MATHEMATICS

Paper 9709/11
Paper 11

Key Messages

Success at this level is highly dependent on candidates demonstrating skilful use of algebraic techniques. On this paper the ability to change the subject of a formula, complete the square, clear the denominator of a fraction and manipulate inequalities were the key to success.

Candidates need to be careful to include the arbitrary constant when carrying out integration without limits.

General Comments

Whilst the presentation of solutions has improved there are still instances of candidates dividing pages into two or even three columns. This makes it very difficult for Examiners to indicate clearly where marks are awarded and should be actively discouraged.

Comments on Specific Questions

Question 1

The expansion of $(a + b)^n$ was used effectively to give the terms in x and x^2, although it was common to see $(ax)^2$ becoming ax^2. The candidates who appreciated the meaning of 'coefficient' usually equated their two expressions to reach a solution. Some equated the terms and incorrectly found a in terms of x.

Answer: $a = \frac{2}{3}$

Question 2

Those candidates who used calculators (in either degrees or radians mode) together with the correct order of operations were quickly able to reach the correct solution. Attempts to simplify the equation prior to using the calculator were almost always incorrect. A neat non-calculator solution was to use the 1,3,√10 triangle to give $\sin(\tan^{-1} 3) = 3/\sqrt{10}$.

Answer: $x = 1 + 3\sqrt{10}$ (or 1.95)

Question 3

The use of $\sin^2\theta + \cos^2\theta = 1$ to give an equation in terms of $\cos\theta$ was the preferred route of the majority. Although most candidates appreciated the need to clear the fraction only a few went on to do this correctly and obtain a value of $\cos^2\theta$. The negative value of $\cos\theta$ was often omitted. A few candidates offered solutions in radians when the range was given in degrees.

Answers: x = 30° and 150°

Question 4

Most candidates scored well on this question. Those who opted to substitute (8, – 4) to find k and then substituted k and (b, $2b$) to find b usually gained all the first 4 marks. Those who chose to equate the gradient of AB with the gradient of the line mostly failed to find a second equation in b and k and made no further progress. Credit was given in part **(ii)** for working out the mid-point using the candidate's value of b.

Answers: **(i)** $k = 3$, $b = 2$ **(ii)** (5, 0)

Question 5

This type of problem provides the opportunity for candidates to use their skills in algebraic manipulation to group like terms, select coefficients and solve inequalities. A few realised that the coefficient of x and the constant both consisted of 2 terms and then used the discriminant to find the critical values. Most solutions ended after the elimination of y. Those who realised a set of values of k was required rarely illustrated this with the correct diagram or inequalities.

Answer: $k < 2$, $k > 6$

Question 6

The use of a simple diagram showing triangle ABC with the right angle at A would have helped many candidates realise that vector **AB** was required in part **(i)** and which sides to use in part **(ii)**.

In part **(i)** those who found **AB** (or **BA**) were able to use the scalar product or Pythagoras' theorem to show the right angle at A. Candidates who found **OA**.**OB** might have realised that a non-zero result indicated they had selected the wrong vectors but none did. It was quite common to see |**OA**| and |**AB**| found in part **(i)** although they were not needed until part **(ii)**.

Those who had realised AB should be used in part **(i)** were usually successful in answering part **(ii)** using ½ × base × height for the area. Very occasionally angle AOB was found and '½absin$C$' was used successfully.

Answer: 12.5 (or exact equivalent)

Question 7

Most candidates were able to quote the correct formulae in both parts either from the formulae booklet or from memory. In part **(i)** candidates should have realised the magnitude of r cannot exceed 1 and in part **(ii)** that the value of n must be a positive integer.

In part **(i)** the two sums to infinity were usually found correctly with the successful candidates then eliminating S followed by a. Most of those who chose other elimination routes did not have the algebraic skills to reach the solution. Although it was indicated that the sum to infinity of the second series was three times the sum to infinity of the first series, candidates regularly formulated this the other way round.

In part **(ii)** the equations from the n^{th} term formula were often found correctly. The straightforward elimination of d to leave a fractional equation in n was often overlooked in favour of finding d in terms of n from one equation and substituting this into the other. This route often led to algebraic errors although a few candidates persevered to find d and then n successfully.

Answers: **(i)** $\frac{2}{5}$ **(ii)** 23

Question 8

In part **(i)** candidates appreciated which lengths formed the perimeter and the arc length formula was used effectively to find arc *AB.* The given perpendicular, *AD*, should have prompted more candidates to find *OD* correctly. The arc length *CD* was often written as $4cos\alpha\alpha$ and wrongly simplified to $4cos^2\alpha$. The candidates who used brackets or more conventional ordering of terms within the expression avoided this confusion. Some used $\alpha = \theta$ without stating this and produced an answer in terms of θ.

In part **(ii)** the formula for sector area was often used correctly to find the area of sector *OAB*. The candidates who were able to find $cos^2\left(\frac{\pi}{3}\right)$ and simplify this in the expression for the area of sector *OCD* produced the best answers.

Answers: **(i)** $8 + 4\alpha cos\alpha + 4\alpha - 8cos\alpha$ **(ii)** $\frac{\pi}{3}$ or $k = \frac{1}{3}$

Question 9

Part **(i)** was well answered by the majority using $\frac{-1}{f'(2)}$ to find the gradient of the normal and then correctly substituting to find the normal equation.

In part **(ii)** most candidates realise that integration of f(x) was required but this was very often incomplete because of the omission of the constant of integration.

In part **(iii)** equating f′(x) to zero and the subsequent solution were often seen. Calculation of f′ (1) or the change in sign of f′(x) were the most commonly used methods to identify the stationary point. When using the change of sign of f(x) candidates should use values of x close to that of the turning point. In this case, taking values of x below zero (for which the function was undefined) gave a fortuitously correct sign of f′(x).

Answers: **(i)** $7y + 2x = 46$ **(ii)** $y = x^2 + \frac{2}{x} + 1$ **(iii)** $x = 1$, minimum

Question 10

The completion of the square in part **(i)** was generally done well as would be expected for this basic quadratic expression.

In part **(ii)** there was some confusion between x and f(x) but candidates with a completed square form of f(x) usually realised that c was their value of b from part **(i)**.

Equating f(x) to 9 and 65 in part **(iii)** gave the required values for those candidates who had correctly completed the square or went back to the original expression. Some reached the correct values for p and q but selected a negative alternative for p even though the question stated that both p and q were positive.

Part **(iv)** involved changing the subject of the completed square form. Some candidates who already had this form restarted the process. Those who used the required form generally changed the subject using inverse operations in the correct order. The range of f(x) indicated the positive square root should be taken but $\pm\sqrt{x+16}$ was often seen in the final answer.

Answers: **(i)** $(x - 1)^2 - 16$ **(ii)** -16 **(iii)** $p = 6$, $q = 10$ **(iv)** $f^{-1}(x) = 1 + \sqrt{x+16}$

Question 11

Part **(ii)** was attempted by most candidates whilst part **(i)** was often left un-attempted.

In part **(i)** those candidates who realised the gradients at Q of both curves were required usually made some progress through the required differentiation and substitution. The use of the gradients to find the angle each tangent made with the x-axis was not often seen and very few reached a final answer. Those who used a vector approach to find the angle were very few in number but they were generally successful.

The techniques required to answer part **(ii)** were much more familiar to candidates and many were able to work to a final solution. The integration of $(4x + 1)^{1/2}$ caused some problems and some candidates wrongly assumed there was no point in substituting the lower limit after their integration.

Answers: **(i)** 29.7° **(ii)** 1

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Paper 9709/12
Paper 12

Key Message

Centres and candidates should be aware that, generally, if a question is subdivided into parts labelled **(i)**, **(ii)** and **(iii)**, this implies that there is a link between the parts. If a candidate is unsure how to proceed with parts **(ii)** or **(iii)** of a question then a good strategy is to look back to the earlier parts and see what has already been achieved. Some candidates would benefit from taking extra care with their calculations in order to avoid errors such as: if $y = x^2 + 1$ then $x^2 = y + 1$.

General Comments

Many good and excellent scripts were seen and the standard of presentation was usually good. The paper seemed to give all candidates the opportunity to show what they had learned and understood on a number of questions, although these were spread throughout the paper. Other questions provided more of a challenge, even for candidates of good ability. Most candidates appeared to have had sufficient time to complete the paper although there was evidence of some candidates rushing towards the end of the paper.

Comments on specific questions

Question 1

Nearly every candidate attempted this question but, although it was straightforward for the better candidates, unfortunately many others attempted either to calculate the area rather than the volume, or the volume about the x-axis rather than the y-axis. On the paper 'y-axis' was printed in bold type and candidates should be aware that this indicates that care should be taken to avoid a mistake. Some candidates tried to find x and then struggled to square $\sqrt{y-1}$ whereas those who found x^2 directly made better progress. There was often confusion over which limits to use, although candidates who attempted the volume generally remembered to use π.

Answer: 8π

Question 2

Only a minority of candidates achieved full marks on this question.

Part **(i)** was done correctly by most candidates. A few candidates used degrees, and some used the sine or cosine rules rather than right-angled triangle trigonometry.

In part **(ii)** a pleasing number of candidates were able to find the angle ABO in radians. Also many candidates were able to find the area of a sector, although some candidates calculated the arc length instead or used $\frac{1}{2}r^2\theta$ with θ in degrees. Some candidates did not realise the need to use the area of the triangle or were unable to find it. Only the more able candidates were able to put together their calculated areas in the correct configuration to produce the required result for the shaded region. Candidates would benefit from further practice when dealing with areas of more unusual regions.

Answer: 13.1

Question 3

Almost every candidate was able to answer the first part correctly but the second part was more challenging. Those who were able to make successful progress realised the link between the two parts of the question and used the first part to help with the second. A common error in method was to assume that the question required candidates to multiply $1 + 5x + 10x^2$ by $1 + px + x^2$ and then equate coefficients. Candidates would benefit from reading the question carefully before starting to answer it. For those with the correct approach, a common error was only to consider one term, usually $10(px + x^2)^2$. Another common error was to multiply out $10(px + x^2)^2$ incorrectly, omitting to square the p in the term $10p^2x^2$.

Answer: $p = 3$

Question 4

Many candidates were able to differentiate correctly, with only some forgetting to multiply by –2 or losing a minus sign on one of the two terms. Many were then unsure as to how to proceed with part **(ii)** of the question and the link between the two parts was missed by many. Those who were most successful wrote down an expression for dy/dx in terms of dy/dt and dx/dt before substituting the given values in to this expression and then equating it with the answer from part **(i)**.

Some candidates misunderstood the information in the question and thought that the given values were x and y rather than the rates of change of these variables. There was some premature approximation with $\frac{8}{3} = 2.7$ sometimes seen.

Answer: 0 or 3

Question 5

There were many completely correct solutions to both parts of this question. In part **(i)** the most favoured route was to replace tan$x$ by sin$x$/cos$x$ then multiply through by cos$x$. Although the answer was given, most answers showed clearly how it was reached using the identity $\cos^2 x + \sin^2 x = 1$. A few candidates forgot to multiply every term by cos$x$ and therefore failed to obtain the given answer.

In part **(ii)** nearly all candidates used the quadratic result from part **(i)** and most obtained the correct values of $\frac{1}{2}$ and $\frac{-1}{3}$. Most candidates dealt well with the negative cosine although some included the base angle (71.5°) in their list of solutions and others incorrectly rounded to 109.4°. Most answers were presented in degrees as the question suggested, and given to 1 decimal place, although some candidates rounded to 109° or 110° and lost a mark. Candidates need to be aware that, as the rubric on the question paper states, answers should be given to one decimal place in the case of angles, rather than 3 significant figures as for other answers.

Answer: **(ii)** 60°, 109.5°

Question 6

In part **(i)** the majority of candidates realised they had to consider $b^2 - 4ac$, but a number did not differentiate but attempted to find the discriminant of the cubic equation for y. Those who realised the need for differentiation sometimes obtained $2x$ rather than $2ax$.

In part **(ii)** a significant minority of candidates equated y to zero and solved rather than differentiating first. The majority differentiated and then solved the quadratic equation correctly but some simply gave the two values of x without looking for the correct inequality.

Answer: **(ii)** $1 < x < 3$

Question 7

In part **(i)** only some candidates calculated **AM** correctly. Their answers were often obtained by adding a number of vectors together, for example realising that **OM** was half of **OX** and proceeding to add that vector to **AO**. More candidates would benefit from this approach rather than simply stating an answer. Some candidates found the vector for a point immediately above the midpoint of *OC* so that their **i** component was –8 instead of –6. The majority of candidates were able to state **AC** correctly. In part **(ii)**, mistakes in **(i)** meant there was a limited number of fully correct answers. The majority of candidates who made their working clear did achieve the method marks for multiplying two moduli together and linking them correctly.

Answers: **(i)** $-6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, $-8\mathbf{i} + 8\mathbf{j}$, **(ii)** 45.4°

Question 8

A minority of candidates answered this question well and found no difficulty in obtaining full marks. Many others found it challenging, particularly the first part. In part **(a)** the most successful approach was substituting $n = 1$ to calculate the first term. Some candidates then assumed that S_2 was the second term and incorrectly calculated the difference as 29. A number of good candidates attempted to equate the correct formulae, $S_n = \frac{n}{2}\{2a + (n-1)d\} = 32n - n^2$ and spent a considerable amount of time working through algebra with little progress towards a solution. Others, when they had correctly reached the stage $64n - 2n^2 = n(2a + dn - d)$, abandoned the question as they did not realise the need to equate the coefficients. Some made algebraic/arithmetic errors when comparing terms, and hence were unable to solve their equations. Candidates who set their work out in a concise and logical manner were more successful.

In part **(b)** most candidates used the correct formula for the sum to infinity, and were accurate in using a formula for the sum of 2 terms. A few misinterpreted the question and believed that the second term was 12.8, so wrote $ar = 12.8$ (or $ar^2 = 12.8$) and then rearranged to substitute. Candidates who recognised that $a + ar = 12.8$ were often more successful in progressing further. Most candidates recognised that they needed to solve their simultaneous equations, though many substituted to produce an equation in r^3 which they failed to solve successfully. The most successful candidates realised that a factor of $(1 - r)$ could be cancelled to form a quadratic equation in r. Some candidates tried to use the formulae for an A.P whilst others incorrectly wrote $S_\infty = \frac{a}{1+r}$ or $S_2 = \frac{a(1-r)^2}{1-r}$.Some candidates achieved the correct value for r (0.6) but forgot to then find "*a*".

Answers: **(a)** $a = 31$, $d = -2$, **(b)** $a = 8$

Question 9

Many candidates found the first two parts of this question straightforward but only a few candidates completed the third part successfully.

In part **(i)** most knew how to find the gradient of *AB* though some inverted the formula or made errors when subtracting negative values. A correct unsimplified $\frac{-9}{3}$ was sometimes followed by "$= \frac{1}{3}$", or "–2". Some then used this gradient and found the equation of *AB*, not *AD*, but most went on to use $mm' = -1$ and then find the equation of *AD*. For some candidates the perpendicular to *AB* had gradient 3.

