

## CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

### MARK SCHEME for the October/November 2014 series

#### **9709 MATHEMATICS**

**9709/12**

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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## **Mark Scheme Notes**

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\nabla$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

|     |                                                                                                                                                                                         |
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| AEF | Any Equivalent Form (of answer is equally acceptable)                                                                                                                                   |
| AG  | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                     |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)                                                                                                  |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)                                                                                             |
| CWO | Correct Working Only – often written by a 'fortuitous' answer                                                                                                                           |
| ISW | Ignore Subsequent Working                                                                                                                                                               |
| MR  | Misread                                                                                                                                                                                 |
| PA  | Premature Approximation (resulting in basically correct work that is insufficiently accurate)                                                                                           |
| SOS | See Other Solution (the candidate makes a better attempt at the same question)                                                                                                          |
| SR  | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

### **Penalties**

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
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| MR–1 | A penalty of MR–1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA–1 | This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.                                                                                                                                                                                                                                                                                                                            |

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| <p>1 <math>\text{Vol} = (\pi) \int x^2 dy = (\pi) \int (y-1) dy</math></p> <p>Integral is <math>\frac{1}{2}y^2 - y</math> or <math>\frac{(y-1)^2}{2}</math></p> <p>Limits for <math>y</math> are 1 to 5</p> <p><math>\rightarrow 8\pi</math> or 25.1(AWRT)</p>                                                                                                                                                                                                                                     | <p>M1<br/>A1<br/>B1</p> <p>A1<br/>[4]</p>                        | <p>Use of <math>\int x^2</math> – not <math>\int y^2</math> – ignore <math>\pi</math><br/>co</p> <p>Sight of an integral sign with 1 and 5</p> <p>co<br/>(no <math>\pi</math> max 3/4)</p>                                                        |
| <p>2 (i) <math>\tan \theta = \frac{5}{12}</math><br/><math>\rightarrow (\theta = 0.3948)</math></p> <p>(ii) Other angle in triangle <math>= \frac{1}{2}\pi - 0.3948</math><br/>Area of triangle <math>AOB = \frac{1}{2} \times 12 \times 5 (= 30)</math><br/>Use of <math>\frac{1}{2}r^2\theta</math> once<br/>Shaded area = sector + sector – triangle<br/><math>= \frac{1}{2} \times 12^2 \times 0.3948 + \frac{1}{2}5^2\theta - 30</math><br/><br/><math>= 28.43 + 14.70 - 30 = 13.1</math></p> | <p>M1<br/>[1]</p> <p>B1<br/>B1<br/>M1<br/>DM1<br/>A1<br/>[5]</p> | <p>Any valid trig method ag</p> <p>Unsimplified OK<br/>co<br/>With <math>\theta</math> in radians and <math>r = 5</math> or 12</p> <p>Sum of 2 sectors – triangle or any other valid method using the given angle and a different one.<br/>co</p> |
| <p>3 (i) <math>(1+x)^5 = 1 + 5x + 10x^2</math></p> <p>(ii) <math>(1+px+x^2)^5</math><br/><math>(1+) 5(px+x^2) + 10(px+x^2)^2</math><br/><br/>Coeff of <math>x^2 = 5 + 10p^2</math><br/><math>= 95 \rightarrow p = 3</math></p>                                                                                                                                                                                                                                                                     | <p>B2,1<br/>[2]</p> <p>M1<br/>DM1<br/>A1<br/>[3]</p>             | <p>Loses 1 for each error</p> <p>Replace <math>x</math> by <math>(px+x^2)</math> in their expansion</p> <p>Considers 2 terms<br/>co – no penalty for <math>\pm 3</math></p>                                                                       |
| <p>4 <math>y = \frac{12}{3-2x}</math></p> <p>(i) Differential <math>= -12(3-2x)^{-2} \times -2</math></p> <p>(ii) <math>\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 0.4 \div 0.15</math><br/><math>\rightarrow \frac{24}{(3-2x)^2} = \frac{8}{3}</math><br/><br/><math>\rightarrow x = 0</math> or 3</p>                                                                                                                                                                                    | <p>B1 B1<br/>[2]</p> <p>M1<br/>M1<br/>A1 A1<br/>[4]</p>          | <p>co co (even if 1st B mark lost)</p> <p>Chain rule used correctly (AEF)</p> <p>Equates their <math>\frac{dy}{dx}</math> with their <math>\frac{8}{3}</math> or <math>\frac{3}{8}</math></p> <p>co co</p>                                        |

