1. A line has equation \( y = 2x - 7 \) and a curve has equation \( y = x^2 - 4x + c \), where \( c \) is a constant. Find the set of possible values of \( c \) for which the line does not intersect the curve. [3]

2. Find the coefficient of \( x \) in the expansion of \( \left( \frac{x}{3} + \frac{9}{x} \right)^7 \). [4]

3. (i) Express \( 3x^2 - 6x + 2 \) in the form \( a(x + b)^2 + c \), where \( a \), \( b \) and \( c \) are constants. [3]

(ii) The function \( f \), where \( f(x) = x^3 - 3x^2 + 7x - 8 \), is defined for \( x \in \mathbb{R} \). Find \( f'(x) \) and state, with a reason, whether \( f \) is an increasing function, a decreasing function or neither. [3]

4. The diagram shows a metal plate \( OABCDEF \) consisting of 3 sectors, each with centre \( O \). The radius of sector \( COD \) is \( 2r \) and angle \( COD \) is \( \theta \) radians. The radius of each of the sectors \( BOA \) and \( FOE \) is \( r \), and \( AOED \) and \( CBOF \) are straight lines.

(i) Show that the area of the metal plate is \( r^2(\pi + \theta) \). [3]

(ii) Show that the perimeter of the metal plate is independent of \( \theta \). [4]

5. Relative to an origin \( O \), the position vectors of the points \( A \) and \( B \) are given by

\[
\overrightarrow{OA} = \left( \begin{array}{c} p - 6 \\ 2p - 6 \\ 1 \end{array} \right) \quad \text{and} \quad \overrightarrow{OB} = \left( \begin{array}{c} 4 - 2p \\ p \\ 2 \end{array} \right),
\]

where \( p \) is a constant.

(i) For the case where \( OA \) is perpendicular to \( OB \), find the value of \( p \). [3]

(ii) For the case where \( OAB \) is a straight line, find the vectors \( \overrightarrow{OA} \) and \( \overrightarrow{OB} \). Find also the length of the line \( OA \). [4]
A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A: The height reached is reduced by 0.04 metres each time the ball bounces.

Model B: The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

7 (a) Show that the equation \( \frac{1}{\cos \theta} + 3 \sin \theta \tan \theta + 4 = 0 \) can be expressed as

\[ 3 \cos^2 \theta - 4 \cos \theta - 4 = 0, \]

and hence solve the equation \( \frac{1}{\cos \theta} + 3 \sin \theta \tan \theta + 4 = 0 \) for \( 0^\circ \leq \theta \leq 360^\circ \). [6]

(b) The diagram shows part of the graph of \( y = a \cos x - b \), where \( a \) and \( b \) are constants. The graph crosses the \( x \)-axis at the point \( C(\cos^{-1} c, 0) \) and the \( y \)-axis at the point \( D(0, d) \). Find \( c \) and \( d \) in terms of \( a \) and \( b \). [2]

8 The function \( f \) is defined by \( f(x) = 3x + 1 \) for \( x \leq a \), where \( a \) is a constant. The function \( g \) is defined by \( g(x) = -1 - x^2 \) for \( x \leq -1 \).

(i) Find the largest value of \( a \) for which the composite function \( gf \) can be formed. [2]

For the case where \( a = -1 \),

(ii) solve the equation \( fg(x) + 14 = 0 \), [3]

(iii) find the set of values of \( x \) which satisfy the inequality \( gf(x) \leq -50 \). [4]
9 A curve passes through the point $A (4, 6)$ and is such that $\frac{dy}{dx} = 1 + 2x^{-\frac{1}{2}}$. A point $P$ is moving along the curve in such a way that the $x$-coordinate of $P$ is increasing at a constant rate of 3 units per minute.

(i) Find the rate at which the $y$-coordinate of $P$ is increasing when $P$ is at $A$. [3]

(ii) Find the equation of the curve. [3]

(iii) The tangent to the curve at $A$ crosses the $x$-axis at $B$ and the normal to the curve at $A$ crosses the $x$-axis at $C$. Find the area of triangle $ABC$. [5]

10 The function $f$ is defined by $f(x) = 2x + (x + 1)^{-2}$ for $x > -1$.

(i) Find $f'(x)$ and $f''(x)$ and hence verify that the function $f$ has a minimum value at $x = 0$. [4]

![Diagram of curve with points A and B]

The points $A (-\frac{1}{2}, 3)$ and $B (1, 2\frac{1}{4})$ lie on the curve $y = 2x + (x + 1)^{-2}$, as shown in the diagram.

(ii) Find the distance $AB$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [6]