UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1604442140

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

JC12 11_9709_13/RP

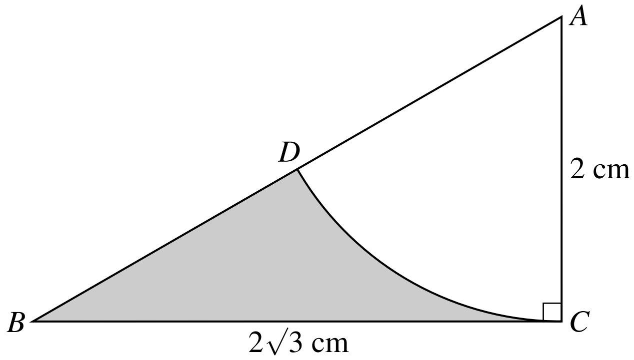
UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Find the coefficient of x^3 in the expansion of $\left(2 - \frac{1}{2}x\right)^7$. [3]

2 It is given that $f(x) = \dfrac{1}{x^3} - x^3$, for $x > 0$. Show that f is a decreasing function. [3]

3 Solve the equation $7\cos x + 5 = 2\sin^2 x$, for $0° \leqslant x \leqslant 360°$. [4]

4



In the diagram, D lies on the side AB of triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. [4]

5 The first term of a geometric progression is $5\frac{1}{3}$ and the fourth term is $2\frac{1}{4}$. Find

(i) the common ratio, [3]

(ii) the sum to infinity. [2]

6 The functions f and g are defined for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$ by

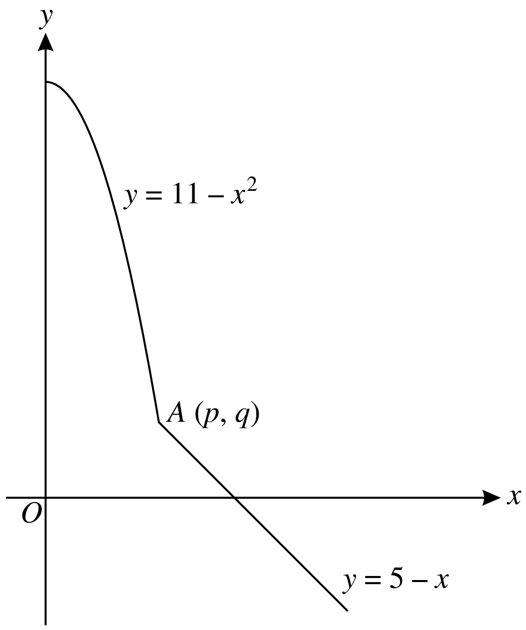
$$f(x) = \tfrac{1}{2}x + \tfrac{1}{6}\pi,$$
$$g(x) = \cos x.$$

Solve the following equations for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) $gf(x) = 1$, giving your answer in terms of π. [2]

(ii) $fg(x) = 1$, giving your answers correct to 2 decimal places. [4]

7



(i) The diagram shows part of the curve $y = 11 - x^2$ and part of the straight line $y = 5 - x$ meeting at the point $A\ (p, q)$, where p and q are positive constants. Find the values of p and q. [3]

(ii) The function f is defined for the domain $x \geqslant 0$ by

$$\mathrm{f}(x) = \begin{cases} 11 - x^2 & \text{for } 0 \leqslant x \leqslant p, \\ 5 - x & \text{for } x > p. \end{cases}$$

Express $\mathrm{f}^{-1}(x)$ in a similar way. [5]

8 A curve is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(3x + 4)^{\frac{3}{2}} - 6x - 8.$$

(i) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

(ii) Verify that the curve has a stationary point when $x = -1$ and determine its nature. [2]

(iii) It is now given that the stationary point on the curve has coordinates $(-1, 5)$. Find the equation of the curve. [5]

9 The position vectors of points A and B relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} p \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix},$$

where p is a constant.

(i) In the case where OAB is a straight line, state the value of p and find the unit vector in the direction of $\overrightarrow{OA}$. [3]

(ii) In the case where OA is perpendicular to AB, find the possible values of p. [5]

(iii) In the case where $p = 3$, the point C is such that $OABC$ is a parallelogram. Find the position vector of C. [2]

10 A straight line has equation $y = -2x + k$, where k is a constant, and a curve has equation $y = \dfrac{2}{x-3}$.

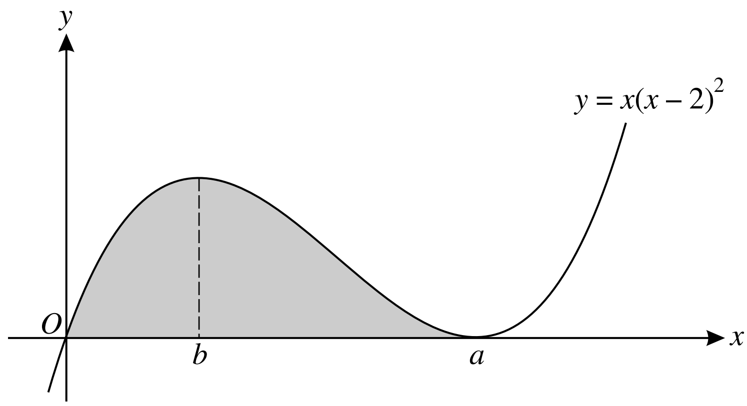
(i) Show that the x-coordinates of any points of intersection of the line and curve are given by the equation $2x^2 - (6 + k)x + (2 + 3k) = 0$. [1]

(ii) Find the two values of k for which the line is a tangent to the curve. [3]

The two tangents, given by the values of k found in part **(ii)**, touch the curve at points A and B.

(iii) Find the coordinates of A and B and the equation of the line AB. [6]

11



The diagram shows the curve with equation $y = x(x - 2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x-coordinate of the maximum point is b, where a and b are constants.

(i) State the value of a. [1]

(ii) Find the value of b. [4]

(iii) Find the area of the shaded region. [4]

(iv) The gradient, $\dfrac{dy}{dx}$, of the curve has a minimum value m. Find the value of m. [4]