In part **(ii)** most candidates appreciated that they had to find the equation of another line and solve simultaneously to obtain the point of intersection, but in a number of cases they found the equation of *AB* rather than *CD* and ended up with (2, 6), one of the points already given, as their answer. A number found the equation of *BC* and then its intersection with *AD* rather than the point *D*.

In part **(iii)** the most straightforward method was to add the vector **BC** to the co-ordinates of *A* but this was rarely seen. Some candidates found the equation of *AE* and solved it with *CD* or used the mid-point of *AC* and doubled its distance from *B,* but these were uncommon. Candidates who produced a sketch or successfully used the given diagram made better progress; more candidates would benefit from doing this.

Answer: **(i)** $y - 6 = \frac{1}{3}(x - 2)$; **(ii)** *D* (6.5, 7.5); **(iii)** 15

Question 10

This was a fairly complicated two stage integration question which some candidates found difficult. Many others were able to tackle the whole question successfully and obtain full marks.

Part **(i)** was answered correctly by most candidates. Some concluded the stationary point was a minimum, despite finding a negative value for the second differential. Others used the correct value of x but made a mistake in simplification, or wrote the wrong equation, and got a positive value for the second differential.

In part **(ii)** the majority of candidates integrated correctly but many forgot the necessary constant and lost marks. Other candidates got confused and considered the value of x to be 1 instead of 2, and therefore found a wrong value for the constant. A few candidates wrote a constant after the first integration step but did not try to find its value.

In part **(iii)** many candidates realised that they needed to integrate for a second time and some who did not write a constant in the first integration did so in the second one. Those candidates who wrote the first constant after the integration, but did not find its value, tended to make it disappear during the process of the second integration or did not write another constant but used the same one, and ended up writing $cx + c$ instead of $ax + c$. A few of the candidates who had done all the calculations correctly up to that point forgot to find the coordinates of the stationary point at the end of the question.

Answers: **(ii)** $-12x^{-2} - 4x + 11$; **(iii)** (2, 12)

Question 11

There was some evidence that a few candidates had found the paper slightly too long as this question was sometimes incomplete or seemed to have been rushed.

In part **(i)** most candidates correctly rearranged to $\frac{1}{2}$ and then found the angle of 60°. Some wrote the angle as 120° and did not put it in terms of π, or divided by 2 and gave the answer as 30°.

In part **(ii)** some candidates attempted to work from the given domain to obtain the range whilst others attempted to use a number of different values. In the latter case candidates sometimes used too small a number of x values and only a minority of candidates obtained the correct inequality although some obtained the lower limit of 2 alone.

In part **(iii)** some candidates tried to work out a series of points to be able to plot the curve and others tried to use transformations, but only the strongest candidates sketched the curve correctly.

In part **(iv)** many candidates were able to make a good attempt at rearranging to make x the subject of the formula and hence find the inverse function. Common errors were to make a mistake with the signs or to forget to divide by –4. Some candidates divided by 2 instead of multiplying by 2 at the end. For weaker candidates a common error was not to divide by 4 but to instead divide by (4cos) or to multiply in the bracket by 2 rather than at the end.

Answer: $\frac{2}{3}\pi$; $2 \le f(x) \le 10$; $f^{-1}(x) = 2\cos^{-1}\left(\frac{6-x}{4}\right)$

MATHEMATICS

Paper 9709/13
Paper 13

General Comments

Many candidates were able to demonstrate their mathematical ability on this paper. Strangely enough it was questions in the first part of the paper, particularly **Questions 2, 3** and **4**, that seemed to pose more difficulty than some of the later questions.

Key Messages

- Once again, careless algebraic manipulation was responsible for the loss of many marks. Candidates need to be more aware of situations in which they need to insert brackets themselves (as in **Question 6**) before carefully removing them for the process of simplification. Errors involving brackets were also made in **Questions 1, 7, 9** and **11**.
- Candidates need to be more aware that, for example (**Question 5**), $\sin^2\theta = \frac{3}{4}$ is a quadratic equation which therefore requires two roots, and these are shown using the ± symbol.

Comments on specific questions

Question 1

This question was answered well by most candidates and proved to be a good start to the paper. Full marks were not always achieved, however, with the main error being writing down ax^2 and ax^3 instead of $(ax)^2$ and $(ax)^3$. Almost all candidates who avoided this error went on to score all 4 marks for this question.

Answer: $\frac{3}{2}$

Question 2

This question was often left until the end of the examination and was sometimes omitted altogether, reflecting a lack of confidence in some candidates when dealing with this topic. Success in both parts depended on deriving a correct expression for the length of *AB* and a significant proportion were not able to proceed correctly from $\tan\left(\frac{\pi}{6}\right) = \frac{3}{AB}$ and then to express *AB* in terms of √3. It was also important to find angle *COA* correctly and errors here were not uncommon.

Answers: **(i)** $6\sqrt{3} + 2\pi$; **(ii)** $9\sqrt{3} - 3\pi$

Question 3

Only a relatively small proportion of candidates provided a fully correct response to part **(i)**, many giving the answer in the more familiar form $a(x+b)^2 + c$. The answer $9(x-\frac{2}{3})^2 + 1$, which received partial credit, occurred at least as commonly as the correct answer. In order to gain any credit in part **(ii)** it was necessary to construct valid reasoning in order to conclude that it is an increasing function. Unfortunately, relatively few made clear the connection between part **(i)** and the derivative of the function and often proceeded, unsuccessfully, to work with the second derivative or to substitute values into the derivative.

Answers: **(i)** $(3x-2)^2 + 1$; **(ii)** Increasing since derivative = $(3x-2)^2 + 1$ which is greater than 0

Question 4

This question was quite often left to the end by candidates or omitted altogether. Most errors arose from an assumption in part **(i)** that progression *R* followed the same pattern exhibited by *P* and *Q* and therefore the terms were 4, 1, $\frac{1}{4}, \frac{1}{16}$, ….. Candidates who made this assumption disregarded the fact that the sums to infinity of the three progressions were in arithmetic progression and these candidates usually were unable to score any marks. A common error in part **(ii)** was to write down the first three terms without actually finding the sum of these terms. Those candidates who did attempt to find the sum could choose whether to simply add the three terms together or to use the formula.

Answers: **(i)** 5; **(ii)** 4.96

Question 5

There was a variety of methods used in part **(i)** to prove the identity and three of the most common methods usually gave the required result after a few lines of working. As in all cases where the answer is given, candidates have to be especially careful not to omit vital lines of working even if they think that a line of working is 'obvious'. Part **(ii)** was generally well done although frequently the last two solutions were missed out due to the omission of the ± symbol earlier.

Answer: **(ii)** 60°, 120°, 240°, 300°

Question 6

Most candidates knew what was required and how to proceed but algebraic and arithmetic errors were responsible for the loss of a relatively large number of marks. In part **(i)** a significant number of candidates did not transfer the details of the points accurately to the process of finding the gradient of *AB* and manipulative errors were commonplace, especially if candidates missed out brackets in subtracting terms such as $3a + 9$ and $2a - 1$. Also a surprising number of candidates misread their own writing of '*a*' and '9', which in many cases became indistinguishable. Most candidates were able to apply the right method for finding the gradient of the line perpendicular to *AB*. Manipulative errors were again commonplace in part **(ii)**. Square root signs appeared to compound the problems considerably and the expansion of brackets was sometimes not performed accurately.

Answers: **(i)** $\frac{-(a+4)}{a+10}$; **(ii)** 4, –18

Question 7

In part **(i)** most candidates were able to find the scalar product accurately and to set this to zero (although setting to 1 and –1 were also occasionally seen). Most candidates were able to solve the simple quadratic equation without difficulty. Part **(ii)** was more unusual and some candidates produced rather confused work when trying to deal with the square roots which were present in the magnitudes of the two vectors. Arithmetic and algebraic slips were fairly common ($\sqrt{10+p^2}$ became $\sqrt{10p^2}$, for example) and these often resulted in the formation of an equation in which the quadratic term did not disappear. The final part was sometimes marred by arithmetic or sign errors when finding ***OB*** – ***OA*** and, although candidates generally knew the method for finding a unit vector, the method mark was not infrequently the only mark scored in part **(iii)**.

Answers: **(i)** –1, 4; **(ii)** 15; **(iii)** 1/5(–4**i** + 3**j**)

Question 8

In part **(i)** almost all candidates stated that the stationary point was a minimum but many failed to say that this was because the second derivative is positive when $x = 3$. In part **(ii)** nearly all candidates performed the two integration steps accurately but a significant proportion forgot the constant of integration each time. Of those who remembered the constants of integration, when it came to evaluating the first constant a large proportion substituted $f'(3) = 7$, instead of $f'(3) = 0$. It is important for candidates to realise that when a function is integrated twice two different constants of integration need to be used and therefore two different conditions need to be input for their evaluation. In this question some candidates used '*c*' each time and finished with an expression for $f(x)$ in which two terms involved c and proceeded to input $y = 7$ when $x = 3$ but, of course, finished with an incorrect value for c.

Answers: **(i)** Minimum since $f''(3) > 0$; **(ii)** $f(x) = 18x^{-1} + 2x - 5$

Question 9

This question was very well done, even by weaker candidates who had not scored so well elsewhere in the paper. There are various ways of solving the equation $3\sqrt{x} = x + 2$, but the most common one used was to square both sides of the equation and this was the point at which there was potential for making an error. If an error was made here, the roots of the resulting equation were not rational numbers and this gave awkward limits in part **(ii)**. In part **(ii)**, the integration was usually performed accurately, with most candidates choosing to also integrate the straight line function rather than compute the area of the trapezium. For those candidates who chose to subtract before arriving at two separate areas, there was potential for errors in handling brackets.

Answers: **(i)** (1, 3), (4, 6); **(ii)** $\frac{1}{2}$

Question 10

In part **(a)(i)** the majority of candidates were able to handle the technique involving composite functions and were able to form the required set of simultaneous equations. The major problem for a significant minority was posed by the handling of the cube root. Most candidates were able to simplify the first equation to $a + b = 8$ and many were able to recognise that $(9a + b)^{\frac{-2}{3}} = 16$ led to the equation $9a + b = 64$. Having obtained the two simultaneous equations in a and b, most candidates were able to solve these successfully. A few candidates cubed the second equation to arrive at $(9a + b)^2 = 16^3$ and, while correct, the algebra involved was more complex and led to a higher proportion of errors. This route also generated a second pair of solutions ($a = -9$, $b = 17$) which most candidates spotted should be rejected because of the negative value of a. In part **(a)(ii)**, once again the major stumbling block arose in handling the cube root. Most candidates were entirely familiar with the required technique for finding the inverse function and it was exclusively algebraic errors that prevented many from obtaining a correct expression. Most produced their expression for $f^{-1}(x)$ as a function of x and partial credit was available to candidates who left a and b in their expression rather than numerical equivalents. The notation for the domain, however, was often not handled successfully. The domain of any function is required as an expression involving x and in this case the correct answer of $x \geq 1$ was expected.

In part **(b)**, for $\frac{dy}{dx}$, most candidates obtained $\frac{1}{3}(7x^2 + 1)^{\frac{-2}{3}}$ correctly, but this has to be multiplied by $14x$ and this element was often seen as 7 or $7x$, or was omitted altogether. Most candidates realised that the chain rule expression of $\left(\frac{dy}{dt}\right) = \left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$ was required but marks were gained for the correct use of this expression rather than its statement alone. Inevitably, there were elements of confusion particularly among those who found the expression difficult to express correctly. Most candidates were able to substitute $x = 3$ and $\frac{dx}{dt} = 8$ into their formulation of the chain rule. It was pleasing to see many of the stronger candidates produce entirely correct solutions with impeccable notation throughout.

Answers: **(a)(i)** $a = 7$, $b = 1$; **(ii)** $f^{-1}(x) = \frac{1}{7}(x^3 - 1)$ for $x \geq 1$; **(b)** 7

MATHEMATICS

Paper 9709/21
Paper 21

Key Messages

Candidates should be encouraged to check that they are working to the required level of accuracy as specified either on the front of the examination paper or within a particular question if different.

Candidates should also be encouraged to check that they have actually answered the question and not stopped short of the final answer. Centres should check that candidates know the meaning of 'exact answer'.

General Comments

Well prepared candidates were able to show their knowledge of the syllabus and apply appropriate techniques to aid solution of the questions. The performance of candidates seemed to improve the farther they progressed in the paper, often finding the later questions more accessible than the earlier ones.

Comments on Specific Questions

Question 1

Even though candidates had been instructed to make use of the trapezium rule, some did not and attempted integration of the separate terms. Even when the trapezium rule was being used, few candidates took the modulus into account, including negative values in their calculations. Some candidates thought that they had to square $2^x - 8$ in order to deal with the modulus. As a result, there were very few correct solutions seen.

Answer: 27

Question 2

Very few candidates were able to deal with relating the given graph to the given equation. There were very few correct statements seen, with incorrect use of both ln and e. The main problem was with candidates not realising that the values of 1.70 and 2.18 from the graph were actually values representing ln*y* and not *y*. Few correct solutions were seen as those candidates that had used a correct method often used prematurely rounded figures in their calculations, thus obtaining an inaccurate final answer.

Answer: $a = 3.45$, $b = 1.85$

Question 3

(a) There was very little recognition of the fact that candidates needed to make use of the double angle formula $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$ before attempting integration. Candidates should take care to use the variable in the question; often x was used rather than θ. Few correct solutions were seen.

(b) Most candidates recognised that integration produced an answer of the form $k\ln(2x + 3)$, with the occasional error in the calculation of the value of k. Unfortunately many candidates did not appreciate the fact that they were required to give an exact answer after applying the limits, in most cases correctly. The all too common answer of 1.35 lost candidates the final accuracy mark.

Answer: **(a)** $2\sin\theta + 2\theta\ (+\ c)$, **(b)** $\frac{1}{2}\ln 15$

Question 4

(i) This question was attempted reasonably well by most candidates. Common errors involved the numerical coefficient of sin2*x* after differentiation. Many candidates with otherwise correct solutions failed to gain the final accuracy mark because they did not appreciate the meaning of the word 'exact'.

(ii) The need for implicit differentiation was recognised by most candidates, with the main errors arising from not recognising that differentiation of 6*xy* required the product rule. Some careless errors in simplification were also seen.

Answer: **(i)** $-\frac{11\sqrt{3}}{2}$, **(ii)** $-\frac{5}{6}$

Question 5

(i) Many completely correct solutions to this part were seen, showing that most candidates had a good understanding of the factor theorem and the solution of the resulting simultaneous equations.

(ii) Many completely correct solutions were seen for the factorisation of the given expression. Some chose to factorise by observation, whilst others used algebraic long division. Those candidates who opted to use synthetic division often did not make appropriate adjustments to obtain factors that were equivalent to the given expression. Centres need to make candidates aware that this must be done as many incorrect answers of $\left(x-\frac{6}{5}\right)(x+2)(x+3)$ were seen.

Very few candidates were able to make the correct link between the given expression and the given equation, i.e. using $x = 5^y$. As a result, few correct solutions were seen as candidates had no correct equations to work from.

