1 Use logarithms to solve the equation
\[ 5^{x+3} = 7^{x-1}, \]
giving the answer correct to 3 significant figures. [4]

2 A curve has equation
\[ y = \frac{3x + 1}{x - 5}. \]
Find the coordinates of the points on the curve at which the gradient is \(-4\). [5]

3 (i) Express \(8 \sin \theta + 15 \cos \theta\) in the form \(R \sin(\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). Give the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence solve the equation
\[ 8 \sin \theta + 15 \cos \theta = 6 \]
for \(0^\circ \leq \theta \leq 360^\circ\). [4]

4 (i) By sketching a suitable pair of graphs, show that the equation
\[ \ln x = 4 - \frac{1}{2}x \]
has exactly one real root, \(\alpha\). [2]

(ii) Verify by calculation that \(4.5 < \alpha < 5.0\). [2]

(iii) Use the iterative formula \(x_{n+1} = 8 - 2 \ln x_n\) to find \(\alpha\) correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 (a) Find \(\int (\tan^2 x + \sin 2x) \, dx\). [3]

(b) Find the exact value of \(\int_0^1 3e^{1-2x} \, dx\). [4]

6 (i) Find the quotient and remainder when
\[ x^4 + x^3 + 3x^2 + 12x + 6 \]
is divided by \((x^2 - x + 4)\). [4]

(ii) It is given that, when
\[ x^4 + x^3 + 3x^2 + px + q \]
is divided by \((x^2 - x + 4)\), the remainder is zero. Find the values of the constants \(p\) and \(q\). [2]

(iii) When \(p\) and \(q\) have these values, show that there is exactly one real value of \(x\) satisfying the equation
\[ x^4 + x^3 + 3x^2 + px + q = 0 \]
and state what that value is. [3]
The parametric equations of a curve are

\[ x = 6 \sin^2 t, \quad y = 2 \sin 2t + 3 \cos 2t, \]

for \( 0 \leq t < \pi \). The curve crosses the \( x \)-axis at points \( B \) and \( D \) and the stationary points are \( A \) and \( C \), as shown in the diagram.

(i) Show that \( \frac{dy}{dx} = \frac{2}{3} \cot 2t - 1 \). [5]

(ii) Find the values of \( t \) at \( A \) and \( C \), giving each answer correct to 3 decimal places. [3]

(iii) Find the value of the gradient of the curve at \( B \). [3]