin{pmatrix}-11\\10\\7\end{pmatrix}$ or equivalent	A1	
			Substitute coordinates of a relevant point in $\mathbf{r}.\mathbf{n} = d$ to find d	DM1	
			Obtain equation $-11x + 10y + 7z = -73$ or equivalent with integer coefficients	A1	
		Or 2	Using relevant vectors, form correctly a two-parameter equation for the plane	M1	
			Obtain $\mathbf{r} = \begin{pmatrix}5\\1\\-4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix} + \mu\begin{pmatrix}2\\5\\-4\end{pmatrix}$ or equivalent	A1	
			State three equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	DM1	
			Obtain $11x - 10y - 7z = 73$ or equivalent with integer coefficients	A1	[5]

9 **(i)** State or imply form $\dfrac{A}{3-x}+\dfrac{Bx+C}{1+x^2}$ — B1

Use relevant method to determine a constant — M1

Obtain $A = 6$ — A1

Obtain $B = -2$ — A1

Obtain $C = 1$ — A1 [5]

(ii) Either Use correct method to obtain first two terms of expansion of $(3-x)^{-1}$ or $\left(1-\dfrac{1}{3}x\right)^{-1}$ or $\left(1+x^2\right)^{-1}$ — M1

Obtain $\dfrac{A}{3}\left(1+\dfrac{1}{3}x+\dfrac{1}{9}x^2+\dfrac{1}{27}x^3\right)$ — A1

Obtain $(Bx + C)(1 - x^2)$ — A1

Obtain sufficient terms of the product $(Bx + C)(1 - x^2)$, $B, C \neq 0$ and add the two expansions — M1

Obtain final answer $3-\dfrac{4}{3}x-\dfrac{7}{9}x^2+\dfrac{56}{27}x^3$ — A1

Or Use correct method to obtain first two terms of expansion of $(3-x)^{-1}$ or $\left(1-\dfrac{1}{3}x\right)^{-1}$ or $\left(1+x^2\right)^{-1}$ — M1

Obtain $\dfrac{1}{3}\left(1+\dfrac{1}{3}x+\dfrac{1}{9}x^2+\dfrac{1}{27}x^3\right)$ — A1

Obtain $(1 - x^2)$ — A1

Obtain sufficient terms of the product of the three factors — M1

Obtain final answer $3-\dfrac{4}{3}x-\dfrac{7}{9}x^2+\dfrac{56}{27}x^3$ — A1 [5]

10 **(a)** Expand and simplify as far as $\mathrm{i}w^2 = -8\mathrm{i}$ or equivalent — B1

Obtain first answer $\mathrm{i}\sqrt{8}$, or equivalent — B1

Obtain second answer $-\mathrm{i}\sqrt{8}$, or equivalent and no others — B1 [3]

(b) **(i)** Draw circle with centre in first quadrant — M1

Draw correct circle with interior shaded or indicated — A1 [2]

(ii) Identify ends of diameter corresponding to line through origin and centre — M1

Obtain $p = 3.66$ and $q = 7.66$ — A1

Show tangents from origin to circle — M1

Evaluate $\sin^{-1}\left(\dfrac{1}{4}\sqrt{2}\right)$ — M1

Obtain $\alpha=\dfrac{1}{4}\pi-\sin^{-1}\left(\dfrac{1}{4}\sqrt{2}\right)$ or equivalent and hence 0.424 — A1

Obtain $\beta=\dfrac{1}{4}\pi+\sin^{-1}\left(\dfrac{1}{4}\sqrt{2}\right)$ or equivalent and hence 1.15 — A1 [6]