1 Solve the inequality $|x + 1| < |3x + 5|$. [4]

2

The diagram shows the curve $y = x^4 + 2x - 9$. The curve cuts the positive $x$-axis at the point $P$.

(i) Verify by calculation that the $x$-coordinate of $P$ lies between 1.5 and 1.6. [2]

(ii) Show that the $x$-coordinate of $P$ satisfies the equation

$$x = \frac{3}{\sqrt[3]{9/x - 2}}.$$ [1]

(iii) Use the iterative formula

$$x_{n+1} = \frac{3}{\sqrt[3]{9/x_n - 2}}$$

to determine the $x$-coordinate of $P$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3 The equation of a curve is $y = \frac{1}{2}e^{2x} - 5e^x + 4x$. Find the exact $x$-coordinate of each of the stationary points of the curve and determine the nature of each stationary point. [6]

4 (i) The polynomial $x^3 + ax^2 + bx + 8$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(x - 3)$ the remainder is 14, and that when $p(x)$ is divided by $(x + 2)$ the remainder is 24. Find the values of $a$ and $b$. [5]

(ii) When $a$ and $b$ have these values, find the quotient when $p(x)$ is divided by $x^2 + 2x - 8$ and hence solve the equation $p(x) = 0$. [4]

5 The parametric equations of a curve are

$$x = \cos 2\theta - \cos \theta, \quad y = 4 \sin^2 \theta,$$

for $0 \leq \theta \leq \pi$.

(i) Show that $\frac{dy}{dx} = \frac{8 \cos \theta}{1 - 4 \cos \theta}$. [4]

(ii) Find the coordinates of the point on the curve at which the gradient is $-4$. [4]
6  (a) Find

(i) \( \int \frac{e^{2x} + 6}{e^{2x}} \, dx \), \[3\]

(ii) \( \int 3 \cos^2 x \, dx \). \[3\]

(b) Use the trapezium rule with 2 intervals to estimate the value of

\[ \int_1^2 \frac{6}{\ln(x + 2)} \, dx, \]

giving your answer correct to 2 decimal places. \[3\]

7  (i) Express \( 3 \cos \theta + \sin \theta \) in the form \( R \cos (\theta - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), giving the exact value of \( R \) and the value of \( \alpha \) correct to 2 decimal places. \[3\]

(ii) Hence solve the equation

\[ 3 \cos 2x + \sin 2x = 2, \]

giving all solutions in the interval \( 0^\circ \leq x \leq 360^\circ \). \[5\]