Answer: **(i)** $a = 19$, $b = -36$, **(ii)** 0.113

Question 6

(i) Most candidates recognised that they should either differentiate the quotient or an equivalent expression rewritten as a product, then the derivative needed to be equated to zero. Only errors in the differentiation of e^{3x} with respect to x and/or subsequent simplification of the resulting equation stopped candidates from obtaining full marks for this part.

(ii) Often candidates attempted to use the equation of the curve rather than the gradient function equated to zero, as obtained in **(i)**. Of those candidates who made use of the correct equation, most failed to re-write it in the form $f(x) = x - \frac{2}{3}\left(1 + e^{-3x}\right)$ or equivalent and to look for a sign change between f(0.7) and f(0.8). Candidates should be reminded of the need to draw a conclusion from their work by stating that a change of sign indicates a root or solution in the given interval.

(iii) Most candidates were able to gain marks for using the iterative formula correctly. Candidates should also be reminded to ensure they are working to the required level of accuracy and that their final answer is also to the required level of accuracy. Some candidates lost valuable marks by failing to do this.

Answer: **(iii)** 0.739

Question 7

(i) Most candidates recognised the need to use the correct trigonometric identity and obtained the required equation. Apart from the occasional slip in factorisation, most candidates were also able to obtain the exact value of tan*α*. Some candidates failed to discount the value that would have given a solution outside the required interval. Often candidates went on to find the value of *α* and, though this was unnecessary, it was not penalised.

(ii) Very few correct solutions were seen. $\text{Tan}(\alpha+\beta)=\frac{1}{6}$ was often stated or implied, with a correct use of the appropriate compound angle formula, but a substitution of $\tan\alpha=\frac{2}{3}$ was rarely made. This part of the question allowed the more able, well prepared candidates to demonstrate their abilities.

Answer: **(i)** $\frac{2}{3}$, **(ii)** $-\frac{20}{9}$

MATHEMATICS

Paper 9709/22
Paper 22

Key Messages

Candidates should be encouraged to check that they are working to the required level of accuracy as specified either on the front of the examination paper or within a particular question if different.

Candidates should also be encouraged to check that they have actually answered the question as required and not stopped short of the final answer.

General Comments

This was a paper which enabled the well prepared candidate to perform well, demonstrating a good understanding of the syllabus content and how to apply the associated skills learned. It was also evident that some candidates had not done enough preparation and as a result performed very poorly.

Comments on Specific Questions

Question 1

This was a question designed for putting candidates at ease, and most were able to gain at least one of the three available marks. Most candidates opted for an approach which involved squaring each side of the equation in order to deal with the modulus. Some errors in factorisation occurred and some candidates were unable to attempt factorisation. Those who adopted the linear equation approach often made sign errors. This resulted in only one correct value (6) being obtained.

Answer: $-\frac{4}{5}, 6$

Question 2

(i) Many candidates did not understand what was expected of them. Too many introduced logarithms and then embarked upon fruitless and unnecessary working. Others, having decided to work in terms of the exponential function, did not realise that integration produced terms which still involved e^{-x} and e^{-3x} but needed different coefficients as a result of that integration. Those candidates that were able to obtain some form of correct integration often failed to simplify their answers correctly through poor algebraic skills.

(ii) Very few correct responses were seen, even though a follow through mark on a form of $k + pe^{-x} + qe^{-3x}$ was available for candidates who had made an arithmetical slip. Many candidates left their answer in a form containing $e^{-\infty}$, not realising that the limit of an exponential function of the form e^{-mx} is zero as $x \to \infty$.

Answer: **(i)** $3 - e^{-a} - 2e^{-3a}$, **(ii)** 3

Question 3

The great majority of candidates were able to gain some credit for differentiation. Correct implicit differentiation led to many correct responses. Most were able to obtain the equation of the normal, with the occasional algebraic or arithmetical slip in working calculations. Many candidates did not leave their answer in the required form, thus resulting in the loss of the final accuracy mark.

Answer: $2x - 3y + 4 = 0$

Question 4

(a) Many candidates performed poorly in this question, often incorrectly 'splitting' logarithms e.g. $\ln(x - 4) = \ln x - \ln 4$ or similar. Some candidates were able to gain credit for use of $2\ln(x - 4) = \ln(x - 4)^2$, but often subsequent errors in working led to incorrect answers of $x = 7$ and $x = 2$. Those who did obtain a correct solution as far as the final part often lost a mark by including the incorrect solution of $x = 2$ as well.

(b) Many good attempts at solution of the inequality were thwarted by a non-integer value or an incorrect integer value, the most common response being 68.4. This stresses the need for candidates to ensure that they have given their final answer in the form required.

Answer: **(a)** 8, **(b)** 69

Question 5

(i) All too often incorrect differentiation was followed by poor solution of a resulting trigonometric equation. Those candidates who were able to differentiate correctly often had difficulty in producing solutions of trigonometric equations, for example by failing to use the double angle formula for $\sin 2x$ correctly with appropriate factorisation.

(ii) Poor integration was just as problematic as the differentiation in part **(i)**. However, many candidates did manage to gain method marks but lost the final accuracy mark as their otherwise 'correct' answer was not in an exact form.

Answer: **(i)** $\frac{\pi}{3}$, **(ii)** $\frac{3\sqrt{3}}{4}$

Question 6

(i) This part was done well by the majority of candidates. Most were able to make a reasonable effort at algebraic long division, with many correct solutions seen. On occasion, candidates did not equate the quotient from the division to zero and rearrange to obtain the given result.

(ii) Using iterative formulae continues to be a strength of many candidates, the main problems arising when candidates failed to work to the required level of accuracy or failed to give their final answer to the required level of accuracy.

Answer: **(ii)** 2.13

Question 7

(i) Well prepared candidates were able to gain full marks with ease. Errors usually involved the angle with common incorrect responses being $\alpha = 22.62°$ or $-67.38°$. Again, candidates should be urged to ensure that they are working to the required level of accuracy as specified in the question.

(ii) Many candidates produced one angle only as a solution to the given equation, but most did realise that they had to use the result form part **(i)** to help them.

(iii) Very few correct solutions were seen. Many candidates misunderstood the question and, even though they realised that the expression could be re-written making use of part **(i)**, mistakenly thought that they had to solve the equation $-7 = 5\cos\frac{\phi}{2} - 12\sin\frac{\phi}{2}$. Some candidates attempted to use calculus in order to find the maximum value rather than deduce it from part **(i)**. Often, these candidates were able to progress as far as obtaining an equation in the form $\tan\alpha = k$, but were unable to get any further.

Answer: **(i)** $13\cos(x + 67.38°)$, **(ii)** 240.6°, 344.6°, **(iii)** 20, 585.2°

MATHEMATICS

Paper 9709/23
Paper 23

Key Messages

Candidates should be encouraged to check that they are working to the required level of accuracy as specified either on the front of the examination paper or within a particular question if different.

Candidates should also be encouraged to check that they have actually answered the question and not stopped short of the final answer. Centres should check that candidates know the meaning of 'exact answer'.

General Comments

Well prepared candidates were able to show their knowledge of the syllabus and apply appropriate techniques to aid solution of the questions. The performance of candidates seemed to improve the farther they progressed in the paper, often finding the later questions more accessible than the earlier ones.

Comments on Specific Questions

Question 1

Even though candidates had been instructed to make use of the trapezium rule, some did not and attempted integration of the separate terms. Even when the trapezium rule was being used, few candidates took the modulus into account, including negative values in their calculations. Some candidates thought that they had to square $2^x - 8$ in order to deal with the modulus. As a result, there were very few correct solutions seen.

Answer: 27

Question 2

Very few candidates were able to deal with relating the given graph to the given equation. There were very few correct statements seen, with incorrect use of both ln and e. The main problem was with candidates not realising that the values of 1.70 and 2.18 from the graph were actually values representing ln*y* and not *y*. Few correct solutions were seen as those candidates that had used a correct method often used prematurely rounded figures in their calculations, thus obtaining an inaccurate final answer.

Answer: $a = 3.45$, $b = 1.85$

Question 3

(a) There was very little recognition of the fact that candidates needed to make use of the double angle formula $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$ before attempting integration. Candidates should take care to use the variable in the question; often x was used rather than θ. Few correct solutions were seen.

(b) Most candidates recognised that integration produced an answer of the form $k\ln(2x + 3)$, with the occasional error in the calculation of the value of k. Unfortunately many candidates did not appreciate the fact that they were required to give an exact answer after applying the limits, in most cases correctly. The all too common answer of 1.35 lost candidates the final accuracy mark.

Answer: **(a)** $2\sin\theta + 2\theta\ (+\ c)$, **(b)** $\frac{1}{2}\ln 15$

Question 4

(i) This question was attempted reasonably well by most candidates. Common errors involved the numerical coefficient of $\sin 2x$ after differentiation. Many candidates with otherwise correct solutions failed to gain the final accuracy mark because they did not appreciate the meaning of the word ‘exact’.

(ii) The need for implicit differentiation was recognised by most candidates, with the main errors arising from not recognising that differentiation of $6xy$ required the product rule. Some careless errors in simplification were also seen.

Answer: **(i)** $-\frac{11\sqrt{3}}{2}$, **(ii)** $-\frac{5}{6}$

Question 5

(i) Many completely correct solutions to this part were seen, showing that most candidates had a good understanding of the factor theorem and the solution of the resulting simultaneous equations.

(ii) Many completely correct solutions were seen for the factorisation of the given expression. Some chose to factorise by observation, whilst others used algebraic long division. Those candidates who opted to use synthetic division often did not make appropriate adjustments to obtain factors that were equivalent to the given expression. Centres need to make candidates aware that this must be done as many incorrect answers of $\left(x-\frac{6}{5}\right)(x+2)(x+3)$ were seen.

Very few candidates were able to make the correct link between the given expression and the given equation, i.e. using $x = 5^y$. As a result, few correct solutions were seen as candidates had no correct equations to work from.

Answer: **(i)** $a = 19$, $b = -36$, **(ii)** 0.113

Question 6

(i) Most candidates recognised that they should either differentiate the quotient or an equivalent expression rewritten as a product, then the derivative needed to be equated to zero. Only errors in the differentiation of e^{3x} with respect to x and/or subsequent simplification of the resulting equation stopped candidates from obtaining full marks for this part.

(ii) Often candidates attempted to use the equation of the curve rather than the gradient function equated to zero, as obtained in **(i)**. Of those candidates who made use of the correct equation, most failed to re-write it in the form $f(x) = x - \frac{2}{3}\left(1+e^{-3x}\right)$ or equivalent and to look for a sign change between f(0.7) and f(0.8). Candidates should be reminded of the need to draw a conclusion from their work by stating that a change of sign indicates a root or solution in the given interval.

(iii) Most candidates were able to gain marks for using the iterative formula correctly. Candidates should also be reminded to ensure they are working to the required level of accuracy and that their final answer is also to the required level of accuracy. Some candidates lost valuable marks by failing to do this.

Answer: **(iii)** 0.739

Question 7

(i) Most candidates recognised the need to use the correct trigonometric identity and obtained the required equation. Apart from the occasional slip in factorisation, most candidates were also able to obtain the exact value of $\tan\alpha$. Some candidates failed to discount the value that would have given a solution outside the required interval. Often candidates went on to find the value of α and, though this was unnecessary, it was not penalised.

(ii) Very few correct solutions were seen. $\mathrm{Tan}\left(\alpha+\beta\right)=\frac{1}{6}$ was often stated or implied, with a correct use of the appropriate compound angle formula, but a substitution of $\tan\alpha=\frac{2}{3}$ was rarely made. This part of the question allowed the more able, well prepared candidates to demonstrate their abilities.

Answer: **(i)** $\frac{2}{3}$, **(ii)** $-\frac{20}{9}$

MATHEMATICS

Paper 9709/31
Paper 31

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. On this occasion there seemed to be rather more candidates than usual who were so ill equipped that they made very little progress of value. Most questions proved to be accessible to well prepared candidates, and discriminated successfully. The questions or parts of questions that were generally done well were **Question 3** (factor and remainder theorem), **Question 7 (i)** (differential equation), **Question 8 (i)** (trigonometry) and **Question 9** (partial fractions). Those that were done least well were **Question 2** (trapezium rule), **Question 5** (complex numbers), and **Question 8 (iii)** (cubic equation).

In general the presentation of work was good, though there were some unhelpfully untidy scripts. Some candidates seemed to spend a considerable amount of time and space on certain questions, e.g. **Question 5**, and this may have contributed to the impression that candidates did not have always quite enough time to attempt all the questions on this paper. Candidates need to be advised that it is important to ration the time spent on questions early in the examination period to guard against difficulties later on.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

This apparently simple question was not often completed correctly. The answer was required to be given to 3 decimal places and in this particular case the correct answer involved 5 digits. Candidates who left the numerical calculation until they had an expression for x such as $\frac{2\ln 3}{\ln 3 - 1}$ were in a good position to state the correct answer. Those who had earlier introduced numerical approximations to logarithms, such as $\ln 3 = 1.0986$, usually lost the accuracy needed to reach the correct answer. Apart from this issue of the final answer there was a considerable amount of poor work with logarithms and indices. For example, some took logarithms to different bases, natural logarithms on the left and common logarithms on the right; others took $\ln(3^{x-2})$ to be $\frac{\ln(3^x)}{\ln(3^2)}$.

Answer: 22.281

Question 2

In general this was poorly answered. In part **(i)** some candidates appeared to be completely unfamiliar with the trapezium rule. Also the concept of a cosecant was not always properly understood. Common errors seen in the attempts to apply the trapezium rule included the use of abscissae (x-coordinates) instead of ordinates (y-coordinates); incorrect weightings of the ordinates; 3 or 5 ordinates instead of 4; an incorrect value for the interval width; and ordinates calculated at unequal intervals. There were some good graphs in part **(ii)** but only a few candidates adequately explained why the use of the trapezium rule gave an overestimate of the true value of the area. In teaching the effect of using the trapezium rule to estimate areas it might be helpful to contrast it with that of using the mid-ordinate rule.

Answers: **(i)** 1.95; **(ii)** overestimate

Question 3

This was well answered overall. The factor and remainder theorems were used appropriately by most candidates and marks were only lost through the occasional slip in the algebra. Those who tried to form the equations in *a* and *b* by long division rarely completed the question correctly.

Answer: $a = 12$, $b = -20$

Question 4

(i) The method of parametric differentiation appeared to be well understood and nearly all candidates knew how to form an expression for $\frac{dy}{dx}$. However, finding the derivatives of *x* and *y* with respect to *t* proved difficult for some candidates. In particular the chain rule was not always applied correctly.

(ii) While many gained the first mark by presenting a correct equation for the tangent in some form, only a few correctly simplified it to the given form.

Question 5

Part **(i)** was only moderately well answered. Having substituted in *w* for *z*, a surprising number of candidates made errors and did not reach the fraction $\frac{1+2\mathrm{i}}{1+\mathrm{i}}$. Others needlessly multiplied the numerator and denominator of *w* by $\mathrm{i}z-2$ before substituting for *z*. This used up valuable time. The use of a conjugate to simplify a complex fraction seems to be well known, but candidates need to be careful to avoid slips when finally converting a number of the form $\frac{a+\mathrm{i}b}{c}$ to the form $x+\mathrm{i}y$.

