READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
1. Solve the equation \(|0.4x - 0.8| = 2\). [3]

2. (i) Given that \(\frac{1 + 4y}{3 + 2y} = 5\), find the value of \(2y\). [3]
(ii) Use logarithms to find the value of \(y\) correct to 3 significant figures. [2]

3. The definite integral \(I\) is defined by \(I = \int_{0}^{2} (4e^{\frac{1}{2}x} + 3) \, dx\).

   (i) Show that \(I = 8e - 2\). [3]

   (ii) Sketch the curve \(y = 4e^{\frac{1}{2}x} + 3\) for \(0 \leq x \leq 2\). [2]

   (iii) State whether an estimate of \(I\) obtained by using the trapezium rule will be more than or less than \(8e - 2\). Justify your answer. [1]

4. The polynomial \(p(x)\) is defined by \(p(x) = 4x^3 + ax^2 + ax + 4\), where \(a\) is a constant.

   (i) Use the factor theorem to show that \((x + 1)\) is a factor of \(p(x)\) for all values of \(a\). [2]

   (ii) Given that the remainder is \(-42\) when \(p(x)\) is divided by \((x - 2)\), find the value of \(a\). [2]

   (iii) When \(a\) has the value found in part (ii), factorise \(p(x^2)\) completely. [4]

5. The diagram shows the curve \(y = \frac{4 \ln x}{x^2 + 1}\) and its stationary point \(M\). The \(x\)-coordinate of \(M\) is \(m\).

   (i) Find an expression for \(\frac{dy}{dx}\) and hence show that \(m = e^{0.5(1+m^2)}\). [5]

   (ii) Use an iterative formula based on the equation in part (i) to find the value of \(m\) correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]
6 (i) Show that \( \frac{\cos 2\theta}{1 + \cos 2\theta} = 1 - \frac{1}{2} \sec^2 \theta \). \[2\]

(ii) Solve the equation \( \frac{\cos 2\alpha}{1 + \cos 2\alpha} = 13 + 5 \tan \alpha \) for \( 0 < \alpha < \pi \). \[4\]

(iii) Find the exact value of \( \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1 + \cos x} \, dx \). \[4\]

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The diagram shows the curve with parametric equations

\[
x = 4 \sin \theta, \quad y = 1 + 3 \cos (\theta + \frac{1}{6}\pi)
\]

for \( 0 \leq \theta < 2\pi \).

(i) Show that \( \frac{dy}{dx} \) can be expressed in the form \( k(1 + (\sqrt{3}) \tan \theta) \) where the exact value of \( k \) is to be determined. \[5\]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the positive y-axis. Give your answer in the form \( y = mx + c \), where the constants \( m \) and \( c \) are exact. \[5\]