Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **7** printed pages.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- **Note:** B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR – 1 A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA – 1 This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1	Use law of the logarithm of a product, power or quotient	**M1***	
	Obtain a correct linear equation, e.g. $(3x-1)\ln 4 = \ln 3 + x\ln 5$	**A1**	
	Solve a linear equation for x	**DM1***	
	Obtain answer $x = 0.975$	**A1**	[4]
2	State a correct un-simplified version of the x or x^2 or x^3 term	**M1**	
	State correct first two terms $1 + x$	**A1**	
	Obtain the next two terms $\frac{3}{2}x^2 + \frac{5}{2}x^3$	**A1 A1**	[4]
	[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{2} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]		
3	Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x \, dx$	**M1***	
	Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	**A1**	
	Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	**A1**	
	Use limits correctly having integrated twice	**DM1***	
	Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	**A1**	[5]
4	State or imply derivative of $(\ln x)^2$ is $\dfrac{2\ln x}{x}$	**B1**	
	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form, e.g. $\dfrac{2\ln x}{x^2} - \dfrac{(\ln x)^2}{x^2}$	**A1**	
	Equate derivative (or its numerator) to zero and solve for $\ln x$	**M1**	
	Obtain the point (1, 0) with no errors seen	**A1**	
	Obtain the point $(e^2, \ 4e^{-2})$	**A1**	[6]
5	**(i)** *EITHER*: Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	**B1**	
	Use correct double angle formulae to express LHS in terms of $\sin \theta$ and/or $\cos \theta$	**M1**	
	Obtain a correct expression in terms of $\sin \theta$ alone	**A1**	
	Reduce correctly to the given form	**A1**	
	OR: Use correct double angle formula to express RHS in terms of $\cos 2\theta$	**M1**	
	Express $\cos^2 2\theta$ in terms of $\cos 4\theta$	**B1**	
	Obtain a correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$	**A1**	
	Reduce correctly to the given form	**A1**	[4]
	(ii) Use the identity and carry out a method for finding a root	**M1**	
	Obtain answer 68.5°	**A1**	
	Obtain a second answer, e.g. 291.5°	**A1√**	
	Obtain the remaining answers, e.g. 111.5° and 248.5°, and no others in the given interval	**A1√**	[4]
	[Ignore answers outside the given interval. Treat answers in radians as a misread.]		

6	(i)	Separate variables correctly and attempt integration of at least one side	B1	
		Obtain term $\ln x$	B1	
		Obtain term of the form $k\ln(3+\cos 2\theta)$, or equivalent	M1	
		Obtain term $-\frac{1}{2}\ln(3+\cos 2\theta)$, or equivalent	A1	
		Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution with terms $a\ln x$ and $b\ln(3+\cos 2\theta)$, where $ab \neq 0$	M1	
		State correct solution in any form, e.g. $\ln x = -\frac{1}{2}\ln(3+\cos 2\theta) + \frac{3}{2}\ln 3$	A1	
		Rearrange in a correct form, e.g. $x = \sqrt{\left(\dfrac{27}{3+\cos 2\theta}\right)}$	A1	[7]
	(ii)	State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60])	B1	[1]
7	(i)	State or imply the form $A + \dfrac{B}{2x+1} + \dfrac{C}{x+2}$	B1	
		State or obtain $A = 2$	B1	
		Use a correct method for finding a constant	M1	
		Obtain one of $B = 1$, $C = -2$	A1	
		Obtain the other value	A1	[5]
	(ii)	Integrate and obtain terms $2x + \frac{1}{2}\ln(2x+1) - 2\ln(x+2)$	B3√	
		Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	M1	
		Obtain the given answer after full and correct working	A1	[5]
8	(i)	Use correct quotient or chain rule	M1	
		Obtain correct derivative in any form	A1	
		Obtain the given answer correctly	A1	[3]
	(ii)	State a correct equation, e.g. $-e^{-a} = -\operatorname{cosec} a \cot a$	B1	
		Rearrange it correctly in the given form	B1	[2]
	(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1	
		Complete the argument correctly with correct calculated values	A1	[2]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.317	A1	
		Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1,3175)	A1	[3]

