Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2015**

1 hour 45 minutes

4520717899

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

CAMBRIDGE
International Examinations

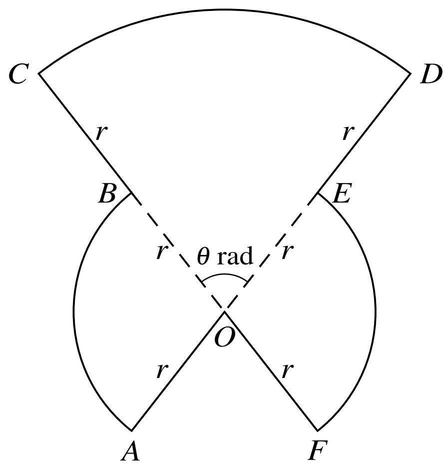
1 A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]

2 Find the coefficient of x in the expansion of $\left(\frac{x}{3} + \frac{9}{x^2}\right)^7$. [4]

3 **(i)** Express $3x^2 - 6x + 2$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f, where $f(x) = x^3 - 3x^2 + 7x - 8$, is defined for $x \in \mathbb{R}$. Find $f'(x)$ and state, with a reason, whether f is an increasing function, a decreasing function or neither. [3]

4



The diagram shows a metal plate $OABCDEF$ consisting of 3 sectors, each with centre O. The radius of sector COD is $2r$ and angle COD is θ radians. The radius of each of the sectors BOA and FOE is r, and $AOED$ and $CBOF$ are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$. [3]

(ii) Show that the perimeter of the metal plate is independent of θ. [4]

5 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} p-6 \\ 2p-6 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4-2p \\ p \\ 2 \end{pmatrix},$$

where p is a constant.

(i) For the case where OA is perpendicular to OB, find the value of p. [3]

(ii) For the case where OAB is a straight line, find the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Find also the length of the line OA. [4]

6 A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A : The height reached is reduced by 0.04 metres each time the ball bounces.

Model B : The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

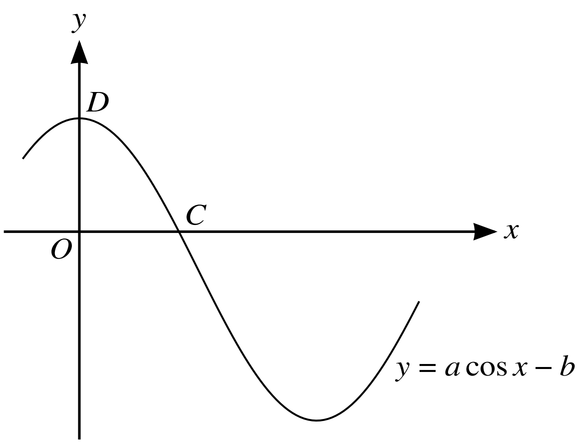
(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

7 **(a)** Show that the equation $\frac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ can be expressed as

$$3\cos^2\theta - 4\cos\theta - 4 = 0,$$

and hence solve the equation $\frac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ for $0° \leqslant \theta \leqslant 360°$. [6]

(b)



The diagram shows part of the graph of $y = a\cos x - b$, where a and b are constants. The graph crosses the x-axis at the point $C\,(\cos^{-1}c,\ 0)$ and the y-axis at the point $D\,(0,\ d)$. Find c and d in terms of a and b. [2]

8 The function f is defined by $\mathrm{f}(x) = 3x + 1$ for $x \leqslant a$, where a is a constant. The function g is defined by $\mathrm{g}(x) = -1 - x^2$ for $x \leqslant -1$.

(i) Find the largest value of a for which the composite function gf can be formed. [2]

For the case where $a = -1$,

(ii) solve the equation $\mathrm{fg}(x) + 14 = 0$, [3]

(iii) find the set of values of x which satisfy the inequality $\mathrm{gf}(x) \leqslant -50$. [4]

9 A curve passes through the point $A\ (4,\ 6)$ and is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + 2x^{-\frac{1}{2}}$. A point P is moving along the curve in such a way that the x-coordinate of P is increasing at a constant rate of 3 units per minute.

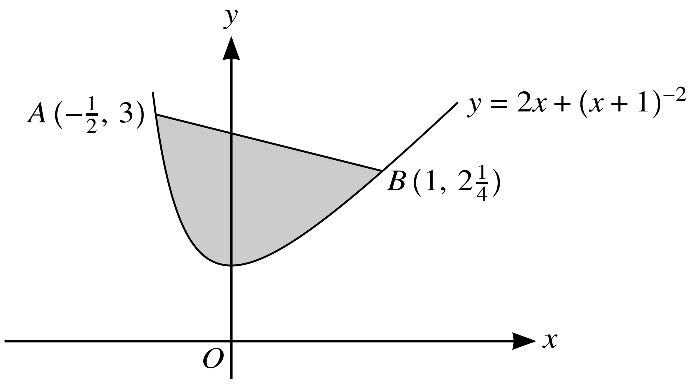
(i) Find the rate at which the y-coordinate of P is increasing when P is at A. [3]

(ii) Find the equation of the curve. [3]

(iii) The tangent to the curve at A crosses the x-axis at B and the normal to the curve at A crosses the x-axis at C. Find the area of triangle ABC. [5]

10 The function f is defined by $\mathrm{f}(x) = 2x + (x+1)^{-2}$ for $x > -1$.

(i) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$ and hence verify that the function f has a minimum value at $x = 0$. [4]



The points $A\ (-\frac{1}{2},\ 3)$ and $B\ (1,\ 2\frac{1}{4})$ lie on the curve $y = 2x + (x+1)^{-2}$, as shown in the diagram.

(ii) Find the distance AB. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [6]