In part **(ii)** most candidates lost all possible marks by substituting a number for *z*, usually $-1+\mathrm{i}$, and then solving for *z*. However there were some good attempts by a minority. Most of these involved trying to solve the quadratic $\mathrm{i}z^2+z-\mathrm{i}=0$. A common error here was to take $\sqrt{-3}$ to be $\pm 3\mathrm{i}$ rather than $\pm\mathrm{i}\sqrt{3}$. Those who rewrote the quadratic as $z^2-\mathrm{i}z-1=0$ were rewarded with an easier route to the final answer. A small number chose the longer and more hazardous method of working with equations in *x* and *y*, from which there were some correct solutions.

Answers: **(i)** $\frac{3}{2}+\frac{1}{2}\mathrm{i}$; **(ii)** $-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}$

Question 6

(i) Though some could make no progress at all, the majority did try to integrate by parts. The error of taking the derivative of $\ln 2x$ to be $\frac{1}{2x}$ was quite common but many attempts reached the correct indefinite integral and formed a correct equation in *a*. This equation was often converted correctly to the given form and it was good to see candidates showing sufficient working to reach a given answer.

(ii) This was done well and most candidates obtained full marks. Some misunderstood the formula and the 'exp' notation, multiplying by e instead of finding the exponential.

Answer: **(ii)** 1.94

Question 7

Part **(i)** was generally very well answered. Most candidates separated variables, integrated correctly, substituted limits and reached a correct expression for ln*R*. The transition to an expression for *R* was sometimes spoiled by the misconception of taking exp(*a* + *b*) to be exp(*a*) + exp(*b*), but generally much sound work was seen here.

Part **(ii)** was poorly done. Very few candidates saw that it could be answered by setting the initial form of $\frac{dR}{dx}$ equal to zero, solving for *x* and then calculating *R*.

Answers: **(i)** $R = 44.7xe^{-0.57x}$, **(ii)** 28.8

Question 8

Part **(i)** was well answered. Most candidates showed sufficient working and scored full marks. A small minority mistakenly took $\sin(2\theta + \theta)$ to be $\sin 2\theta + \sin\theta$, and a few expanded correctly but then stopped. Candidates found part **(ii)** less straightforward. Having substituted correctly they seemed to find eliminating the surds and introducing $\sin 3\theta$ a lengthy and difficult task. In part **(iii)** candidates were asked to solve a cubic equation in *x*. Most solved the equation $\sin 3\theta = \frac{3}{4}$ for *θ*, but then almost invariably failed to calculate the corresponding values of *x*. Of those who did solve for *x*, the negative root proved elusive.

Answers: **(iii)** 0.322, 0.799, – 1.12

Question 9

Part **(i)** was very well answered. A minority chose to work with an inappropriate form of partial fractions, but most solutions were either fully correct or evaluated all but one constant correctly. When setting up the preliminary identity involving the numerator $x^2 - 8x + 9$, candidates should check carefully that they have not miscopied the numerator and that they have multiplied the numerator of each partial fraction by the correct factor or combination of factors. The unwanted presence of an extra factor of (2 – *x*) in the identity was a regular source of error. In part **(ii)** most could state and use a correct expansion of $(1-x)^{-1}$. They were less successful with $(2-x)^{-1}$ and $(2-x)^{-2}$. Errors such as taking $(2-x)^{-1}$ to be equivalent to $2(1-\frac{1}{2}x)^{-1}$ or $\frac{1}{2}(1-x)^{-1}$ were quite frequently encountered. As a result there were errors in most solutions.

Answers: **(i)** $\frac{2}{(1-x)} - \frac{1}{(2-x)} + \frac{3}{(2-x)^2}$, or $\frac{2}{(1-x)} + \frac{x+1}{(2-x)^2}$ **(ii)** $\frac{9}{4} + \frac{5}{2}x + \frac{39}{16}x^2$

Question 10

(i) Taking a general point of the line *l* to be *P* and the point on the line with coordinates (4, – 9, 9) to be *B*, most of the successful solutions worked with the scalar product of a direction vector for the line and vector $\overrightarrow{AP}$ or $\overrightarrow{AB}$. Attempts using $\overrightarrow{OP}$ or $\overrightarrow{OB}$ were frequently seen and gained no credit. Successful solutions using a suitable vector product were also seen.

(ii) Most successful attempts obtained two equations in *a* and *b* by using the fact that the point (4, – 9, 9) lies in the plane and that the normal to the plane is perpendicular to the direction vector of the line. Another approach was to use (4, – 9, 9) and then find and use a second point lying on *l*. Substituting the components of $\overrightarrow{OP}$ in the equation of the plane also yields the two equations in *a* and *b*, but very few candidates made useful progress beyond the substitution.

Answers: **(ii)** *a* = 2, *b* = – 2

MATHEMATICS

Paper 9709/32
Paper 32

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. On this occasion there seemed to be rather more candidates than usual who were so ill equipped that they made very little progress of value. Most questions proved to be accessible to well prepared candidates, and discriminated successfully. The questions or parts of questions that were generally done well were **Question 3** (factor and remainder theorem), **Question 7 (i)** (differential equation), **Question 8 (i)** (trigonometry) and **Question 9** (partial fractions). Those that were done least well were **Question 2** (trapezium rule), **Question 5** (complex numbers), and **Question 8 (iii)** (cubic equation).

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Answers: **(i)** $\dfrac{2}{(1-x)} - \dfrac{1}{(2-x)} + \dfrac{3}{(2-x)^2}$, or $\dfrac{2}{(1-x)} + \dfrac{x+1}{(2-x)^2}$ **(ii)** $\dfrac{9}{4} + \dfrac{5}{2}x + \dfrac{39}{16}x^2$

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(i) Taking a general point of the line l to be P and the point on the line with coordinates (4, – 9, 9) to be B, most of the successful solutions worked with the scalar product of a direction vector for the line and vector $\overrightarrow{AP}$ or $\overrightarrow{AB}$. Attempts using $\overrightarrow{OP}$ or $\overrightarrow{OB}$ were frequently seen and gained no credit. Successful solutions using a suitable vector product were also seen.

(ii) Most successful attempts obtained two equations in a and b by using the fact that the point (4, – 9, 9) lies in the plane and that the normal to the plane is perpendicular to the direction vector of the line. Another approach was to use (4, – 9, 9) and then find and use a second point lying on l. Substituting the components of $\overrightarrow{OP}$ in the equation of the plane also yields the two equations in a and b, but very few candidates made useful progress beyond the substitution.

Answers: **(ii)** a = 2, b = – 2

MATHEMATICS

Paper 9709/33
Paper 33

General Comments

The standard of work on this paper was generally very good. Most candidates were able to make progress in most questions, and it was comparatively rare to see scripts which did not offer a full set of solutions. Many candidates scored full marks on the first three questions (modular inequality, parametric equations, and factor theorem), **Question 7** (vectors) was well done, and there were a large number of fully correct solutions to **Question 10** (integration by substitution and partial fractions). Those that were done least well were **Question 5(ii)** (demonstrating a given result for complex numbers), **Question 6** (approximate values for integrals), **Question 8** (differential equation), and **Question 9(i)** (curve sketching).

In general the presentation of work was good. Some candidates are still presenting scripts with work in double columns, which is more difficult to read and mark and should be discouraged. There were five questions on this paper (**Question 2**, **Question 4**, **Question 5**, **Question 9** and **Question 10**) where candidates were asked to demonstrate a given answer. In several instances candidates did not show sufficient working to support their answer. Candidates should be aware that when an answer is given in the question they need to be particularly thorough in their working in order to demonstrate that they have reached the required answer.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a complete understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only ‘correct answer’.

Comments on Specific Questions

Question 1

The majority of candidates preferred to form a quadratic inequality by squaring both sides of the modular inequality. This approach was usually successful, but candidates who made a slip in their algebra and reached an incorrect quadratic inequality often scored no further mark because they used their calculator to solve it and showed no working. They should be aware that method marks cannot be awarded when the answer is incorrect and no method is shown. A good number of candidates opted to draw a sketch and solve linear inequalities, usually very successfully.

Answer: $-\frac{4}{5} < x < 6$

Question 2

Some candidates worked through the differentiation of $\tan\theta$ as a quotient, but it was acceptable to quote the answer as a known result. Most errors in solutions to this question arose when differentiating $\cos^2\theta\sin\theta$. Some candidates focused on the $\cos^2\theta$ and appeared to forget the $\sin\theta$, and the incorrect result $\frac{d}{d\theta}\left(\cos^2\theta\right) = \sin^2\theta$ was a common error. Some candidates tried to avoid dealing with the $\cos^2\theta$ by rewriting it in terms of $\cos 2\theta$. This was often successful, but it did result in additional work at the next stage when expressing the final answer in terms of $\cos\theta$. Another popular approach was to express y in terms of $\sin\theta$, but some of these candidates then had difficulty in differentiating $\sin^3\theta$. The given answer was too tempting for some candidates: many correct but incomplete solutions reached the "correct" result, and many incorrect solutions also claimed to reach the given answer.

Question 3

There were many fully correct solutions to part **(i)**. The most concise solutions were usually from candidates who started by substituting $x = -1$ and $x = -2$ and then went on to solve the resulting simultaneous equations. Those who tried to expand three brackets sometimes made slips in the working. In part **(ii)** most candidates set out to use algebraic division, but dividing by a quadratic factor often caused problems with terms not lined up correctly and then slipping into the incorrect columns. A few candidates were not expecting a linear remainder and tried to continue with the division, resulting in a quotient involving algebraic fractions. Similarly, some candidates who used the method of undetermined coefficients did not allow for a linear remainder. A small number of candidates confused the quotient with the remainder. Some candidates tried to find the remainder when using a complex linear factor $x \pm \mathrm{i}$. They often reached a correct term, $\mathrm{i} - 13$, but rarely related this back to the value of x substituted.

Answers: **(i)** $a = 11$, $b = 5$ **(ii)** $x - 13$

Question 4

Part **(i)** was usually correct, but some candidates did not make it clear that they had substituted $\cos 60° = \frac{1}{2}$, and a very small number of candidates used incorrect expressions for $\cos(\theta \pm 60°)$. In part **(ii)** some candidates went all the way through the working again, but many were able to apply the result from part **(i)** to get to $\frac{\cos 2x}{\cos x} = 3$ very quickly. Most candidates reached the correct quadratic equation in $\cos x$ and solved it. The question asks for an exact answer; decimal solutions were common but not accepted. Candidates should also think about the context of their answers: one of the roots here should have been discarded because it was too large to be a value for $\cos x$.

Answer: **(ii)** $\frac{1}{4}\left(3 - \sqrt{17}\right)$

Question 5

There were many correct solutions to part **(i)**, but some candidates did not follow the request in the question to show all of their working and give exact values for x and y. Several candidates overlooked the i in the numerator.

Many candidates lost marks in part **(ii)** because they simply wrote down the given answer without explaining that they were using the general result $\arg(wz) = \arg w + \arg z$ and they did not explain what $\tan^{-1}\left(\frac{3}{5}\right)$ and $\tan^{-1}\left(\frac{1}{4}\right)$ represented or where the $\frac{1}{4}\pi$ comes from.

Answer: **(i)** $-\frac{7}{17} + \frac{23}{17}\mathrm{i}$

Question 6

(i) This topic has not been examined recently, and some candidates appeared to be unsure of the correct method. Amongst those candidates who demonstrated an understanding of the basic structure of the method, common errors involved working with the wrong number of intervals, not starting from $x = 0$ and using 0, 1, 2 and 3 as values for x rather than 0, 0.1, 0.2 and 0.3. Poor use of brackets in writing down the formula often led to calculation errors.

(ii) There were many completely correct solutions, but errors in the binomial expansion were common. Some candidates ignored the negative index and some expanded in powers of x rather than x^2. A common error was to state $\frac{1}{(1+3x^2)^2} = \frac{1}{1+6x^2+9x^4} = 1+\frac{1}{6}x^2+\frac{1}{9}x^4$. Having obtained an expansion, candidates were expected to integrate it to obtain their second approximation to the value of the integral, but several applied the trapezium rule for a second time. If they had applied the rule correctly in part **(i)** they were usually successful here.

Answer: **(i)** 0.255 **(ii)** 0.259

Question 7

This question on vectors proved to be more accessible to candidates than some in the recent past. Part **(i)** was particularly well answered, with most candidates recognising the need to verify that their solution for λ and μ obtained using two of the component equations needed to be verified in the third equation. In part **(ii)** many candidates recognised that they needed to use their answer from part **(i)** to find the position vector of the point of intersection and then find the length of the vector. This approach was usually successful. Although the question asks for the values for a, some candidates expressed their position vector in terms of λ, but they usually went back to a for their final answer. Several candidates tried to use a remembered formula for the distance of a point from a line, but this was not helpful here.

Answer: **(ii)** –2, 3

Question 8

The majority of candidates recognised the need to start by separating the variables, and this step was usually completed correctly. There were many correct answers for $\int y^{-\frac{1}{2}}\mathrm{d}y$, but $\ln\left(\sqrt{y}\right)$ was a common incorrect answer. Integrating $x\sin\frac{1}{3}x$ proved to be more difficult. A few candidates made serious errors such as ignoring the x or rewriting the function as $\sin\left(\frac{1}{3}x^2\right)$, but most did attempt integration by parts. The most common problem here was making an error with the factors of 3, often dividing rather than multiplying. There were several sign errors, both through misquoting the formula for integration by parts and through errors in the integration. The term $3x\cos\frac{1}{3}x$ often became $3\cos\frac{1}{3}x$ in the course of the working. For several candidates the constant of integration was an afterthought, so when they rearranged their answer to express y in terms of x, the constant was added on the end rather than within the squared bracket.

It was comparatively rare to see a fully correct answer to part **(ii)**. Most candidates gained some marks for a correct process of using the given values to obtain a particular solution to the differential equation. Some candidates who lost the final mark in part **(i)** were able to score full marks here by working with their correct expression for $\sqrt{y}$, but by this stage in the question many candidates were working with incorrect expressions for y. Several candidates were working in degree mode rather than radians.

Answers: **(i)** $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ **(ii)** 203

Question 9

(i) The response to this part of the question was weak. The purpose of a sketch is to illustrate the main features of a function. Many candidates attempted to draw accurate graphs which were usually unsuccessful because of the range of values of *y* involved. Curves were often drawn as straight lines. Although many candidates correctly identified $y = 40 - x^3$ as the second sketch required, they often drew little more than a small straight line segment to represent it, and several candidates drew something which looked more like $y = 40 - x^2$. In order to demonstrate that there is only one root, a candidate needed to demonstrate what happened to their two curves outside the first quadrant.

(ii) The question asks for verification by calculation, so candidates were expected to offer values, not just statements about results being positive or negative. They were asked to demonstrate the presence of a root, so their conclusion was expected to refer to the root.

