



**Cambridge International Examinations**  
Cambridge Ordinary Level

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**ADDITIONAL MATHEMATICS**

**4037/11**

Paper 1

**May/June 2016**

MARK SCHEME

Maximum Mark: 80

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**Published**

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## Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

Question	Answer	Marks	Guidance
<b>1 (i)</b>	-27	<b>B1</b>	
<b>(ii)</b>	$9 - 8k = 0$ $k = \frac{9}{8}$  <b>Or</b> $\frac{dy}{dx} = 4x - 3$ when $\frac{dy}{dx} = 0$ , $x = \frac{3}{4}$ so $k = \frac{9}{8}$  <b>Or</b> completing the square $y = 2\left(x - \frac{3}{4}\right)^2 + k - \frac{9}{8}$ $k = \frac{9}{8}$	<b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b>	for use of discriminant with a complete method to get to $k =$  for a complete method to get to $k =$  for a complete method to get to $k =$
<b>2 (a)</b>	$2^{4(3x-1)} = 2^{3(x+2)}$ or $4^{2(3x-1)} = 4^{\frac{3}{2}(x+2)}$ or $8^{\frac{4}{3}(3x-1)} = 8^{x+2}$ or $16^{3x-1} = 16^{\frac{3}{4}(x+2)}$  leading to $x = \frac{10}{9}$ cao	<b>B1</b>  <b>M1</b> <b>A1</b>	<b>B1</b> for a correct statement  for equating indices
<b>(b)</b>	$p = \frac{5}{3}$ $q = -2$	<b>B1</b> <b>B1</b>	

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Question	Answer	Marks	Guidance
3	<p>On <math>x</math>-axis, <math>2x^2 - 7 = 1</math>  <math>x = 2</math></p> $\frac{dy}{dx} = \frac{4x}{2x^2 - 7}$ <p>When <math>x = 2</math>, <math>\frac{dy}{dx} = 8</math></p> <p>Gradient of normal <math>= -\frac{1}{8}</math></p> <p>Equation of normal <math>y = -\frac{1}{8}(x - 2)</math></p> <p>Required form <math>x + 8y - 2 = 0</math></p>	<p><b>M1</b> <b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>for equating to 1</p> <p>for attempt at perpendicular through <i>their</i> (2, 0), must be using <math>y = 0</math></p> <p>must be equated to zero with integer coefficients</p>
4 (a)	$\mathbf{A}^2 = \begin{pmatrix} 7 & -2 \\ -3 & 6 \end{pmatrix}$ $\mathbf{A}^2 - 2\mathbf{B} = \begin{pmatrix} 1 & -2 \\ -5 & 2 \end{pmatrix}$	<p><b>B1</b></p> <p><b>M1</b> <b>A1</b></p>	<p>for their <math>\mathbf{A}^2 - 2\mathbf{B}</math></p>
(b)	$\begin{pmatrix} 4 & 1 \\ 10 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ <p>so <math>\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 &amp; -1 \\ -10 &amp; 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}</math></p> <p>leading to <math>\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}</math></p> <p><math>x = 1</math>  <math>y = -3</math></p>	<p><b>M1</b> <b>DM1</b></p> <p><b>A1</b> <b>A1</b></p>	<p>for pre-multiplication by <i>their</i> inverse matrix</p> <p><b>DM1</b> for attempt at matrix multiplication</p> <p>Allow in matrix form</p>
5 (i)	$\frac{d}{dx} \left( \frac{e^{4x}}{4} - xe^{4x} \right) = e^{4x} - ((x \times 4e^{4x}) + e^{4x})$ $= -4xe^{4x}$	<p><b>B1</b></p> <p><b>M1</b> <b>A1</b> <b>A1</b></p>	<p>for <math>\frac{d}{dx} \left( \frac{e^{4x}}{4} \right) = e^{4x}</math></p> <p>for attempt to differentiate a product</p> <p>for a correct product</p> <p>for correct final answer</p>
(ii)	$\int_0^{\ln 2} xe^{4x} dx = -\frac{1}{4} \left[ \frac{e^{4x}}{4} - xe^{4x} \right]_0^{\ln 2}$ $= -\frac{1}{4} \left( \left( \frac{16}{4} - 16 \ln 2 \right) - \frac{1}{4} \right)$ $= 4 \ln 2 - \frac{15}{16}$	<p><b>B1FT</b></p> <p><b>B1</b> <b>M1</b> <b>A1</b></p>	<p><b>FT</b> for use of <i>their</i> <math>\frac{1}{p} \times \left( \frac{e^{4x}}{4} - xe^{4x} \right)</math>, must be numerical <math>p</math>, but <math>\neq 0</math></p> <p>for <math>e^{4 \ln 2} = 16</math></p> <p>for correct use of limits, must be an integral of the correct form</p>

