Cambridge International examinations
Cambridge International Advanced Level

Mathematics

Paper 5 Mechanics 2 (M2)

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

Read these Instructions First

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Do Not Write in any BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use 10 m s\(^{-2}\).
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
1 A small ball \( B \) is projected with speed 12 m s\(^{-1}\) at an angle of 30° above the horizontal from a point \( O \) on horizontal ground. At the instant 0.8 s after projection, \( B \) is 0.5 m vertically above the top of a vertical post.

(i) Calculate the height of the top of the post above the ground. \([3]\)

(ii) Show that \( B \) is at its greatest height 0.2 s before passing over the post. \([2]\)

2 One end of a light elastic string of natural length 0.4 m is attached to a fixed point \( O \). The other end of the string is attached to a particle of weight 5 N which hangs in equilibrium 0.6 m vertically below \( O \).

(i) Find the modulus of elasticity of the string. \([2]\)

The particle is projected vertically upwards from the equilibrium position and comes to instantaneous rest after travelling 0.3 m upwards.

(ii) Calculate the speed of projection of the particle. \([3]\)

(iii) Calculate the greatest extension of the string in the subsequent motion. \([3]\)

3 The point \( O \) is 8 m above a horizontal plane. A particle \( P \) is projected from \( O \). After projection, the horizontal and vertically upwards displacements of \( P \) from \( O \) are \( x \) m and \( y \) m respectively. The equation of the trajectory of \( P \) is

\[ y = 2x - x^2. \]

(i) Find the value of \( x \) for the point where \( P \) strikes the plane. \([2]\)

(ii) Find the angle and speed of projection of \( P \). \([3]\)

(iii) Calculate the speed of \( P \) immediately before it strikes the plane. \([2]\)
A uniform object is made by drilling a cylindrical hole through a rectangular block. The axis of the cylindrical hole is perpendicular to the cross-section $ABCD$ through the centre of mass of the object. $AB = CD = 0.7\,\text{m}$, $BC = AD = 0.4\,\text{m}$, and the centre of the hole is 0.1 m from $AB$ and 0.2 m from $AD$ (see diagram). The hole has a cross-section of area $0.03\,\text{m}^2$.

(i) Show that the distance of the centre of mass of the object from $AB$ is 0.212 m, and calculate the distance of the centre of mass from $AD$. [4]

The object has weight 70 N and is placed on a rough horizontal surface, with $AD$ in contact with the surface. A vertically upwards force of magnitude $F$ N acts on the object at $C$. The object is on the point of toppling.

(ii) Find the value of $F$. [2]

The force acting at $C$ is removed, and the object is placed on a rough plane inclined at an angle $\theta^\circ$ to the horizontal. $AD$ lies along a line of greatest slope, with $A$ higher than $D$. The plane is sufficiently rough to prevent sliding, and the object does not topple.

(iii) Find the greatest possible value of $\theta$. [2]

A particle $P$ of mass 0.4 kg is placed at rest at a point $A$ on a rough horizontal surface. A horizontal force, directed away from $A$ and with magnitude $0.6t$ N, acts on $P$, where $t$ s is the time after $P$ is placed at $A$. The coefficient of friction between $P$ and the surface is 0.3, and $P$ has displacement from $A$ of $x$ m at time $t$ s.

(i) Show that $P$ starts to move when $t = 2$. Show also that when $P$ is in motion it has acceleration $(1.5t - 3)\,\text{m}\,\text{s}^{-2}$. [3]

(ii) Express the velocity of $P$ in terms of $t$, for $t \geq 2$. [4]

(iii) Express $x$ in terms of $t$, for $t \geq 2$. [3]

[Question 6 is printed on the next page.]
$OA$ is a rod which rotates in a horizontal circle about a vertical axis through $O$. A particle $P$ of mass $0.2$ kg is attached to the mid-point of a light inextensible string. One end of the string is attached to the rod at $A$ and the other end of the string is attached to a point $B$ on the axis. It is given that $OA = OB$, angle $OAP = angle OBP = 30^\circ$, and $P$ is $0.4$ m from the axis. The rod and the particle rotate together about the axis with $P$ in the plane $OAB$ (see diagram).

(i) Calculate the tensions in the two parts of the string when the speed of $P$ is $1.2$ m s$^{-1}$. [6]

The angular speed of the rod is increased to $5$ rad s$^{-1}$, and it is given that the system now rotates with angle $OAP = angle OBP = 60^\circ$.

(ii) Show that the tension in the part $AP$ of the string is zero. [6]