(iii) There were a few slips in using the iterative formula, but this part of the question was usually answered well. Most errors here were due to inappropriate accuracy in the iterations. The question specifies accuracy to a number of decimal places, but some candidates appeared to be working to that number of significant figures and lost marks as a result.

(iv) A small number of candidates spotted the link between $x^3 + \ln(x+1) = 40$ and $(e^y - 1)^3 + y = 40$, but many either worked through a second set of iterations or made no progress with this part.

Answers: **(i)** $y = 40 - x^3$, **(iii)** 3.377, **(iv)** 1.48

Question 10

Some candidates did not recognise e^{2x} as $(e^x)^2$, but the majority completed the first stage of the substitution successfully. Some candidates created extra work for themselves by working with $\frac{u^2}{u(u+1)(u+2)}$ rather than simplifying the fraction before moving on. Much of the work on partial fractions was correct, but was sometimes longer than necessary when candidates did not take the opportunity to simplify the algebra. Candidates with correct partial fractions usually integrated correctly. Some candidates did not change the limits on the integral as part of the substitution process and tried to use 0 and ln4 or 0 and 4 incorrectly. Many answers were clearly presented with candidates giving correct demonstrations of how they had reached the given answer, but some solutions did not contain sufficient working to be convincing. Many candidates who did not recognise the opportunity to use partial fractions still contrived to reach the given answer via an invalid route.

MATHEMATICS

Paper 9709/41
Paper 41

General Comments

The standard of the work seen on this paper was very disappointing. There were some very good attempts at the answers to the paper but a significant number of candidates were clearly not at all well prepared for the examination.

Some candidates lost marks due to not giving answers to 3 significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used.

When an answer is given in the question, candidates must be extremely careful to show all of their working in order to justify the given answer. On this paper this was particularly the case in **Question 3**.

Comments on Specific Questions

Question 1

Most candidates performed reasonably well on this question. They were able to use the fact that the driving force is given by $\frac{P}{v}$ and used this to set up an equation using Newton's second law which involved the unknown resistance. A number of well presented correct answers were seen. One error that occurred was the misreading of 22.5 kW as 22.5 W

Answer: 290

Question 2

A variety of methods were used in order to attempt to solve this problem. Some candidates resolved forces parallel to the plane and produced a solution directly. Others either used the method of triangle of forces or Lami's theorem. Those who resolved forces in directions other than parallel to the plane often forgot to include the normal reaction in their workings. Some candidates mixed up the definitions of mass and weight.

Answer: 36, 31.2

Question 3

Very few candidates achieved full marks on this question, generally because they were not able to convincingly explain the inequalities which occurred in the given answers.

(i) In this part of the question, since the block remained at rest, the solution involved finding an equation for F and an equation for R and then using $F < \mu R$. Most candidates realised that it was necessary to resolve forces parallel to the slope in order to obtain an equation involving F and to resolve forces perpendicular to the slope to find R. Many found the value of $\frac{17}{24}$ given in the question but used it with an equality rather than explaining the required inequality.

(ii) In this part of the question the majority of candidates realised that as there was motion down the plane then $F = \mu R$ must be used but the inequality which was seen in the given answer came about by stating that the force acting down the plane was greater than zero and this again was often not explained clearly. Also it may have surprised some candidates to see a value of μ in the question where $\mu > 1$ which is not often seen but is quite possible.

Question 4

(i) Most candidates were able to find the speed of particle *P* at *A* (namely 3.3 ms^{-1}) by using one of the constant acceleration formulae. However many also attempted to find the speed of *Q* at *B* by the same method even though the acceleration was given as a function of time, *t*, and so this speed had to be found by using integration. The required answer is then found by matching the speeds at *A* and at *B*.

(ii) Most candidates correctly found the distance *OA* (46 m) using one of the constant acceleration formulae but again many did not realise that it was necessary to use integration to find the distance *OB*.

Answer: 0.1, 69.3

Question 5

(i) This part of the question was generally well done by candidates. Most realised that the tensions in the two strings *BP* and *BQ* were equal to the weights of the particles *P* and *Q* respectively and that the frictional force was $\mu \times 0.25\,g$. The majority of candidates used the fact that the three forces were in equilibrium with the only unknown in this equation being the required coefficient of friction.

(ii) Most candidates attempted either to apply Newton's second law to both particles *P* and *B* or to apply it to the system. Those who applied the equations to both particles were able to set up a pair of simultaneous equations in the acceleration, *a,* and the tension, *T,* which could then be solved. Those who only applied their equation to the system could only find the acceleration, *a*, and the question could not be completed without applying Newton's second law to one or other of *P* and *B* in order to determine the required tension.

Answer: 0.4, 2.22, 1.56

Question 6

(i) Most candidates made good attempts at this part of the question with the majority writing down Newton's second law for the particle and using the given acceleration to correctly determine the required resistance.

(ii) The problem which caused the greatest loss of marks here was the poor quality of diagram. Candidates would be advised to sketch a graph which is large enough to accurately display all required information and to label any crucial points on the graph and on the axes. Most candidates correctly attempted to use the constant acceleration formulae to determine the required velocity and time values but often did not display them on their graph as was asked for in the question.

Answer: 13.5

Question 7

This question was not structured and so candidates had a choice of approach, either using work/energy methods or using Newton's second law in order to determine the value of the angle.

Most attempted to use Newton's second law by writing down the equation of motion parallel to the plane. Firstly the constant acceleration of the block had to be determined and most made a good attempt at this. The method then involved combining the three force terms, namely, the component of the driving force, the resistance and the weight component and then equating this to mass × acceleration. The equation can then be solved for α. Some of those who used this method often forgot to include the weight component in their equation but were still able to gain some marks for their attempt.

The alternative work/energy method is more involved and requires the evaluation of the change in potential energy and the change in kinetic energy in addition to finding the work done both by the resistance and by the pulling force. The work/energy equation relating these then had to be set up and a solution could then be found for α. Some candidates failed to include all of the necessary terms in the equation and also some made sign errors when setting up the work/energy equation.

Answer: 35.3

MATHEMATICS

Paper 9709/42
Paper 42

General Comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to 3 significant figures as requested and also due to premature approximation within their calculations leading to the final answer, particularly in **Question 3**. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

In **Questions 2** and **6**, sines and cosines of angles were given so that candidates could perform exact calculations. However, many candidates often then proceeded to find the relevant angles to 1 decimal place and immediately lost accuracy and in some cases marks.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction.

Comments on Specific Questions

Question 1

(i) Most candidates attempted to use the constant acceleration equations for the vertical motion in order to find the time of flight. However, many only found the time taken to reach the highest point, whereas this value should be doubled to find the total time required. Overall this question was well done by most candidates.

(ii) The majority of candidates scored marks in this part even if they had made errors in part **(i)** since the answers here were followed through.

Answer: 2.2, 24.2, 22

Question 2

(i) Most candidates scored both marks in this part of the question. They generally showed that the components in the positive x-direction of the forces 25 and 30 were 24 and –24 respectively and hence the result followed. As in all questions where the answer is given, it is even more important to show all working in such cases in order to guarantee full marks.

(ii) The majority of candidates found the resultant force correctly but many made errors in determining the direction of the force. A common error when finding the tangent of the required angle for the direction of the force was to assume that 5/0 was equal to 5 and hence deriving an incorrect angle.

(iii) Very few candidates managed to complete the question correctly. Even though the correct answer to part **(ii)** had been found, the replacement force was often merely stated as –5 N

Answer: 5 N in positive y-direction, 30 N in the negative y-direction.

Question 3

(i) This is another case where the answer is given and so candidates are advised to show very clear working when obtaining such an answer. Most candidates used the fact that the power of the train's engine is given by Fv where F is the driving force and v is the speed of the train. It was necessary to write down this equation in the two cases at A and at B and link them with the given information. It is then possible to reduce these equations to a single equation involving the speed at B. One error that occurred frequently was that the driving force F was stated as $200000a$ and this led to erroneous equations.

(ii) This part of the question was extremely well done. Almost all candidates attempted to find the gain in kinetic energy. The work/energy equation was then used to produce the answer. The only error that occasionally occurred was an incorrect sign in the work/energy equation which should read as:

WD by the train's engine = KE gain + WD against resistance

Answer: 4.64×10^6

Question 4

(i) Most candidates scored both marks on this part of the question by resolving horizontally for the two forces X and 40. However, some attempted to find X by resolving vertically and ignoring the normal reaction. Overall this part of the question was very well done.

(ii) The majority of candidates failed to find the new value of X in this part and continued to use the value they had found in part **(i)**. The normal reaction, R, was found by resolving vertically leading to a four term equation from which $R = 98.038$ N could be determined. Once this had been achieved then the coefficient of friction could be determined by using $F = \mu R$ with the given value of $F = 10$.

Answer: 23.1, 0.102

Question 5

(i) (a) This part of the question involved two stages of working, namely determining the frictional force by using $F = \mu R$ and then finding the work done by this force as it moved through a distance of 0.9 m. Most performed these calculations correctly although some introduced the tension, T, in the string and instead obtained the work done by $T - F$.

(i) (b) The loss in potential energy of the system was solely due to the loss of height of particle B and was evaluated using $0.3 \times g \times 0.9$. Some candidates quoted the potential energy loss as zero after considering only particle A.

(i) (c) The most straightforward method to determine the gain in kinetic energy was simply to use the work/energy equation as:

Kinetic energy gain = Potential energy loss – WD against resistance.

Many candidates used this method although some again used the wrong sign within the equation.

An alternative method which some candidates used involved writing down Newton's second law for the two particles and determining the acceleration of the particles before the string broke. This acceleration was then used in the constant acceleration equation $v^2 = u^2 + 2as$ with $u = 0$ and the velocity at the instant that the string broke could be found. The formula for kinetic energy now had to be used to answer the question. Clearly this involved far more effort than using the work/energy equation. Within this method candidates often made errors in finding a and in some cases used $a = g$ even though at this stage the string was unbroken. Another error that was frequently seen was finding the kinetic energy of only one of the particles.

(ii) The approach to this part depended on the method that had been used in part **(i) (c)**. Those who used the longer method would already have calculated the speed of the particles at the break. Those who used the work/energy method would at this stage only know the kinetic energy and so would have to use the kinetic energy formula to determine the required speed as the string broke. Once again this was frequently calculated for a single particle of mass 0.3 kg. At this stage the equation $v^2 = u^2 + 2as$ could be used, with u as the speed at the break and with $a = g$, and hence the speed as A hit the floor could be found.

Answer: 1.89, 2.7, 0.81, 3.67

Question 6

Overall candidates found this question to be the most difficult on the paper.

(i) (a) Most candidates found the acceleration correctly by resolving the weight parallel to the plane. In a few cases g was missing.

(i) (b) Most candidates made reasonable attempts at this part of the question. The acceleration was almost always found by using Newton's second law applied to the particle in a direction parallel to the plane where the two force terms were the weight component and friction. Errors occurred in cases where, when using $F = \mu R$, the normal reaction, R, was taken as mg rather than the component of the weight perpendicular to the plane.

(ii) The majority of candidates did not score many marks beyond this part of the question. Having found the acceleration in the two cases, it was necessary to apply the constant acceleration equations to the particle with the given initial and final conditions. It was then possible to find the distance travelled in each of the sections AB and BC, noting that the sum of the distances $AB + BC = 5$. Only a few candidates managed to complete this successfully.

An alternative was to use the work/energy method for the whole motion from A to C although this did not use the accelerations found in the first two parts. This involved using:

Loss of PE = Gain in KE + WD against friction

A number of excellent solutions using this method were seen.

(iii) This part of the question was answered slightly better in that candidates took their values for AB and BC and attempted to find the time taken in each stage using one of the constant acceleration equations to find t over each of the two parts of the motion. Even with incorrect values of AB and BC candidates were able to score the method mark.

Answer: 2.8, -2, 2.5, 2.21

Question 7

(i) This part was reasonably well done by most candidates. The value of v was found by most by substituting into the given formula. A few candidates found the correct value but gave an incorrect sign. In determining the acceleration, a number of candidates gave the answer as a negative value even though the question asked for the magnitude of acceleration. A number of candidates wrongly used the given formula and differentiated it and then evaluated this expression at $t = 3$ which gave the acceleration at $t = 3$ rather than the acceleration over the first three seconds which was asked for in the question.

(ii) This part was well done by almost all candidates. Most differentiated the given expression for v and set $\frac{dv}{dt} = 0$ to find $t = 10$ and substituted this value of t into the given expression for v. Others completed the square in the expression for v to produce the result. It was also possible to see from the graph that the maximum value occurred at $t = 10$. Overall very few errors were seen here.

(iii) (a) Many misread this part of the question when looking at the average speed from A to B, thinking that the part of the graph from $t = 3$ to $t = 5$ was a straight line when in fact it was part of the given curve. This led to an incorrect evaluation of the distance travelled over the first 5 seconds of the motion. In fact the distance consisted of two parts, namely from $t = 0$ to $t = 3$ which can be evaluated simply from the area of a triangle, and from $t = 3$ to $t = 5$ where the evaluation of the area has to be performed by integration.

(iii) (b) This part of the question involved firstly evaluating the distance BC using integration of the given expression for v with limits from $t = 5$ to $t = 15$ although some candidates wrongly evaluated an integral with limits of $t = 3$ to $t = 15$. Secondly the distance found for AB from the previous part had to be combined with the value of BC and the average speed found from the definition as $(AB + BC)/15$. There were some very good answers seen but many lost marks due to the initial misreading of the graph.

Answer: -4.8, 1.6, 5, 2.35, 3.00

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

- Candidates are reminded to maintain sufficient accuracy in their working to achieve 3 significant figure accuracy in their final answer (e.g. **Question 2** and **Question 3(ii)**).
- The use of a force diagram is recommended to help ensure that all forces are considered when resolving or when applying Newton's Second Law or when setting up a work/energy equation.

General comments

Question 1, **Question 2** and **Question 5** were found to be easier whilst **Question 4(iii)**, **Question 6(i)** and **Question 7** were found to be more challenging. There was much work of a very high standard, with clearly presented, accurate solutions supported by appropriate diagrams and working. Some candidates, however, are very brief in their working, giving solutions which could benefit from more evidence of the method used, particularly when the result is incorrect.

Comments on specific questions

Question 1

The majority of candidates found this a straightforward question and gained full marks.

(i) Although candidates generally applied $P = Fv$, erroneous solutions were seen with $P = mav$ leading to 12600W and $P = Rv$ leading to 14400W. A few candidates gave their answer as 27kW without stating the required value of P.

(ii) This part of the question was answered less well than part **(i)**. Some candidates attempted to use constant acceleration formulae even though the acceleration was not constant. Another error was to leave out the resisting force to obtain $\frac{27000}{25} = 1400a$, leading to an acceleration of 0.771 ms^{-2}.

