A golf ball $B$ is projected from a point $O$ on horizontal ground. $B$hits the ground for the first time at a point 48 m away from $O$ at time 2.4 s after projection. Calculate the angle of projection. [3]

A particle $P$ of mass 0.2 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 64 N. The other end of the string is attached to a fixed point $A$ on a smooth horizontal surface. $P$ is placed on the surface at a point 0.8 m from $A$. The particle $P$ is then projected with speed 10 m s$^{-1}$ directly away from $A$.

(i) Calculate the distance $AP$ when $P$ is at instantaneous rest. [3]

(ii) Calculate the speed of $P$ when it is 1.0 m from $A$. [3]

A small ball of mass $m$ kg is projected vertically upwards with speed 14 m s$^{-1}$. The ball has velocity $v$ m s$^{-1}$ upwards when it is $x$ m above the point of projection. A resisting force of magnitude $0.02mv$ N acts on the ball during its upward motion.

(i) Show that, while the ball is moving upwards, \( \frac{500}{v+500} - 1 \) \( \frac{dv}{dx} = 0.02 \). [3]

(ii) Find the greatest height of the ball above its point of projection. [3]

A particle $P$ is projected with speed 50 m s$^{-1}$ at an angle of 30$^\circ$ above the horizontal from a point $O$ on a horizontal plane.

(i) Calculate the speed of $P$ when it has been in motion for 4 s, and calculate another time at which $P$ has this speed. [5]

(ii) Find the distance $OP$ when $P$ has been in motion for 4 s. [2]

Two light elastic strings each have one end attached to a fixed horizontal beam. One string has natural length 0.6 m and modulus of elasticity 12 N; the other string has natural length 0.7 m and modulus of elasticity 21 N. The other ends of the strings are attached to a small block $B$ of weight $W$ N. The block hangs in equilibrium $d$ m below the beam, with both strings vertical (see diagram).

(i) Given that the tensions in the strings are equal, find $d$ and $W$. [4]

The small block is now raised vertically to the point 0.7 m below the beam, and then released from rest.

(ii) Find the greatest speed of the block in its subsequent motion. [4]
6 A horizontal disc with a rough surface rotates about a fixed vertical axis which passes through the centre of the disc. A particle $P$ of mass 0.2 kg is in contact with the surface and rotates with the disc, without slipping, at a distance 0.5 m from the axis. The greatest speed of $P$ for which this motion is possible is $1.5 \, \text{m} \, \text{s}^{-1}$.

(i) Calculate the coefficient of friction between the disc and $P$. [2]

$P$ is now attached to one end of a light elastic string, which is connected at its other end to a point on the vertical axis above the disc. The tension in the string is equal to half the weight of $P$. The disc rotates with constant angular speed $\omega \, \text{rad} \, \text{s}^{-1}$ and $P$ rotates with the disc without slipping. $P$ moves in a circle of radius 0.5 m, and the taut string makes an angle of $30^\circ$ with the horizontal.

(ii) Find the greatest and least values of $\omega$ for which this motion is possible. [5]

(iii) Calculate the value of $\omega$ for which the disc exerts no frictional force on $P$. [2]

7 A uniform lamina $A\,B\,C$ is in the form of a major segment of a circle with centre $O$ and radius 0.35 m. The straight edge of the lamina is $AB$, and angle $AOB = \frac{2}{3}\pi$ radians (see diagram).

(i) Show that the centre of mass of the lamina is 0.0600 m from $O$, correct to 3 significant figures. [6]

The weight of the lamina is 14 N. It is placed on a rough horizontal surface with $A$ vertically above $B$ and the lowest point of the arc $BC$ in contact with the surface. The lamina is held in equilibrium in a vertical plane by a force of magnitude $F$ N acting at $A$.

(ii) Find $F$ in each of the following cases:

(a) the force of magnitude $F$ N acts along $AB$; [2]

(b) the force of magnitude $F$ N acts along the tangent to the circular arc at $A$. [3]