CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2014 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a power	M1	
		Obtain a correct linear equation in any form, e.g. $x = (x-2)\ln 3$	A1	
		Obtain answer $x = 22.281$	A1	**[3]**
2	**(i)**	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 1.95	A1	**[3]**
	(ii)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
		Justify a statement that the estimate will be an overestimate	B1	**[2]**
3		Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$	B1	
		Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21	M1	
		Obtain a correct equation, e.g. $8a + 4b + 5 = 21$	A1	
		Solve for a or for b	M1	
		Obtain $a = 12$ and $b = -20$	A1	**[5]**
4	**(i)**	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(ii)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**
5	**(i)**	Substitute $z = 1 + i$ and obtain $w = \frac{1+2i}{1+i}$	B1	
		EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent	M1	
		Simplify numerator to $3 + i$ or denominator to 2	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	**[4]**

	(ii)	*EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$	B1	
		Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y	M1	
		OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts	B1	
		Solve for x and y	M1	
		Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$	A1	
		Obtain final answer $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\mathrm{i}$	A1	[4]
6	**(i)**	Integrate and reach $bx\ln 2x - c\int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent	M1*	
		Obtain $x\ln 2x - \int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent	A1	
		Obtain integral $x\ln 2x - x$, or equivalent	A1	
		Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
		Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
		Obtain the given answer	A1	[6]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.94	A1	
		Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945).	A1	[3]
7	**(i)**	Separate variables correctly and attempt to integrate at least one side	B1	
		Obtain term $\ln R$	B1	
		Obtain $\ln x - 0.57x$	B1	
		Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$	M1	
		Obtain correct solution in any form	A1	
		Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$	A1	[6]
	(ii)	Equate $\dfrac{\mathrm{d}R}{\mathrm{d}x}$ to zero and solve for x	M1	
		State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
		Obtain $R = 28.8$ (allow 28.9)	A1	[3]
8	**(i)**	Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ	M1	
		Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$	M1	
		Obtain a correct expression in terms of $\sin\theta$ in any form	A1	
		Obtain the given identity	A1	[4]
		[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]		

	(ii)	Substitute for x and obtain the given answer	B1	**[1]**
	(iii)	Carry out a correct method to find a value of x	M1	
		Obtain answers 0.322, 0.799, –1.12	A1 + A1 + A1	**[4]**
		[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]		
9	**(i)**	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	B1	
		Use a correct method to determine a constant	M1	
		Obtain one of $A = 2$, $B = -1$, $C = 3$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	**[5]**
		[The alternative form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2-x)^2}$, where $A = 2$, $D = 1$, $E = 1$ is marked B1M1A1A1A1 as above.]		
	(ii)	Use correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2-x)^{-1}$, $(2-x)^{-2}$, $(1-\tfrac{1}{2}x)^{-1}$ or $(1-\tfrac{1}{2}x)^{-2}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1$\surd$ + A1$\surd$ + A1$\surd$	
		Obtain final answer $\dfrac{9}{4}+\dfrac{5}{2}x+\dfrac{39}{16}x^2$, or equivalent	A1	**[5]**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for M1. The $\surd$ is on A,B,C.]		
		[For the A,D,E form of partial fractions, give M1 A1$\surd$ A1$\surd$ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]		
		[In the case of an attempt to expand $(x^2-8x+9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
10	**(i)**	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i}-17\mathbf{j}+4\mathbf{k}+\lambda(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	B1	
		Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1	
		Solve and obtain $\lambda = 3$	A1	
		Carry out a complete method for finding the length of AP	M1	
		Obtain the given answer 15 correctly	A1	
		*OR*1: Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i}+17\mathbf{j}-4\mathbf{k}$	B1	
		Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i}+17\mathbf{j}-4\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	M1	
		Obtain correct answer, e.g. $-30\mathbf{i}+6\mathbf{j}+33\mathbf{k}$	A1	
		Divide the modulus of the product by that of the direction vector	M1	
		Obtain the given answer correctly	A1	
		*OR*2: State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
		Use a scalar product to find the projection of BA (or AB) on l	M1	
		Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}}$	A1	
		Use Pythagoras to find the perpendicular		

	Obtain the given answer correctly	A1	
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the cosine of *ABP*	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}.\sqrt{306}}$	A1	
	Use trig. to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Find a second point *C* on *l* and use the cosine rule in triangle *ABC* to find the cosine of angle *A, B*, or *C*, or use a vector product to find the area of *ABC*	M1	
	Obtain correct answer in any form	A1	
	Use trig. or area formula to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point *P* on *l* with parameter λ in any form	B1	
	Use correct method to express AP^2 (or *AP*) in terms of λ	M1	
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2+(-17+\lambda)^2+(4-2\lambda)^2$	A1	
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
	Obtain the given answer correctly	A1	[5]
(ii) *EITHER*:	Substitute coordinates of a general point of *l* in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in *a* and *b*	M1*	
	Obtain a correct equation, e.g. $4a-9b-27+1=0$	A1	
	Obtain a second correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a=2$ and $b=-2$	A1	
OR:	Substitute coordinates of a point of *l* and obtain a correct equation, e.g. $4a-9b=26$	B1	
	EITHER: Find a second point on *l* and obtain an equation in *a* and *b*	M1*	
	Obtain a correct equation	A1	
	OR: Calculate scalar product of a direction vector for *l* and a vector normal to the plane and equate to zero	M1*	
	Obtain a correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a=2$ and $b=-2$	A1	[5]