Answer: 27000, 0.2

Question 2

Whilst many candidates achieved full marks, there was some confusion when applying Newton's Second Law along two inclined planes, with for example $0.65g$ or $T\sin\alpha$ included in the equation of motion for P. Candidates usually formed and solved a pair of simultaneous equations in T and a as expected. Although the question only required the tension, many found the acceleration first and sometimes used an approximation of 3.6 ms^{-2} leading to an accuracy error with a tension of 3.94 N instead of 3.95 N. A few candidates formed a single equation directly in T, rather than a pair of simultaneous equations. Candidates need to be aware that if they are too economical in the working shown and then they make an error, it is difficult to award any marks without evidence of a correct method.

Answer: 3.95

Question 3

This question was sometimes omitted as candidates who were unable to attempt part **(i)** usually made no attempt at part **(ii)**.

(i) Those who resolved horizontally and vertically at O were generally able to find $W\cos\alpha$ and $W\sin\alpha$ although some candidates believed that the tensions in the strings were the same whilst others did not realise that the two tensions were W N and 7N. Since $W\cos\alpha = 3.8$ was given in the question, candidates were expected to show sufficient work leading to 3.8 rather than leaving the examiner to find and substitute a value for $\cos\beta$. A few candidates formed a triangle of forces and used the sine and cosine rules as an alternative method of solution.

(ii) Candidates who had obtained a value for $W\sin\alpha$ were usually successful in finding $\tan\alpha$ and thus values for α and W. It was less common to see W found from using the identity $\sin^2\alpha + \cos^2\alpha = 1$. The value of W was often seen as 6.76 rather than 6.77 following the use of an insufficiently accurate value for α.

Answer: 5.6, 6.77, 55.8

Question 4

Parts **(i)** and **(ii)** were usually well attempted. However, candidates were less successful in obtaining the correct answer of 34.7 m in part **(iii)**.

(i) The most common solution used $v = u +$ at to obtain the value of v and then used $v = -0.1t^2 + 2.4t - k$ to obtain k, although some candidates ignored 'hence' in the question and attempted to find k first. Some equated $0.25 \times 8 = 12.8 - k$ to find k without finding v explicitly. $k = -10.8$ was sometimes seen instead of $k = 10.8$.

(ii) The majority of candidates equated $\frac{dv}{dt}$ to zero to find the time at which the maximum velocity occurred. However, some believed that the maximum velocity was when $t = 18$. Others found the correct time but did not continue to find the maximum velocity, while others introduced a sign error using '+10.8' rather than '-10.8' even following $k = 10.8$ in part **(i)**.

(iii) Many candidates did not recognise that they needed to consider the two stages of motion separately and found $\int v\,dt$ with limits of 0 and 18. Others found $\int v\,dt$ and then erroneously substituted $t = 10$. Those who used indefinite integration often considered $t = 0$ rather than $t = 8$ to obtain the constant of integration.

Answer: 2, 10.8, 3.6, 34.7

Question 5

This question, although not straightforward, was answered successfully by many candidates with the correct application of Newton's Second Law to the two situations. Errors seen included the omission of a term e.g. the component of weight (leading to μ = 0.688). $X = 8$ could be obtained in such cases but from incorrect work. Candidates should beware of dimensional errors such as the use of 8sin5° instead of 8$g$sin5°. Those who formed equations in X and μ, and attempted to find μ first, overcomplicated the solution by attempting to solve, for example, $\frac{8g\sin5 + 8g\mu\cos5 + 8\times0.15}{7} = \frac{8g\sin5 + 8g\mu\cos5 + 8\times1.15}{8}$. In contrast, those who found X first often presented clear, concise and correct solutions.

Answer: 8, 0.600

Question 6

Most candidates were able to attempt some parts of the question but some were unclear how to attempt to find the masses of the two particles in part **(i)**.

(i) The accelerations for P and for Q were often given correctly as 4 ms^{-2} and -4 ms^{-2} but with the required magnitude not stated. Those who continued and used -4 in one of their equations of motion often concluded with each particle having a mass of 0.5 kg. Some who found $T = 14m$ for particle P and $T = 6m$ for particle Q concluded incorrectly that the mass of P was 0.7 kg and the mass of Q was 0.3 kg.

(ii) Most candidates correctly found $h = 2$, either using the graph and calculating the appropriate area or using $s = \frac{1}{2}at^2$. A few solutions used constant speed instead of constant acceleration to obtain $h = 4\times1 = 4$.

(iii) Very many candidates misinterpreted the distance required. Some calculated the distance travelled by P to the greatest height (2.8 m). Others calculated h and added the distance travelled by P after Q reached the floor (also 2.8 m). A few found the distance to be 3.6 m (2 + 0.8 + 0.8), suggesting some downward motion for P.

Answer: 4, 0.3, 0.7

Question 7

This appeared to be the most challenging question with full marks uncommon.

(i) Regardless of the method used, candidates needed to include the frictional force, the component of weight down the plane and the component of the 35 N force up the plane. Common errors were to omit one force, to assume zero acceleration or, when using the work/energy method, to include $3g\sin\alpha \times 12.5$ as well as the change in potential energy.

(ii) Many candidates used $3g\cos\alpha$ (24 N) instead of $3g\cos\alpha - 35\sin\beta$ (14.2 N) for the normal reaction when applying $\mu = \frac{F}{R}$ and hence 0.57 was a common incorrect value for μ.

(iii) Candidates who attempted to use Newton's Second Law and $v^2 = u^2 + 2as$ frequently found an incorrect value for a, either due to an incorrect value of μ in part **(ii)** or by involving 35 N in their solution. Following $\mu = 0.57$, it was common to see an acceleration of -10.56 ms^{-2} leading to a distance of 0.758 m. Some candidates oversimplified the situation, stating $a = -g\sin\alpha$, despite the rough plane. Those who attempted to form a work/energy equation frequently did so with an erroneous 'work done against friction'.

Answer: 171, 0.963, 0.584

MATHEMATICS

Paper 9709/51
Paper 51

General Comments

A good, clear diagram can often help candidates to think through the question and decide how to solve the problem.

It is pleasing to note that most candidates now use g = 10 ms^{-2} (the acceleration due to gravity), as specified in the rubric, and not 9.8 or 9.81.

Comments on Specific Questions

Question 1

This question proved to be challenging for many candidates.

Many candidates thought that tan 15 = (velocity in y-direction)/(velocity in x-direction) instead of tan 15 = (vertical distance)/(horizontal distance).

Answer: V = 37.3

Question 2

This question was quite well answered.

(i) Some candidates arrived at the given answer but from completely incorrect work.

(ii) A correct moment equation was often seen leading to a correct answer.

Answers: **(i)** Moment of weight of cone about P = 6 Nm, **(ii)** F = 13.0 N

Question 3

This question was generally well done. Some candidates tried to use an energy equation to find the extension at the equilibrium position. This was not possible. $T = mg$ should have been used. An energy equation was used by many candidates for the next part of the question. Common mistakes were a wrong sign or only three terms instead of four seen.

Answer: Speed of P at the instant the string first becomes slack = 1.11 ms^{-1}

Question 4

(i) This part of the question was usually well done. A few candidates used $\frac{1}{4}$ of 1.8 instead of $\frac{1}{3}$ of 1.8 to find the centre of mass of the triangle from CE.

(ii) Most candidates found the greatest value of T by taking moments about A. Only a few candidates went on to find the least value which could be done by taking moments about F.

Answers: **(i)** Distance of centre of mass from AB = 0.488 m, **(ii)** Greatest T = 30.5 N and least T = 5.5 N

Question 5

This question proved to be rather difficult for many candidates.

(i) The justification that B was projected horizontally was often not sufficiently clear or thorough. The given equation y = –0.05x^2 needed to be compared to the trajectory equation

$$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha}$$

When this is done $x\tan\alpha = 0$, so $\tan\alpha = 0$, hence $\alpha = 0$ where α is the angle of projection with the horizontal.

The speed is then found from the equation $-0.05x^2 = \dfrac{-gx^2}{2V^2\cos^2 0}$.

(ii) Very few candidates scored any marks on this part of the question. There are several possible methods for solving this part.

One method is as follows: B's final velocity is $10\tan 60 = 10\sqrt{3}$. Now use the vertical motion to give $(10\sqrt{3})^2 = 2gh$, h = 15 and so y = –15 or 15 below the point of projection. The speed can now be found by using Pythagoras's theorem. This gives $v^2 = 10^2 + (10\sqrt{3})^2$, v = 20ms^1.

Answers: **(i)** α = 0, speed of projection = 10 ms^{-1}, **(ii)** y = –15, speed of B = 20ms^{-1}

Question 6

(i) This part of the question was generally well done.

(ii) Quite often the only mark lost was the last mark because candidates did not express v in terms of x. It was left as x in terms of v.

(iii) x = 7 was used by far too many candidates instead of x = 8.

Answers: **(i)** $3v^{\frac{1}{2}}\dfrac{dv}{dx} = 2$, **(ii)** $v = x^{\frac{2}{3}}$, **(iii)** t = 3

Question 7

This question was quite well done.

(i) Good marks were scored on this part of the question.

(ii) Most candidates used Newton's Second Law with the correct radial acceleration. If a mistake occurred it was when attempting some of the trigonometry.

(iii) This part of the question was fairly well done. Again the trigonometry involved caused some problems.

Answers: **(i)** $T = \dfrac{3}{\cos\theta}$, m = 0.3; **(ii)** ω = 5 which is independent of θ; **(iii)** θ = 45

MATHEMATICS

Paper 9709/52
Paper 52

General Comments

A good, clear diagram can often help candidates to think through the question and decide how to solve the problem.

It is pleasing to note that most candidates now use g = 10 ms^{-2} (the acceleration due to gravity), as specified in the rubric, and not 9.8 or 9.81.

Comments on Specific Questions

Question 1

This question proved to be challenging for many candidates.

Many candidates thought that tan 15 = (velocity in *y*-direction)/(velocity in *x*-direction) instead of tan 15 = (vertical distance)/(horizontal distance).

Answer: $V = 37.3$

Question 2

This question was quite well answered.

(i) Some candidates arrived at the given answer but from completely incorrect work.

(ii) A correct moment equation was often seen leading to a correct answer.

Answers: **(i)** Moment of weight of cone about P = 6 Nm, **(ii)** F = 13.0 N

Question 3

This question was generally well done. Some candidates tried to use an energy equation to find the extension at the equilibrium position. This was not possible. $T = mg$ should have been used. An energy equation was used by many candidates for the next part of the question. Common mistakes were a wrong sign or only three terms instead of four seen.

Answer: Speed of P at the instant the string first becomes slack = 1.11 ms^{-1}

Question 4

(i) This part of the question was usually well done. A few candidates used $\frac{1}{4}$ of 1.8 instead of $\frac{1}{3}$ of 1.8 to find the centre of mass of the triangle from *CE*.

(ii) Most candidates found the greatest value of T by taking moments about A. Only a few candidates went on to find the least value which could be done by taking moments about F.

Answers: **(i)** Distance of centre of mass from AB = 0.488 m. **(ii)** Greatest T = 30.5 N and least T = 5.5 N

Question 5

This question proved to be rather difficult for many candidates.

(i) The justification that B was projected horizontally was often not sufficiently clear or thorough. The given equation y = $-0.05x^2$ needed to be compared to the trajectory equation

$$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha}$$

When this is done $x\tan\alpha = 0$, so $\tan\alpha = 0$, hence $\alpha = 0$ where α is the angle of projection with the horizontal.

The speed is then found from the equation $-0.05x^2 = \dfrac{-gx^2}{2V^2\cos^2 0}$.

(ii) Very few candidates scored any marks on this part of the question. There are several possible methods for solving this part.

One method is as follows: B's final velocity is $10\tan 60 = 10\sqrt{3}$. Now use the vertical motion to give $(10\sqrt{3})^2 = 2gh$, $h = 15$ and so $y = -15$ or 15 below the point of projection. The speed can now be found by using Pythagoras's theorem. This gives $v^2 = 10^2 + (10\sqrt{3})^2$, v = 20ms^1.

Answers: **(i)** $\alpha = 0$, speed of projection = 10 ms^{-1}, **(ii)** y = –15, speed of B = 20ms^{-1}

Question 6

(i) This part of the question was generally well done.

(ii) Quite often the only mark lost was the last mark because candidates did not express v in terms of x. It was left as x in terms of v.

(iii) $x = 7$ was used by far too many candidates instead of $x = 8$.

Answers: **(i)** $3v^{\frac{1}{2}}\dfrac{dv}{dx} = 2$, **(ii)** $v = x^{\frac{2}{3}}$, **(iii)** t = 3

Question 7

This question was quite well done.

(i) Good marks were scored on this part of the question.

(ii) Most candidates used Newton's Second Law with the correct radial acceleration. If a mistake occurred it was when attempting some of the trigonometry.

(iii) This part of the question was fairly well done. Again the trigonometry involved caused some problems.

Answers: **(i)** $T = \dfrac{3}{\cos\theta}$, m = 0.3; **(ii)** $\omega = 5$ which is independent of θ; **(iii)** $\theta = 45$

MATHEMATICS

Paper 9709/53
Paper 53

General Comments

Most candidates produced work that was clear and well presented.

It is pleasing to note that most candidates now use g = 10 ms^{-2} (the acceleration due to gravity), as instructed in the rubric, and not g = 9.8 or 9.81.

A good clear diagram can often help with thinking through a question and deciding how to solve the problem.

Comments on Specific Questions

Question 1

This question was generally well done and proved to be a good source of marks. A number of candidates found the angle to be 59°. This was the angle with the vertical and not the horizontal.

Answer: angle of projection = 31.0°

Question 2

Good marks were often scored on this question.

(i) A few candidates tried to set up an equation involving the tension instead of applying an energy equation.

(ii) Most candidates set up an energy equation. Sign errors were a problem for some candidates.

Answers: **(i)** AP = 1.3 m, **(ii)** Speed of P = 9.17 ms^{-1}

Question 3

(i) In this part of the question the answer was given. Only a small number of candidates arrived at the correct result.

(ii) This part of the question was well done. A few candidates failed to realise that, when integrating $\frac{1}{500+v}$, it would produce a logarithmic function, namely ln(500 + *v*).

Answers: **(i)** $\left(\frac{500}{500+v}-1\right)\frac{dv}{dx}$ = 0.02, **(ii)** greatest height = 9.62 m

Question 4

(i) The speed after 4 seconds was often calculated correctly. Finding the other value for the time proved to be more difficult. The easiest way to do this was to use $v = u + at$ for the vertical motion. This resulted in $15 = 50 \sin 30° - gt$ leading to $t = 1$.

(ii) Quite a number of candidates only worked out the vertical distance and then stopped, thinking that this was the required distance.

Answers: **(i)** speed of P after 4 s = 45.8 ms^{-1}, the other time = 1 s; **(ii)** OP = 174 m

Question 5

(i) This part of the question was generally well done. Some candidates tried to use elastic energy and found that they could not progress very far. $T = \dfrac{\lambda x}{l}$ should be used for both strings and then the two tensions equated.

(ii) Too many candidates failed to find a 5 term energy equation.

Answers: **(i)** $d = 0.9$ and $W = 12$, **(ii)** the greatest speed of the block = 1.29 ms^{-1}

Question 6

(i) The coefficient of friction was usually correctly calculated.

