READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
1 Find the gradient of the curve
\[ y = 3e^{4x} - 6\ln(2x + 3) \]
at the point for which \( x = 0 \). \[3\]

2 Solve the equation \( 5\tan 2\theta = 4\cot \theta \) for \( 0^\circ < \theta < 180^\circ \). \[5\]

3 Given that \( 3e^x + 8e^{-x} = 14 \), find the possible values of \( e^x \) and hence solve the equation \( 3e^x + 8e^{-x} = 14 \) correct to 3 significant figures. \[6\]

4 The polynomial \( p(x) \) is defined by
\[ p(x) = 8x^3 + 30x^2 + 13x - 25. \]
(i) Find the quotient when \( p(x) \) is divided by \( x + 2 \), and show that the remainder is 5. \[3\]
(ii) Hence factorise \( p(x) - 5 \) completely. \[3\]
(iii) Write down the roots of the equation \( p(|x|) - 5 = 0 \). \[1\]

5 A curve is defined by the parametric equations
\[ x = 2 \tan \theta, \quad y = 3 \sin 2\theta, \]
for \( 0 \leq \theta < \frac{1}{2}\pi \).
(i) Show that \( \frac{dy}{dx} = 6\cos^4 \theta - 3\cos^2 \theta \). \[4\]
(ii) Find the coordinates of the stationary point. \[3\]
(iii) Find the gradient of the curve at the point \( (2\sqrt{3}, \frac{3}{2}\sqrt{3}) \). \[2\]

6 The equation of a curve is \( y = \frac{3x^2}{x^2 + 4} \). At the point on the curve with positive \( x \)-coordinate \( p \), the gradient of the curve is \( \frac{1}{2} \).
(i) Show that \( p = \sqrt{\left( \frac{48p - 16}{p^2 + 8} \right)} \). \[5\]
(ii) Show by calculation that \( 2 < p < 3 \). \[2\]
(iii) Use an iterative formula based on the equation in part (i) to find the value of \( p \) correct to 4 significant figures. Give the result of each iteration to 6 significant figures. \[3\]
7 (a) Find \[ \int \frac{1 + \cos^4 2x}{\cos^2 2x} \, dx. \] \[ \text{[5]} \]

(b) Without using a calculator, find the exact value of \[ \int_{4}^{14} \left(2 + \frac{6}{3x-2}\right) \, dx, \] giving your answer in the form \( \ln(ae^b) \), where \( a \) and \( b \) are integers. \[ \text{[5]} \]