This document consists of 3 printed pages and 1 blank page.
The diagram shows the graph of the probability density function, \( f \), of a random variable \( X \). Find the median of \( X \). \[3\]

2 The heights of a certain type of plant have a normal distribution. When the plants are grown without fertilizer, the population mean and standard deviation are 24.0 cm and 4.8 cm respectively. A gardener wishes to test, at the 2\% significance level, whether Hiergro fertilizer will increase the mean height. He treats 150 randomly chosen plants with Hiergro and finds that their mean height is 25.0 cm. Assuming that the standard deviation of the heights of plants treated with Hiergro is still 4.8 cm, carry out the test. \[5\]

3 The cost of hiring a bicycle consists of a fixed charge of 500 cents together with a charge of 3 cents per minute. The number of minutes for which people hire a bicycle has mean 142 and standard deviation 35.

(i) Find the mean and standard deviation of the amount people pay when hiring a bicycle. \[3\]

(ii) 6 people hire bicycles independently. Find the mean and standard deviation of the total amount paid by all 6 people. \[3\]

4 A cereal manufacturer claims that 25\% of cereal packets contain a free gift. Lola suspects that the true proportion is less than 25\%. In order to test the manufacturer’s claim at the 5\% significance level, she checks a random sample of 20 packets.

(i) Find the critical region for the test. \[5\]

(ii) Hence find the probability of a Type I error. \[1\]

Lola finds that 2 packets in her sample contain a free gift.

(iii) State, with a reason, the conclusion she should draw. \[2\]
5 A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} 
    \frac{k}{x-1} & 3 \leq x \leq 5, \\
    0 & \text{otherwise,}
\end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{1}{\ln 2}$. [4]

(ii) Find $a$ such that $P(X < a) = 0.75$. [4]

6 In order to obtain a random sample of people who live in her town, Jane chooses people at random from the telephone directory for her town.

(i) Give a reason why Jane’s method will not give a random sample of people who live in the town. [1]

Jane now uses a valid method to choose a random sample of 200 people from her town and finds that 38 live in apartments.

(ii) Calculate an approximate 99% confidence interval for the proportion of all people in Jane’s town who live in apartments. [4]

(iii) Jane uses the same sample to give a confidence interval of width 0.1 for this proportion. This interval is an $x\%$ confidence interval. Find the value of $x$. [4]

7 A random variable $X$ has the distribution Po$(1.6)$.

(i) The random variable $R$ is the sum of three independent values of $X$. Find $P(R < 4)$. [3]

(ii) The random variable $S$ is the sum of $n$ independent values of $X$. It is given that

$$P(S = 4) = \frac{16}{3} \times P(S = 2).$$

Find $n$. [4]

(iii) The random variable $T$ is the sum of 40 independent values of $X$. Find $P(T > 75)$. [4]