READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
1. A six-sided die shows a six on 25 throws out of 200 throws. Test at the 10% significance level the null hypothesis: \( P(\text{throwing a six}) = \frac{1}{6} \), against the alternative hypothesis: \( P(\text{throwing a six}) < \frac{1}{6} \). [5]

2. A researcher is investigating the lengths, in kilometres, of the journeys to work of the employees at a certain firm. She takes a random sample of 10 employees.
   
   (i) State what is meant by 'random' in this context. [1]

   The results of her sample are as follows.
   
   1.5  2.0  3.6  5.9  4.8  8.7  3.5  2.9  4.1  3.0

   (ii) Find unbiased estimates of the population mean and variance. [3]

   (iii) State what is meant by 'population' in this context. [1]

3. Based on a random sample of 700 people living in a certain area, a confidence interval for the proportion, \( p \), of all people living in that area who had travelled abroad was found to be \( 0.5672 < p < 0.6528 \).

   (i) Find the proportion of people in the sample who had travelled abroad. [1]

   (ii) Find the confidence level of this confidence interval. Give your answer correct to the nearest integer. [4]

4. In the past, the time spent by customers in a certain shop had mean 12.5 minutes and standard deviation 4.2 minutes. Following a change of layout in the shop, the mean time spent in the shop by a random sample of 50 customers is found to be 13.5 minutes.

   (i) Assuming that the standard deviation remains at 4.2 minutes, test at the 5% significance level whether the mean time spent by customers in the shop has changed. [5]

   (ii) Another random sample of 50 customers is chosen and a similar test at the 5% significance level is carried out. State the probability of a Type I error. [1]

5. The thickness of books in a large library is normally distributed with mean 2.4 cm and standard deviation 0.3 cm.

   (i) Find the probability that the total thickness of 6 randomly chosen books is more than 16 cm. [4]

   (ii) Find the probability that the thickness of a book chosen at random is less than 1.1 times the thickness of a second book chosen at random. [5]
In each turn of a game, a coin is pushed and slides across a table. The distance, $X$ metres, travelled by the coin has probability density function given by

$$f(x) = \begin{cases} kx^2(2-x) & 0 \leq x \leq 2, \\ 0 & \text{otherwise}, \end{cases}$$

where $k$ is a constant.

(i) State the greatest possible distance travelled by the coin in one turn. [1]

(ii) Show that $k = \frac{3}{4}$. [3]

(iii) Find the mean distance travelled by the coin in one turn. [3]

(iv) Out of 400 turns, find the expected number of turns in which the distance travelled by the coin is less than 1 metre. [3]

A large number of spoons and forks made in a factory are inspected. It is found that 1% of the spoons and 1.5% of the forks are defective. A random sample of 140 items, consisting of 80 spoons and 60 forks, is chosen. Use the Poisson approximation to the binomial distribution to find the probability that the sample contains

(i) at least 1 defective spoon and at least 1 defective fork, [3]

(ii) fewer than 3 defective items. [3]

The random variable $X$ has the distribution $\text{Po}(\lambda)$. It is given that

$$P(X = 1) = p \quad \text{and} \quad P(X = 2) = 1.5p,$$

where $p$ is a non-zero constant. Find the value of $\lambda$ and hence find the value of $p$. [4]