MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/23 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2014 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.
Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking \( g \) equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG  Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO  Correct Working Only – often written by a ‘fortuitous’ answer
ISW  Ignore Subsequent Working
MR  Misread
PA  Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR  Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1  A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1  This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.
1  State or imply correct y-values 6, 4, 0, 8, 24  B1
   Use correct formula, or equivalent, with h = 1 and five y-values  M1
   Obtain 27  A1 [3]

2  State or imply ln y = \ln a + x \ln b  B1
   Equate ln b to numerical gradient of line  M1
   Obtain b = 1.85  A1
   Substitute to find value of a  M1
   Obtain a = 3.45  A1 [5]

3  (a) Express integrand in the form \( p \cos \theta + 2 \)  M1
   State correct \( 2 \cos \theta + 2 \)  A1
   Integrate to obtain \( 2 \sin \theta + 2 \theta + c \)  A1 [3]

   (b) Integrate to obtain form \( k \ln(2x + 3) \)  M1
      Obtain correct \( \frac{1}{2} \ln(2x + 3) \)  A1
      Apply limits correctly  DM1
      Obtain \( \frac{1}{2} \ln 15 \)  A1 [4]

4  (i) Differentiate to obtain form \( k_1 \sin 2x + k_2 \cos x \)  M1
      Obtain correct \(-6 \sin 2x - 5 \cos x \)  A1
      Substitute \( \frac{1}{6} \pi \) to obtain \(-\frac{11}{2} \sqrt{3} \) or exact equivalent  A1 [3]

      (ii) Obtain \( 6y + 6x \frac{dy}{dx} \) as derivative of \( 6xy \)  B1
           Obtain \( 3y^2 \frac{dy}{dx} \) as derivative of \( y^3 \)  B1
           Obtain \( 3x^2 + 6y + 6x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0 \) or equivalent  B1
           Substitute 1 and 2 to find value of gradient dependent on at least one  B1
           Obtain gradient \(-\frac{15}{18} \) or \(-\frac{5}{6} \)  A1 [5]

5  (i) State \(-40 + 4a + b = 0 \) or equivalent  B1
      State \(-135 + 9a + b = 0 \) or equivalent  B1
      Solve a pair of linear simultaneous equations  M1
      Obtain \( a = 19 \) and \( b = -36 \)  A1 [4]

      (ii) Identify \( 5x - 6 \) as a factor  B1
           State \((x + 2)(x + 3)(5x - 6)\)  B1
           State or imply \( 5^v = \frac{6}{5} \), following a positive value from factorisation  B1√
           Apply logarithms and use power law  M1
           Obtain 0.113 only
6 (i) Use quotient rule or equivalent M1

\[ \frac{2x(1 + e^{3x}) - 3x^2 e^{3x}}{(1 + e^{3x})^2} \text{ or equivalent} \] A1

Equate first derivative to zero and attempt rearrangement to \( x = \ldots \) DM1

Obtain \( x = \frac{2}{3}(1 + e^{-3x}) \) with sufficient detail and no errors seen (AG) A1 [4]

(ii) Consider sign of \( x - \frac{2}{3}(1 + e^{-3x}) \) at 0.7 and 0.8 or equivalent M1

Obtain correct values (-0.05 and 0.07 or equivalents) and conclude appropriately A1 [2]

(iii) Use the iterative formula correctly at least once M1

Obtain final answer 0.739 A1

Show sufficient iterations to 5 decimal places to justify result or show a sign change in the interval (0.7385, 0.7395) A1 [3]

7 (i) Use \( \tan^2 \alpha = 1 + \tan^2 \alpha \) B1

Confirm \( 3 \tan^2 \alpha + 4 \tan \alpha - 4 = 0 \) B1

Solve quadratic equation for \( \tan \alpha \) M1

Obtain, finally, \( \tan \alpha = \frac{2}{3} \) only A1 [4]

(ii) State or imply \( \tan(\alpha + \beta) = \frac{1}{6} \) B1

State \( \frac{\beta + \tan \beta}{1 - \frac{2}{3} \tan \beta} = \frac{1}{6} \), following their value of \( \tan \alpha \) B1 \√

Solve equation of form \( \frac{a + bt}{c + dt} \) for \( t \) M1

Obtain \( \tan \beta = -\frac{9}{20} \) A1

Conclude with \( \cot \beta = -\frac{20}{9} \) or exact equivalent A1 [5]