(ii) In this part of the question too many candidates used the same friction that they had found in part **(i)** and as a result only scored 1 of the 5 marks available. The new friction should have been 0.675 N. This comes from using μ = 0.45 and the normal reaction = $0.2\,g - \left(\dfrac{0.2g}{2}\right)\sin 30°$. Many candidates only found either the least or the greatest value of ω.

(iii) This part of the question was generally well done.

Answers: **(i)** coefficient of friction = 0.45, **(ii)** greatest value of $\omega = 3.93 \text{ rad s}^{-1}$ and least value of $\omega = 1.38 \text{ rad s}^{-1}$, **(iii)** $\omega = 2.94 \text{ rad s}^{-1}$ when no friction force on P

MATHEMATICS

Paper 9709/61
Paper 61

Key Messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working so that, in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks. Candidates should label graphs and axes including units and choose appropriate scales.

General comments

It was pleasing to see that at least some of the candidates who took this paper had a good knowledge of the syllabus. This is an improvement on last year. There were still candidates from some Centres who did not even attempt the examination but handed in blank sheets of paper. Most of these candidates did not know what a normal distribution was, did not recognise the binomial situation, had no idea what permutations and combinations were about and were unable to understand the concepts of mean and standard deviation. They did not appear to have had any practice with past papers and consequently a large number of candidates were unable to complete the paper. The paper was straightforward and allowed candidates to demonstrate their knowledge of basic skills.

Comments on specific questions

Question 1

This was a routine question on finding the mean and variance from a set of 10 discrete data points. Nearly all candidates gained full marks on this question apart from the few who thought that $(-2)^2 = -4$.

Answers: 2.7, 27.8

Question 2

The first two parts were attempted by most candidates who had covered the syllabus, although many did not know what the mode was. Part **(iii)** involved reading the question carefully and finding the mean first before finding the probability that the number of phone calls received is more than the mean. Only the better candidates appreciated this point.

Answers: 0.12, 1, 0.41

Question 3

Throwing dice and tossing a coin are both standard questions and this was no different. Many candidates gained full marks on this conditional probability question while others were able to gain credit for making a start.

Answer: 1/5

Question 4

This was a straightforward question for those who knew their descriptive statistics: finding the median and quartiles and drawing a box-and-whisker plot of the data. Unfortunately many candidates did not calculate the median using $\frac{(n+1)}{2}$ or the quartiles as $\frac{(n+1)}{4}$ and $(n + 1) \times \frac{3}{4}$, losing a couple of marks. However, follow-through marks meant that candidates could still gain 3 marks by correctly drawing the axis and labelling it, and drawing the two box-and-whisker graphs. Very few candidates labelled their axis and thus most lost a mark. The comments at the end were good.

Answers: LQ 0.41, Q2 0.52, UQ 0.79, smartphone B is quicker, slightly less variable etc.

Question 5

This question was done well by many candidates who recognised the binomial situation and were able to find p, and hence the variance npq. Many candidates did not spot this and tried, incorrectly, to use the normal distribution. In part **(iii)**, only the best candidates were able to work out that the probability of at least 1 faulty screw needed to be found before the probability of exactly 7 packets containing at least 1 faulty screw. Candidates did manage to gain a method mark for spotting a binomial situation.

Answers: 1.104 (given), 0.887, 0.216

Question 6

This was well done by many candidates who recognised the normal approximation to the binomial. There were still some candidates who forgot to use a continuity correction, and others who did not draw a diagram and thus were unsure whether the required probability was > 0.5 or < 0.5. In part **(ii)** a diagram made the answer obvious but many candidates ploughed through pages of working to arrive at the same result. The fact that this part is only worth 2 marks should show the candidates that not a lot of work is required.

Answers: 30 or 31 sheep, 73.6, 64.2

Question 7

It was pleasing that many candidates performed well on this question, gaining full marks in part **(i).** Part **(ii)** proved more difficult but candidates made good attempts at this, together with part **(iii).**

Answers: 1008, 392, 480

MATHEMATICS

Paper 9709/62

Paper 62

Key Messages

To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working so that, in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks. Candidates should label graphs and axes including units and choose sensible scales.

General comments

Many candidates showed sound knowledge of the syllabus and anyone who had covered the syllabus and done some revision should have been able to tackle the questions.

Comments on specific questions

Question 1

It was good that almost all candidates recognised this as a combinations-type question. There were many varieties of combinations with about half the candidates correctly taking the 2 restrictions out first and the rest after they had made the choices. Numerous wrong answers were seen, for example ${}^{50}C_2 \times {}^{43}C_1$ or ${}^{50}C_{48} - 2$ were.

Answer: ${}^{8}C_{43} = 1712304$

Question 2

This was a routine permutation question and **(i)** was well done by those who recognised it as such. A common mistake in part **(ii)** was for candidates to find the number of arrangements with no restrictions (10!) and then to subtract the number of arrangements with all 4 girls standing together (4! × 7!). They did not take into account the possibility that an arrangement of 3 girls standing together is also not permitted, nor indeed with 2 girls standing together.

Answers: 86400, 604800

Question 3

Candidates either understood this straightforward question in part **(i),** and scored full marks, or tried in vain to use a possibility space diagram for throwing two dice, and then doubled it for 4 dice. Others listed the options 1, 1, 1, 2 and managed to find 4 different ways, but then multiplied by $\frac{1}{36}$ instead of $\left(\frac{1}{6}\right)^4$. In part **(ii)** most candidates recognised the binomial situation and used their probability found in **(i),** resulting in 2 or possibly 3 method marks. Some candidates did not know the meaning of 3 significant figures and gave the answer as 0.012, thus losing an accuracy mark. Others used rounded decimals in their working and lost an accuracy mark because the probabilities in the binomial distribution did not add up to 1.

Answers: $\frac{1}{324}$, 0.0124

Question 4

Many candidates read this question as initially Sharik chose a computer and then chose the right or wrong answers, thus having three extra branches initially with probabilities of $\frac{1}{3}$ each. The tree diagrams were mainly well drawn, with almost every candidate who attempted this question gaining a mark for writing the first two probabilities ($\frac{1}{3}$ and $\frac{1}{2}$) correctly. The table in part **(ii)** was well evaluated but many candidates did not notice, or were unable to do, the final instruction which was to find E(X) from their table.

Answers: **(ii)** P(1) = P(2) = P(3) = $\frac{1}{3}$, E(X) = 2

Question 5

This was entirely on the normal distribution and those candidates who had covered the normal distribution in their revision and practice papers were all able to gain full marks. It was pleasing to see that candidates were able to standardise correctly with very few continuity corrections or squares or square roots. Not everyone realised that this question just wanted $\Phi(0.9659)$, and used $1 - \Phi(0.9659)$ instead. In part **(b)** some candidates found difficulty with the algebra in solving two equations by eliminating one variable, and ended up with $P(z > \frac{2\sigma}{\sigma}) = P(z > \sigma)$.

Answers: 0.833, 7.74, 0.0228

Question 6

This question on descriptive statistics was not done well. Teachers need to stress the importance of labels, accurate graph drawing, and sensible scales. Candidates who choose a scale of 10 small squares to represent 25 units or 30 units or 33 units, in order to fill up the page entirely, are not going to gain full marks for accurate plotting of points. Part **(ii)** was the worst-attempted question part in the entire paper, with only about 5% of candidates realising that 28% of daffodils had a height of h or more meant that 72% had a height of h or less, and that is what a cumulative frequency graph shows. Part **(iii)** was routinely done and many scored full marks though some used class boundaries or lower or upper end points when calculating the mean.

Answers: **(ii)** a single value between 21 and 23 cm, 6.01

Question 7

Once again, some candidates recognised the binomial distribution and some did not. Of those who did, nearly all realised that 'between 4 and 6 inclusive' meant 'P(4) + P(5) + P(6), and so were able to gain one or two method marks. Part **(ii)** was the usual normal approximation to the binomial which was recognised by a large proportion of candidates. Some found the wrong mean and variance but then went on to perform as well as they could, and gained 2 or 3 method marks. Some forgot the continuity correction and some found the wrong area. A diagram would have helped show whether the probability is > 0.5 or < 0.5. For justifying in part **(iii),** quoting the syllabus $np > 5$ and $nq > 5$ with numbers to demonstrate was sufficient. Saying $np > 5$ and $npq > 5$ is not enough to justify the approximation e.g when $n = 30$, $p = 0.8$ and $q = 0.2$.

Answers: 0.606, 0.842, $np > 5$ and $nq > 5$

MATHEMATICS

Paper 9709/63
Paper 63

Key Messages

Candidates should be encouraged to show all necessary working in order to make their approach clear.

To do well in this paper, candidates must work to 4 significant figures or more to achieve the accuracy required in their answers.

Candidates should be encouraged to sketch normal distribution graphs where appropriate.

General comments

Questions 1, 4 and **7** were generally answered more confidently than other questions.

The majority of candidates used the answer booklets provided effectively. However a number of candidates failed to utilise the available space appropriately, either by answering the entire paper on a single page, or by dividing the page into two columns, and both of these made it difficult to follow their reasoning. This also resulted in some candidates making corrections that were very unclear. Some candidates who used supplemental answer booklets or graph paper failed to secure them appropriately to their main answer booklet.

A number of candidates made more than one attempt at a question and then did not indicate which their submitted solution was. A small number of candidates did not replace their deleted solution.

Comments on Specific Questions

Question 1

Most candidates used the normal distribution formula correctly. A few candidates incorrectly used a continuity correction. Unfortunately, a significant number converted 1% to 0.1 and used this throughout. Candidates should be encouraged to utilise the critical values for the normal distribution as provided within the formula booklet, as many did not convert their probability to the necessary degree of accuracy. A number of 2 significant figure answers were seen without more accurate results being stated.

Answer: 4.30

Question 2

Many candidates did not understand the requirements of this question, which was based upon an 'assumed mean' and then calculating the appropriate mean and variance. A surprising number of simple numerical errors were identified within calculations. Many candidates did not work with at least 4 significant figures throughout their solutions, and so lost accuracy marks.

(i) Good candidates created a data table which enabled accurate calculation of the individual values required for all parts of this question. Many candidates misinterpreted the expression and calculated $\Sigma x - 62$. A few candidates calculated the variance at this stage and then attempted to work back to the required value.

(ii) Few correct solutions were seen for this part, with the best methods linking with the table created for part **(i)**. Good candidates stated the individual terms which were to be evaluated. However, most candidates interpreted the expression as requiring their value to part **(i)** to be squared. A number of candidates attempted to expand the expression and then were unable to evaluate the resulting expression.

(iii) Most candidates successfully completed this part of the question. This was due to almost all candidates using the correct formulae with the original data, rather than using their answers from parts **(i)** and **(ii)**. Some candidates who had previously calculated the mean or variance then recalculated at this stage, sometimes with a different result.

Answer: **(i)** 11.5 **(ii)** 75.1 **(iii)** 63.4, 7.32

Question 3

Most candidates attempted this question successfully.

(i) Most candidates correctly interpreted the information in the question to identify the greatest number of books that could be read, although many did not state this in their answer. Candidates should be encouraged to identify clearly their statement. Almost all candidates then attempted to calculate the probability from the binomial distribution for their number of books. A small number of candidates attempted to use the normal distribution even though the binomial distribution was identified within the question.

(ii) The majority of candidates calculated the probability using the binomial distribution method as indicated, even when they had used the normal distribution previously. Some candidates interpreted 'fewer than 10 books' to include 10 books. Most candidates did not appear to realise that they had already calculated one of the probabilities required for 1 – P(10,11,12) and recalculated all terms. A few used the alternative approach of P(0,1,2,3,4,5,6,7,8,9) which often had numerical inaccuracies in their calculations. Some candidates used the normal approximation without checking $np > 5$ and $nq > 5$ which would have shown them this was not an appropriate approximation.

Answers: **(i)** 12, 0.0138 **(ii)** 0.747

Question 4

Candidates of all abilities attempted this question with some success. However, few fully correct solutions were seen.

(i) Good solutions had the data in order and the terms evenly spaced. Better candidates initially produced an unordered stem-and-leaf diagram. Good candidates included the units that were required for the correct interpretation of the key. Many candidates did not show that they understood the necessity for the 'leaf' to be consistently spaced, with corrections often making the leaf inaccurate. Although most candidates recognised that there should be a key, few included the relevant context unit of 'glasses'.

(ii) The best solutions had the relevant critical values clearly identified before any attempt to draw the box-and-whisker plot on graph paper. Most candidates identified the median correctly using $\frac{(n+1)}{2}$ but many then did not use $\frac{(n+1)}{4}$ and $(n + 1) \times \frac{3}{4}$ to identify the quartiles. The majority of candidates chose an appropriate scale to plot their diagram, with 2 cm to 10 glasses being the most accurately used. Few candidates continued the 'whiskers' through the box. A small number of candidates calculated and plotted 'outliers' which is outside the requirements of the syllabus. Few candidates plotted a linear scale with contextual labelling. The solutions of the weakest candidates frequently neither stated the critical values nor had any scale on their graph.

Question 5

Most candidates attempted to use the normal distribution correctly, although at times there was a lack of accuracy in reading from the table. Many candidates could benefit from additional focus on the correct interpretation of the normal distribution function table.

(i) Most candidates were able to standardise correctly, with each section identified clearly. The best solutions often had a sketch of the normal distribution to aid the candidate. Most candidates calculated each proportion independently, rather than recognising that the total of the proportions is 1. A small number of candidates included a continuity correction for data.

(ii) Few candidates identified the appropriate area, with the best solutions again containing a sketch. A large number of solutions had a mixture of *z*-values and probabilities which were then numerically combined. It should be emphasised to candidates that this is not appropriate. Few candidates included the algebraic manipulation required to solve their final equation, with good candidates often failing to achieve the required degree of accuracy.

Answers: **(i)** 0.101, 0.761, 0.138 **(ii)** 1.06

Question 6

(a) Few fully correct solutions to this question were seen. Most candidates identified the possible values for the end dice were either 3,3 or 1,3. Good solutions then considered that the middle 5 dice could be any value, so would provide 6^5 outcomes. This was then used with the possible combinations of the end dice to produce a total. However, many candidates used 5^x, 6!, 5! or probabilities to calculate the number of outcomes of the middle dice.

(b) Many candidates attempted this question well, although some candidates appear to have penalised themselves with the lack of explanation or poor notation resulting in incomplete solutions. Good solutions identified clearly the potential allocation of games to the people, and then calculated the combination of each before summing their answers. Good candidates often recognised that similar game distributions produced the same number of outcomes, and utilised this to reduce the number of calculations. Weak solutions often omitted at least one games combination (frequently 3,3,3). A small number of candidates assumed that for each person the number of games was always picked from 9, as if they had been replaced prior to choosing the stated number. Candidates should appreciate that simply listing the 10 potential allocations is not sufficient for 6 marks.

Answers: **(a)** 23328 **(b)** 4920

Question 7

Many candidates did well on this question and the best solutions had clear workings provided at all stages. However a number of candidates failed to state what was being calculated and this made it difficult to follow their reasoning where the results were incorrect.

