MATHEMATICS

Paper 2 Pure Mathematics 2 (P2)

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
1 Solve the equation $|3x - 1| = |2x + 5|$. \[3\]

2 (i) Find $\int_0^a (e^{-x} + 6e^{-3x}) \, dx$, where $a$ is a positive constant. \[4\]

(ii) Deduce the value of $\int_0^\infty (e^{-x} + 6e^{-3x}) \, dx$. \[1\]

3 A curve has equation

$$3 \ln x + 6xy + y^2 = 16.$$ Find the equation of the normal to the curve at the point $(1, 2)$. Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. \[7\]

4 (a) Find the value of $x$ satisfying the equation $2 \ln(x - 4) - \ln x = \ln 2$. \[5\]

(b) Use logarithms to find the smallest integer satisfying the inequality

$$1.4^x > 10^{10}.$$ \[3\]

5 The diagram shows part of the curve

$$y = 2 \cos x - \cos 2x$$

and its maximum point $M$. The shaded region is bounded by the curve, the axes and the line through $M$ parallel to the $y$-axis.

(i) Find the exact value of the $x$-coordinate of $M$. \[4\]

(ii) Find the exact value of the area of the shaded region. \[4\]
The polynomial \( p(x) \) is defined by

\[
p(x) = x^4 - 3x^3 + 3x^2 - 25x + 48.
\]

The diagram shows the curve \( y = p(x) \) which crosses the \( x \)-axis at \( (\alpha, 0) \) and \( (3, 0) \).

(i) Divide \( p(x) \) by a suitable linear factor and hence show that \( \alpha \) is a root of the equation

\[
x = \sqrt[4]{16 - 3x}.
\]

(ii) Use the iterative formula \( x_{n+1} = \sqrt[4]{16 - 3x_n} \) to find \( \alpha \) correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

7 (i) Express \( 5 \cos \theta - 12 \sin \theta \) in the form \( R \cos(\theta + \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), giving the value of \( \alpha \) correct to 2 decimal places.

(ii) Hence solve the equation \( 5 \cos \theta - 12 \sin \theta = 8 \) for \( 0^\circ < \theta < 360^\circ \).

(iii) Find the greatest possible value of

\[
7 + 5 \cos \frac{\phi}{2} - 12 \sin \frac{\phi}{2}
\]

as \( \phi \) varies, and determine the smallest positive value of \( \phi \) for which this greatest value occurs.