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| <p><b>5</b> <math>1 + \sin x \tan x = 5 \cos x</math></p> <p>(i) Replaces t by s/c<br/> <math>1 + \frac{s^2}{c} = 5c</math><br/> Replace <math>s^2</math> by <math>1 - c^2</math><br/> <math>\rightarrow 6c^2 - c - 1 (= 0)</math></p> <p>(ii) Soln of quadratic <math>\rightarrow (c = -\frac{1}{3} \text{ or } \frac{1}{2})</math><br/> <math>\rightarrow x = 60^\circ \text{ or } 109.5^\circ</math></p>             | <p>M1</p> <p>M1</p> <p>A1<br/>[3]</p> <p>M1<br/>A1 A1<br/>[3]</p>        | <p>Correct formula</p> <p>Correct formula used in appropriate place</p> <p>AG</p> <p>Correct method<br/>co co</p>                                                                                             |
| <p><b>6</b> <math>y = x^3 + ax^2 + bx</math></p> <p>(i) <math>\frac{dy}{dx} = 3x^2 + 2ax + b</math></p> <p>(ii) <math>b^2 - 4ac = 4a^2 - 12b (&lt; 0)</math><br/> <math>\rightarrow a^2 &lt; 3b</math></p> <p>(iii) <math>y = x^3 - 6x^2 + 9x</math><br/> <math>\frac{dy}{dx} = 3x^2 - 12x + 9 &lt; 0</math><br/> <math>= 0 \text{ when } x = 1 \text{ and } 3</math><br/> <math>\rightarrow 1 &lt; x &lt; 3</math></p> | <p>B1</p> <p>M1</p> <p>A1<br/>[3]</p> <p>M1</p> <p>A1<br/>A1<br/>[3]</p> | <p>co</p> <p>Use of discriminant on their quadratic <math>\frac{dy}{dx}</math><br/> or other valid method<br/> co – answer given</p> <p>Attempt at differentiation<br/> co<br/> condone <math>\leq</math></p> |
| <p><b>7</b> (i) <math>\mathbf{AM} = -6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}</math><br/> <math>\mathbf{AC} = -8\mathbf{i} + 8\mathbf{j}</math></p> <p>(ii) <math>\mathbf{AM} \cdot \mathbf{AC} = 48 + 16 = 64</math><br/> <math>64 = \sqrt{128}\sqrt{65}\cos\theta</math><br/> <math>\rightarrow \theta = 45.4^\circ</math></p>                                                                                         | <p>B2,1<br/>B1<br/>[3]</p> <p>M1</p> <p>M1 M1<br/>A1<br/>[4]</p>         | <p>co -1 each error<br/>co</p> <p>Use of <math>x_1y_1 + \text{etc.}</math> with suitable vectors</p> <p>Product of moduli. Correct link.<br/>co</p>                                                           |