(i) The best solutions clearly stated the possible outcomes required for *X* to be 3 prior to making any attempt at calculating. Some good solutions had clear tree diagrams with the potential outcomes clearly identified. Many poor solutions were simply a statement of two 3-term probabilities added together. Candidates should be reminded that in a 'show' question, they must include all the steps necessary to support their calculations.

(ii) Most candidates produced a probability distribution table for three values, although some appear to have considered the number of green apples chosen rather than the number of turns required. Many candidates used the fact that the total of the probabilities is 1 to calculate the final term.

(iii) When attempted, many candidates scored well on this part. Several good methods were seen, with the best using the conditional probability formula. Common errors were either to omit P(O,O,O) from P(at least 2O), or not to consider all three combinations of picking 2 orange peppers. Some candidates ignored the fact that the peppers were taken without replacement and so lost marks.

Answers: **(ii)** P(2) = $\frac{1}{6}$, P(4) = $\frac{1}{2}$ **(iii)** $\frac{1}{3}$

MATHEMATICS

Paper 9709/71

Paper 71

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a wide range of scripts seen. Questions particularly well attempted were **Question 5(i)** and **Question 6**, whilst **Question 3 and Question 4(iii)** proved to be quite demanding for many candidates.

Presentation of work was generally good, with candidates realising the need to show all stages of their working out; there were few occasions this time where solutions lacked essential working. There were, however, cases when an answer at an early stage of a calculation was rounded prematurely, causing inaccuracy in the final answer and an unnecessary loss of marks. This was seen particularly in **Question 6**.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Candidates were required to find the parameters for the distribution A – 2B, and then to find the probability that A – 2B > 0. Many candidates did this successfully, however, some candidates worked with an incorrect distribution of 2A – B. Other candidates made errors in calculating the mean and variance; in particular when calculating the variance, $60^2 + 4 \times 28^2$ was required and errors such as $60^2 + 2 \times 28^2$ and $60^2 - 4 \times 28^2$ were seen. Standardising and the use of tables were generally well attempted.

Answer: 0.335

Question 2

It is important that candidates know the conditions when approximating distributions can be used. In this question the distribution was Binomial, but could be approximated to a Poisson distribution because $n > 50$ and $np < 5$. Some candidates were able to justify their approximation but many did not do so fully. Even without full justification, many candidates successfully used Po(1.5) to find the required probability. There were a few occasions when candidates did not use an approximating distribution, but used the given distribution of Bin(150, 0.01); this was not what the question required.

In part **(ii)**, some candidates incorrectly calculated separate probabilities, for example P(X > 3) and P(X < 7), and then multiplied these together. However, many candidates successfully found P(X = 4, 5, 6) with the correct value of λ (3.5).

Answer: $n > 50$ and $np < 5$, 0.777, 0.398

Question 3

This question, on probability density functions, caused problems for a large number of candidates. It is important that candidates do not just rely on applying standard techniques, but also have an understanding of the underlying theory. Part **(a)** was reasonably well attempted, but in part **(b)**, which tested understanding of the topic, few candidates were able to answer with confidence. A large number of candidates tried to apply standard integration techniques which were not needed. To find a, the area ($\frac{1}{2}\pi a^2$) needed to be equated to 1; many candidates attempted to integrate either the circumference or the area. Similarly, when finding $E(X)$, most candidates attempted incorrect integrations rather than using symmetry. When finding $P(X < c)$, again, few candidates used symmetry, and often used Normal tables despite the fact that this was not a Normal Distribution curve.

Answer: 0.156, 0.798, 0, 0.8

Question 4

Many candidates successfully found the required Confidence Interval. Common errors included incorrect z values and use of 33 rather than $\frac{33}{150}$ in the expression. It should be noted that the answer here must be given as an interval; some candidates gave their final answer as "0.133 or 0.307", or as "0.133 and 0.307", rather than as an interval 0.133 to 0.307.

In part **(ii)** most candidates successfully calculated the unbiased estimate for the population mean. The variance was better attempted than has been the case in previous years; there were only a few cases where candidates found the biased variance, thinking that this was the answer required. There still appears to be some confusion by candidates between the two alternative formulae for the unbiased estimate of the population variance.

Part **(iii)** was not well attempted. Very few candidates suggested generating 4-digit random numbers (each digit from 0 to 9), with repeats and numbers greater than 9526 being discarded. The majority of candidates suggested a time-consuming method of writing the numbers on pieces of paper and drawing 150 at random from a hat, which was particularly impractical in this case and, whilst generating a random sample, did not use random numbers. It is important for candidates to have an appreciation of practical statistical sampling methods, in this case involving the use of random numbers.

Answer: 0.133 to 0.307; 126.9, 11001.17; 4-digit numbers, ignore numbers > 9526, ignore repeats

Question 5

Part **(i)** was well attempted, with many candidates reaching the required probability.

In part **(ii)**, most candidates correctly explained why a one-tail test was appropriate. The test was carried out successfully by a large number of candidates; many candidates clearly showed the required comparison (either as an inequality statement or on a clearly labelled diagram) and in most cases a valid comparison was made. A suitable conclusion with no contradictory statements, and preferably in the context of the question, was then required.

Few candidates appreciated in part **(c)** that the Central Limit Theorem was not required because the population was known to be normally distributed. It was important that candidates were clearly referring to the population as being Normal; answers such as 'it is normally distributed' or 'the distribution is Normal' were not clear enough.

Answer: 0.146; looking for a decrease; there is evidence that the mean time has decreased; no, the population is normally distributed

Question 6

This was a well attempted question, with part **(i)**, in particular, being a good source of marks for many candidates. Parts **(ii)** and **(iii)**, on type I and type II errors, were reasonably well attempted; this is not always the case for such questions. Common errors in all three parts were incorrect values for λ.

In part **(ii)** most candidates identified that a Type I error would be found by $P(X < 2| \lambda = 3.875)$, although the inclusion of $P(X = 3)$ was sometimes seen. A number of candidates, when calculating $\frac{3.1}{12} \times 15$, approximated this answer prematurely and used it in the subsequent calculation. The final answer was thus not to the required level of accuracy, causing a loss of marks. In part **(iii)** many candidates identified that a Type II error would be found by $1 – P(X < 2)$ but a significant number of candidates failed to realise that the time period was still 15 weeks, and used $\lambda = 0.1$ rather than $\lambda = 1.5$.

The explanation required in part **(iv)** was not always fully given. Here candidates were asked to identify why it was impossible in the given situation to make a Type II error. Many good responses were seen indicating that the null hypothesis would be rejected hence a Type II error could not occur, but many responses were not as clear, a number simply indicating that a Type I error would be made.

Answer: 0.186, 0.257, 0.191, He will reject H_o

MATHEMATICS

Paper 9709/72

Paper 72

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a wide range of scripts seen. Questions particularly well attempted were **Question 5(i)** and **Question 6**, whilst **Question 3 and Question 4(iii)** proved to be quite demanding for many candidates.

Presentation of work was generally good, with candidates realising the need to show all stages of their working out; there were few occasions this time where solutions lacked essential working. There were, however, cases when an answer at an early stage of a calculation was rounded prematurely, causing inaccuracy in the final answer and an unnecessary loss of marks. This was seen particularly in **Question 6**.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

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Answer: 0.186, 0.257, 0.191, He will reject H_o

MATHEMATICS

Paper 9709/73
Paper 73

Key Messages

When expressing the hypotheses for a hypothesis test in a verbal form, reference must be made to the population statistic. Thus, for **Question 1(ii)**, the population mean was required and for **Question 5(i)** the population proportion was required.

General Comments

Many candidates showed a sound understanding of the syllabus and answered the questions in an efficient manner. In many cases work was well presented and methods were clear to follow.

Many candidates scored well on **Questions 1(i), 3(i), 4(i)** and **6**, whilst some candidates found **Questions 3(ii), 4(ii)** and **5** more demanding.

Question 3(ii) required knowledge of when it was not necessary to use the Central Limit Theorem**.**

Question 5 required candidates to apply a significance test, distinguish between Type I and Type II errors and to find the probability of a Type I error in the context of a binomial distribution.

Most candidates attempted all of the questions in the available time.

Comments on Specific Questions

Question 1

(i) Most candidates explained that a difference was required, rather than an increase or a decrease.

(ii) Many candidates omitted the hypotheses.

Some candidates suggested that the null hypothesis should be H_0: $\mu = 1.91$. Other candidates suggested that the null hypothesis should be H_0: μ = the sample mean. These were incorrect.

Many candidates wrote down a valid comparison such as $1.91 < 2.054$ or $0.0281 > 0.02$ and stated the correct conclusion in a suitable form and context ("... no evidence ... mean height ...").

Answer: There was no evidence that the mean height was different.

Question 2

(i) Most candidates who used the property that the (triangular) area was equal to 1 found the value of c correctly. Those candidates who used integration needed to state that $f(x) = x$ first. This fact was also required in the following parts. Some candidates incorrectly used $f(x) = x + c$ or $f(x) = kx$ for some value of k other than 1.

Answer: $c = \sqrt{2}$ or $c = 1.41$

(ii) Candidates who used integration needed to use the correct f(x) = x and the given probability of 0.1 and the correct corresponding limits a and 1.

The value of a could also be found by using areas. Either a trapezium area or the difference of two triangle areas could be used.

Answers given in decimal form or in surd form were accepted.

Answer: 0.894

(iii) To find E(X) candidates needed to integrate xf(x) with limits 0 and c. Some candidates used incorrect limits such as 0 and 1 or 0.894 and c. Some candidates misunderstood this part and attempted to find the median by integrating f(x) and equating the result to 0.5.

Answer: 0.943 or $\dfrac{2\sqrt{2}}{3}$

Question 3

(i) Many candidates found the unbiased estimates of μ and σ^2 correctly (90.25 and 56.3924 or the values to 3 significant figures). Other candidates found only the biased sample variance (55.6875).

Some candidates mixed up the two alternative forms for the variance.

Candidates do need to be able to distinguish formula terms such as $\Sigma(t^2)$ and $(\Sigma t)^2$.

Many candidates knew the correct form for the 97% confidence interval and many candidates found the correct interval. Some candidates used an incorrect value for z such as 1.881 or 1.882 instead of 2.17. Other candidates omitted the 80 or did not include the square root.

Answer: the interval 88.4 to 92.1

(ii) To score full marks here a valid reason was required together with the statement that it was not necessary to use the Central Limit Theorem. It was not sufficient just to state “no”.

The correct reason was that the population was given as normally distributed. Some candidates correctly stated both the reason and the conclusion. Other candidates referred to “it” or “the distribution” or “the data”. Such statements were not sufficiently precise for both marks, but an answer such as “no, because it is normally distributed” was allowed to score one mark. Some candidates thought that the answer was “yes, as n is large” or “yes, as the distribution is unknown” or “yes, as I needed to find the variance”. These answers scored no marks.

Answer: The population was given as normally distributed, so it was not necessary to use the Central Limit Theorem.

Question 4

(i) The new variable was the total mass of 4 randomly chosen tomatoes of type A and 6 randomly chosen tomatoes of type B. The mean and variance for this variable were required. Many candidates found these correctly (1280 and 9744). Some candidates made the error of using 4^2 and 6^2 for the variance. Referring to $A_1 + A_2 + ... + A_4$ and $B_1 + ... + B_6$ rather than $4A$ and $6B$ can be helpful. The correct probability was given by the larger area to the left of 1500. A diagram can help with this.

The simplest way to deal with the 1.5kg was to change this to 1500g to obtain consistent units. Some candidates tried to change the mean and variance to kg and usually made an error in the variance.

Answer: 0.987

(ii) It was necessary to create a new variable ($A - 0.90B$) or equivalent. Various errors were seen at this stage, including "$A - B \geq 0.9B$" and "$B - A \leq 0.1$". For the correct combination $A - 0.90B$ some candidates found the correct mean and variance (8 and 1729.44). Other candidates made errors when finding the variance, such as using 0.9 instead of 0.9^2 or by subtracting variances instead of adding them.

"at least 90% of ...." required $A - 0.9B \geq 0$ in the normal distribution with mean 8. The correct probability was given by the larger area to the right of 0. A diagram can help with this.

Answer: 0.576

Question 5

(i) The null hypothesis should be stated as the population proportion = 0.1 (H_0: $p = 0.1$ was acceptable) and the alternative hypothesis H_1 similarly (with ... > 0.1).

The parameters $n = 18$ and $p = 0.1$ required the use of the binomial distribution. Neither a normal distribution nor a Poisson distribution was valid.

The test statistic of 4 plants out of the 18 plants required the use of the tail of the binomial $X \geq 4$. Some candidates found this probability correctly (0.0982). Some rounding errors were seen. Other candidates incorrectly found $P(X \geq 5) = 0.0282$. The use of $P(X = 4)$ was invalid.

It was necessary to write down a suitable comparison with the 8% significance level such as $0.0982 > 0.08$ and then to state the correct conclusion in an appropriate form and context ("... no evidence ... proportion of plants ...").

Other candidates found the critical region ($X \geq 5$) together with the appropriate probabilities and continued accordingly with this valid method. It was necessary to include the comparison of $X = 4$ with the critical region $X \geq 5$ in the explanation.

Answer: no evidence that the proportion of plants reaching 1m has increased

(ii) To score full marks here a valid reason was required together with the statement that a Type II error might have been made. It was not sufficient just to state "Type II error".

The correct reason was that the null hypothesis was not rejected. Many candidates correctly stated both the reason and the conclusion, including those who were following through correctly from an earlier error (in which case the conclusion and the error type might have been reversed).

Answer: H_0 was not rejected, so a Type II error might have been made

(iii) For a Type I error the probability of rejecting H_0 when H_0 was true was required. This required the probability $P(X \geq 5) = 0.0282$ as $0.0282 < 0.08$.

Many candidates incorrectly found $P(X \geq 4)$ or used the wrong end of the binomial distribution.

Other candidates stated 0.08.

Answer: 0.0282

Question 6

Many correct answers for parts **(i)** and **(ii)** were seen.

(i) The new Poisson parameter ($\lambda = 3.84$) for the 8-minute period and P(4) were needed.

A few candidates used an incorrect value for λ. A few candidates found P(0 to 4).

Answer: 0.195

(ii) The new Poisson parameter (λ = 1.44) for the 3-minute period and P(3, 4, 5, ... calls) were needed. The probability was best found from 1 – P(0, 1, 2). Some candidates incorrectly included the P(3) term in this.

Answer: 0.176

(iii) As the Poisson distribution for the large call centre had a mean of 41, which was greater than 15, the normal distribution N(41, 41) was a suitable approximating distribution.

Many candidates correctly stated this and made a reasonable attempt at standardising and finding the probability. Continuity corrections (40.5 and 59.5) were required. Some candidates incorrectly used 41.5 and 58.5. Other candidates did not use continuity corrections.

For the correct calculations the probability was found from $\Phi_2 - (1 - \Phi_1)$ or $\Phi_2 + \Phi_1 - 1$ or equivalent. After using 41.5 this changed to $\Phi_2 - \Phi_1$. Some candidates did not find the area between the values.

Answer: 0.529