Cambridge International A Level

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MATHEMATICS **9709/33**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 *EITHER*: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**

Obtain critical values $x = -5$ and $x = \frac{3}{5}$ **A1**

State final answer $-5 < x < \frac{3}{5}$ **A1**

OR: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality **(B1**

Obtain critical value $x = \frac{3}{5}$ similarly **B2**

State final answer $-5 < x < \frac{3}{5}$ **B1)**

[Do not condone $\le$ for $<$.] [4]

2 **(i)** State or imply $y \ln 3 = (2-x)\ln 4$ **B1**

State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent **B1**

State gradient is $-\dfrac{\ln 4}{\ln 3}$, or exact equivalent **B1**

[3]

(ii) Substitute $y = 2x$ and solve for x, using a log law correctly at least once **M1**

Obtain answer $x = \ln 4 / \ln 6$, or exact equivalent **A1**

[2]

3 **(i)** State answer $R = 3$ **B1**

Use trig formula to find **M1**

Obtain $\alpha = 41.81°$ with no errors seen **A1**

[3]

(ii) Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42° to 2 d.p.) **B1√**

Carry out an appropriate method to find a value of x in the given range **M1**

Obtain answer 216.5° only **A1**

[Ignore answers outside the given interval.] [3]

4 **(i)** State $\dfrac{dx}{dt} = 1 - \sin t$ **B1**

Use chain rule to find the derivative of y **M1**

Obtain $\dfrac{dy}{dt} = \dfrac{\cos t}{1 + \sin t}$, or equivalent **A1**

Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$ **M1**

Obtain the given answer correctly **A1**

[5]

(ii) State or imply $t = \cos^{-1}(\frac{1}{3})$ **B1**

Obtain answers $x = 1.56$ and $x = -0.898$ **B1 + B1**

[3]

5		Separate variables and make reasonable attempt at integration of either integral	**M1**
		Obtain term $\frac{1}{2}e^{2y}$	**B1**
		Use Pythagoras	**M1**
		Obtain terms $\tan x - x$	**A1**
		Evaluate a constant or use $x = 0$, $y = 0$ as limits in a solution containing terms $ae^{\pm 2y}$ and $b\tan x$, ($ab \neq 0$)	**M1**
		Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$	**A1**
		Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where $a > 0$	**M1**
		Obtain answer $y = 0.179$	**A1**
			[8]
6	**(i)**	Use the product rule	**M1**
		Obtain correct derivative in any form	**A1**
		Equate 2-term derivative to zero and obtain the given answer correctly	**A1**
			[3]
	(ii)	Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or compare values of relevant expressions at $p= 2$ and $p = 2.5$	**M1**
		Complete the argument correctly with correct calculated values	**A1**
			[2]
	(iii)	Use the iterative formula correctly at least once	**M1**
		Obtain final answer 2.15	**A1**
		Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145,2.155)	**A1**
			[3]
7	**(i)**	State or imply $du = 2x\,dx$, or equivalent	**B1**
		Substitute for x and dx throughout	**M1**
		Reduce to the given form and justify the change in limits	**A1**
			[3]
	(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**
		Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent	**A1 + A1**
		(deduct A1 for each error or omission)	
		Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**
		Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent	**A1**
			[5]

8	**(i)**	State a correct equation for AB in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent	**B1**
		Equate at least two pairs of components of AB and l and solve for λ or for μ	**M1**
		Obtain correct answer for λ or for μ, e.g. $\lambda = -1$ or $\mu = 2$	**A1**
		Show that not all three equations are not satisfied and that the lines do not intersect	**A1**
			[4]
	(ii)	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l, e.g. $(1-\mu)\mathbf{i} + (-3+2\mu)\mathbf{j} + (-2+\mu)\mathbf{k}$	**B1**
		Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	**M1**
		Solve and obtain $\mu = \frac{3}{2}$	**A1**
		Carry out a method to calculate AP when $\mu = \frac{3}{2}$	**M1**
		Obtain the given answer $\frac{1}{\sqrt{2}}$ correctly	**A1**
		OR 1:Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l	**(B1**
		Use correct method to express AP^2 (or AP) in terms of μ	**M1**
		Obtain a correct expression in any form, e.g. $(1-\mu)^2 + (-3+2\mu)^2 + (-2+\mu)^2$	**A1**
		Carry out a complete method for finding its minimum	**M1**
		Obtain the given answer correctly	**A1)**
		OR 2:Calling (2, –2, –1) C, state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	**(B1**
		Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on l	**M1**
		Obtain correct answer in any form, e.g. $\frac{9}{\sqrt{6}}$	**A1**
		Use Pythagoras to find the perpendicular	**M1**
		Obtain the given answer correctly	**A1)**
		OR 3:State $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form	**(B1**
		Calculate vector product of $\overrightarrow{AC}$ and a direction vector for l, e.g. $(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**
		Obtain correct answer in any form, e.g. $\mathbf{i} + \mathbf{j} - \mathbf{k}$	**A1**
		Divide modulus of the product by that of the direction vector	**M1**
		Obtain the given answer correctly	**A1)**
			[5]
9	**(i)**	*EITHER*: Multiply numerator and denominator of $\frac{u}{v}$ by 2 + i, or equivalent	**M1**
		Simplify the numerator to –5 +5i or denominator to 5	**A1**
		Obtain final answer –1 + I	**A1**
		OR: Obtain two equations in x and y and solve for x or for y	**(M1**
		Obtain $x = -1$ or $y = 1$	**A1**
		Obtain final answer –1 + I	**A1)**
			[3]
	(ii)	Obtain $u + v = 1 + 2\mathrm{i}$	**B1**
		In an Argand diagram show points A, B, C representing u, v and $u + v$ respectively	**B1**$\sqrt{}$
		State that OB and AC are parallel	**B1**
		State that $OB = AC$	**B1**
			[4]

(iii) Carry out an appropriate method for finding angle AOB, e.g. find $\arg(u/v)$ **M1**

Show sufficient working to justify the given answer $\frac{3}{4}\pi$ **A1**

[2]

10 (i) State or imply the form $\dfrac{A}{x+3}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ **B1**

Use a correct method to determine a constant **M1**

Obtain one of the values $A=-3$, $B=1$, $C=2$ **A1**

Obtain a second value **A1**

Obtain the third value **A1**

[Mark the form $\dfrac{A}{x+3}+\dfrac{Dx+E}{(x-1)^2}$, where $A=-3$, $D=1$, $E=1$, B1M1A1A1A1 as above.] [5]

(ii) Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(x-1)^{-1}$, $(1-x)^{-1}$, $(x-1)^{-2}$, or $(1-x)^{-2}$ **M1**

Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction **A1**$\sqrt{}$ **+ A1**$\sqrt{}$ **+ A1**$\sqrt{}$

Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2$, or equivalent **A1**

[5]