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| <p><b>8 (a)</b> <math>S_n = 32n - n^2</math>.<br/> Set <math>n</math> to 1, <math>a</math> or <math>S_1 = 31</math><br/> Set <math>n</math> to 2 or other value <math>S_2 = 60</math><br/> <math>\rightarrow</math> 2nd term = 29 <math>\rightarrow d = -2</math><br/> (or equates formulae – compares coeffs <math>n^2, n</math>)<br/> [M1 comparing, A1 <math>d</math> A1 <math>a</math>]</p> <p><b>(b)</b> <math>\frac{a}{1-r} = 20, \frac{a(1-r^2)}{1-r}, \text{ or } a + ar = 12.8</math></p> <p>Elimination of <math>\frac{a}{1-r}</math> or <math>a</math> or <math>r</math></p> <p><math>\rightarrow (r = 0.6) \rightarrow a = 8</math></p> | <p>B1</p> <p>M1 A1<br/>[3]</p> <p>B1 B1</p> <p>M1</p> <p>DM1 A1<br/>[5]</p>                                                 | <p>co</p> <p>Correct method.<br/>co<br/>[M1 only when coeffs compared]</p> <p>co co</p> <p>‘Correct’ elimination to form equation in <math>a</math> or <math>r</math></p> <p>Complete method leading to <math>a =</math><br/>Condone <math>a = 8</math> and 32</p>                                                                          |
| <p><b>9 (i)</b> <math>m_{AB} = -3</math> or <math>-\frac{9}{3}</math></p> <p><math>m_{AD} = \frac{1}{3}</math></p> <p>Eqn <math>AD</math> <math>y - 6 = \frac{1}{3}(x - 2)</math> or <math>3y = x + 16</math></p> <p><b>(ii)</b> Eqn <math>CD</math> <math>y - 3 = -3(x - 8)</math> or <math>y = -3x + 27</math><br/> Sim Eqns</p> <p><math>\rightarrow D(6\frac{1}{2}, 7\frac{1}{2})</math></p> <p><b>(iii)</b> Use of vectors or mid-point<br/> <math>\rightarrow E(5, 12)</math> or mid-point <math>(5, 4.5)</math><br/> Length of <math>BE = 15</math></p>                                                                                      | <p>B1</p> <p>M1</p> <p>A1<br/>[3]</p> <p>B1 <math>\frac{1}{3}</math><br/>M1</p> <p>A1<br/>[3]</p> <p>B1<br/>B1<br/>[2]</p>  | <p>oe</p> <p>use of <math>m_1 m_2 = -1</math> with grad <math>AB</math></p> <p>co – OK unsimplified</p> <p>OK unsimplified. <math>\frac{1}{3}</math> on <math>m</math> of <math>AB</math>.<br/>Reasonable algebra leading to <math>x =</math> or <math>y =</math> with <math>AD</math> and <math>CD</math></p> <p>May be implied<br/>co</p> |
| <p><b>10</b> <math>\frac{d^2 y}{dx^2} = \frac{24}{x^3} - 4</math></p> <p><b>(i)</b> (If <math>x = 2</math>) it's negative <math>\rightarrow</math> Max</p> <p><b>(ii)</b> <math>\left(\frac{dy}{dx}\right) = -12x^{-2} - 4x + (A)</math><br/> <math>= 0</math> when <math>x = 2</math><br/> <math>\rightarrow A = 11</math></p> <p><b>(iii)</b> <math>(y =) 12x^{-1} - 2x^2 + Ax + (c)</math><br/> <math>y = 13</math> when <math>x = 1 \rightarrow c = -8</math><br/> (If <math>x = 2</math>) <math>y = 12</math></p>                                                                                                                              | <p>B1<br/>[1]</p> <p>B2,1,0</p> <p>M1<br/>A1<br/>[4]</p> <p>B2,1,0 <math>\frac{1}{3}</math></p> <p>M1</p> <p>A1<br/>[4]</p> | <p>www</p> <p>oe one per term</p> <p>Attempt at the constant <math>A</math> after <math>\int n</math><br/>co</p> <p>oe Doesn't need <math>+c</math>, but does need a term <math>A</math> to give “<math>Ax</math>”.<br/>Attempt at <math>c</math> after <math>\int n</math><br/>co</p>                                                      |

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| <p><b>11</b> <math>f: x \mapsto 6 - 4\cos\left(\frac{1}{2}x\right)</math></p> <p><b>(i)</b> <math>6 - 4\cos\left(\frac{1}{2}x\right) = 4 \rightarrow 4\cos\left(\frac{1}{2}x\right) = 2</math></p> $\frac{1}{2}x = \frac{1}{3}\pi \quad x = \frac{2}{3}\pi$ <p><b>(ii)</b> Range is <math>2 \leq f(x) \leq 10</math></p> <p><b>(iii)</b></p> <p><b>(iv)</b> <math>\cos\left(\frac{1}{2}x\right) = \frac{1}{4}(6 - y)</math></p> $\frac{1}{2}x = \cos^{-1}\left(\frac{1}{4}(6 - y)\right)$ $f^{-1}(x) = 2\cos^{-1}\left(\frac{6 - x}{4}\right)$ | <p>M1</p> <p>M1</p> <p>A1<br/>[3]</p> <p>B1 B1<br/>[2]</p> <p>B1<br/>B1<br/>[2]</p> <p>M1</p> <p>M1</p> <p>A1<br/>[3]</p> | <p>Makes <math>\cos\left(\frac{1}{2}x\right)</math> the subject.</p> <p>Looks up "<math>\frac{1}{2}x</math>" before <math>\times 2</math></p> <p>co (120° gets A0 – decimals A0)</p> <p>condone &lt;</p> <p>Point of inflexion at <math>\pi</math><br/>Fully correct</p> <p>Makes <math>\cos\left(\frac{1}{2}x\right)</math> the subject</p> <p>Order of operations correct<br/>(M marks allowed if + for -)</p> <p>oe – needs to be a function of <math>x</math> not <math>y</math></p> |
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