9	**(i)**	*Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$	**B1**	
		Use a correct method for finding the position vector of D	**M1**	
		Obtain answer $3\mathbf{i}+3\mathbf{j}+\mathbf{k}$, or equivalent	**A1**	
		EITHER: Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and BC	**M1**	
		Show that $ABCD$ has a pair of adjacent sides that are equal	**A1**	
		OR: Calculate scalar product $\overrightarrow{AC}.\overrightarrow{BD}$ or equivalent	**M1**	
		Show that $ABCD$ has perpendicular diagonals	**A1**	[5]
	(ii)	*EITHER*: State $a+2b+3c=0$ or $2a+b-2c=0$	**B1**	
		Obtain two relevant equations and solve for one ratio, e.g. $a:b$	**M1**	
		Obtain $a:b:c=-7:8:-3$, or equivalent	**A1**	
		Substitute coordinates of a relevant point in $-7x+8y-3z=d$, and evaluate	**M1**	
		Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	
		*OR*1:Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}+2\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	**M1**	
		Obtain two correct components of the product	**A1**	
		Obtain correct product, e.g. $-7\mathbf{i}+8\mathbf{j}-3\mathbf{k}$	**A1**	
		Substitute coordinates of a relevant point in $-7x+8y-3z=d$ and evaluate d	**M1**	
		Obtain answer $-7x+8y-3z=29$ or equivalent	**A1**	
		*OR*2:Attempt to form a 2-parameter equation with relevant vectors	**M1**	
		State a correct equation, e.g. $\mathbf{r}=2\mathbf{i}+5\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\mu(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	**A1**	
		State 3 equations in x, y, z, λ and μ	**A1**	
		Eliminate λ and μ	**M1**	
		Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	
		*OR*3:Using a relevant point and relevant direction vectors, form a determinant equation for the plane	**M1**	
		State a correct equation, e.g. $\begin{vmatrix} x-2 & y-5 & z+1 \\ 1 & 2 & 3 \\ 2 & 1 & -2 \end{vmatrix}=0$	**A1**	
		Attempt to expand the determinant	**M1**	
		Obtain correct values of two cofactors	**A1**	
		Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	[5]

10	**(a)**	*EITHER*: Use quadratic formula to solve for z	**M1**	
		Use $i^2 = -1$	**M1**	
		Obtain a correct answer in any form, simplified as far as $(-2 \pm i\sqrt{8})/2i$	**A1**	
		Multiply numerator and denominator by i, or equivalent	**M1**	
		Obtain final answers $\sqrt{2} + i$ and $-\sqrt{2} + i$	**A1**	
		OR: Substitute $x + iy$ and equate real and imaginary parts to zero	**M1**	
		Use $i^2 = -1$	**M1**	
		Obtain $-2xy + 2x = 0$ and $x^2 - y^2 + 2y - 3 = 0$, or equivalent	**A1**	
		Solve for x and y	**M1**	
		Obtain final answers $\sqrt{2} + i$ and $-\sqrt{2} + i$	**A1**	[5]
	(b) (i)	*EITHER*: Show the point representing 4 + 3i in relatively correct position	**B1**	
		Show the perpendicular bisector of the line segment joining this point to the origin	**B1√**	[2]
		OR: Obtain correct Cartesian equation of the locus in any form, e.g. $8x + 6y = 25$	**B1**	
		Show this line	**B1√**	
		[This f.t. is dependent on using a correct method to determine the equation.]		
	(ii)	State or imply the relevant point is represented by 2 + 1.5i or is at (2, 1.5)	**B1**	
		Obtain modulus 2.5	**B1√**	
		Obtain argument 0.64 (or 36.9°) (allow decimals in [0.64, 0.65] or [36.8, 36.9])	**B1√**	[3]