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Question	Answer	Marks	Guidance
6 (i)	$2 - \sqrt{5} < f(x) \leq 2$	<b>B2</b>	<b>B1</b> for $\leq 2$ <b>B1</b> for $2 - \sqrt{5} <$ or awrt $-0.24$ Must be using $f$ , $f(x)$ or $y$ , $2 - \sqrt{5} <$ , if not then <b>B1</b> max
(ii)	$f^{-1}(x) = (2 - x)^2 - 5$ Domain $2 - \sqrt{5} < x \leq 2$ Range $y$ or $-5 \leq f^{-1}(x) < 0$	<b>M1</b> <b>A1</b>  <b>B1</b> <b>B1</b>	for a correct method to find the inverse  Must be using the correct variables for the B marks
(iii)	$fg(x) = f\left(\frac{4}{x}\right)$ $= 2 - \sqrt{\frac{4}{x}} + 5$ leading to $x = -4$	<b>M1</b> <b>DM1</b>  <b>A1</b>	for correct order of functions for solution of equation
7 (i)	Finding an angle of $68.2^\circ$ or $21.8^\circ$ $\frac{4.5}{\sin 68.2} = \frac{2.4}{\sin \alpha}$ leading to $\alpha = 29.7^\circ$ (allow $\pm 0.1$ ) Direction is $82.1^\circ$ to the bank, upstream (allow $\pm 0.1^\circ$ )	<b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b>	for the sine rule
(ii)	$\frac{4.5}{\sin 68.2} = \frac{2.4}{\sin 29.7} = \frac{v_r}{\sin 82.1}$ leading to $v_r = 4.8$  time taken = $\frac{80.78}{4.8} = 16.8$  Alternative method: Finding an angle of $68.2^\circ$ or $21.8^\circ$ $4.5^2 = 2.4^2 + v_r^2 - (2 \times 2.4 \times v_r \cos 68.2)$ leading to $v_r = 4.8$  Use of sine rule to obtain angle and direction to obtain direction is $82.1^\circ$ to the bank, upstream  Use of time taken = $\frac{80.78}{4.8} = 16.8$	<b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b>  <b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b>	for the sine rule for resultant velocity for attempt to find $AB$ and hence the time taken  for correct use of the cosine rule for resultant velocity for use of the sine rule for $\alpha = 29.7^\circ$ for $82.1^\circ$ for attempt to find $AB$ and hence the time taken

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Question	Answer	Marks	Guidance
8	(i) $y - 6 = -\frac{4}{12}(x + 8)$ ( $3y + x = 10$ )	M1 A1	for a correct method allow unsimplified
	(ii) $y - 7 = 3(x + 1)$ ( $y = 3x + 10$ )	DM1 A1	for attempt at a perpendicular line using (-1, 7) allow unsimplified
	(iii) point of intersection $(-2, 4)$ which is the midpoint of $AB$	M1 M1 A1	for attempt to find the point of intersection using simultaneous equations for attempt to find midpoint for all correct
	Alternative method: Midpoint $(-2, 4)$ Verification that this point lies on $CP$ .	M1 M1 A1	for attempt to find midpoint for full verification for all correct
	(iv) $CP = \sqrt{10}$ or 3.16	B1	
	(v) Area = $\frac{1}{2} \times \sqrt{10} \times 4\sqrt{10}$ = 20	M1 A1	for correct method <b>using CP</b> for 19.9 – 20.1

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Question	Answer	Marks	Guidance
9 (i)	$2 \cos x \cot x = \cot x + 2 \cos x$ $2 \cos x \frac{\cos x}{\sin x} + 1 = \frac{\cos x}{\sin x} + 2 \cos x$	M1	for use of $\cot x = \frac{\cos x}{\sin x}$ for both terms
	$2 \cos^2 x + \sin x = \cos x + 2 \cos x \sin x$ $2 \cos^2 x - 2 \cos x \sin x = \cos x - \sin x$ $2 \cos x (\cos x - \sin x) = \cos x - \sin x$ $(2 \cos x - 1)(\cos x - \sin x) = 0$	DM1  DM1  A1	for multiplication throughout by $\sin x$  for attempt to factorise  for completely correct solution www
(ii)	<p>Alternative method:</p> $a \cos^2 x - a \cos x \sin x - b \cos x + b \sin x = 0$ $a \cos x \cot x - a \cos x - b \cot x + b = 0$ $a = 2, b = 1$	M1  DM1 DM1  A1	for expansion of RHS  for division by $\sin x$ for comparing like terms to obtain both $a$ and $b$ for both correct www
	$(2 \cos x - 1)(\cos x - \sin x) = 0$ $\cos x = \frac{1}{2}, \tan x = 1$ $x = \frac{\pi}{3}, x = \frac{\pi}{4}$	M1  A1, A1	for either  A1 for each, penalise extra solutions within the range by withholding the last A mark
10 (i)	$f(-2) = -32 - 2k + p = 0$ $f'\left(\frac{1}{2}\right) = \frac{12}{4} + k = 0$ leading to $k = -3$ and $p = 26$	M1  M1  A1, A1	for attempt at $f(-2)$  for attempt at $f'\left(\frac{1}{2}\right)$  A1 for each
	(ii)	B1FT  B1	FT for <i>their</i> $\frac{p}{2}$ all correct
(iii)	Showing that $4x^2 - 8x + 13 = 0$ has no real roots  so $x = -2$ only www	M1,  A1	M1 for a valid attempt at solution of equation leading to no solution or consideration of the discriminant

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11 (i)	$AB = 2r \sin \theta$ or $\sqrt{r^2 + r^2 - 2r^2 \cos 2\theta}$ or $\frac{r \sin 2\theta}{\sin\left(\frac{\pi}{2} - \theta\right)}$ or $\frac{r \sin 2\theta}{\cos \theta}$	B1	
(ii)	$2r \sin \theta + 2r\theta = 20$ $r = \frac{10}{\theta + \sin \theta}$	M1 A1	for use of (i) + arc length = 20, oe must be convinced
(iii)	$\frac{dr}{d\theta} = -\frac{10(1 + \cos \theta)}{(\theta + \sin \theta)^2}$ When $\theta = \frac{\pi}{6}$ , $\frac{dr}{d\theta} = -17.8$	M1 A2,1,0 A1	for a correct attempt to differentiate –1 each error allow awrt –17.8
(iv)	$\frac{dr}{dt} = 15$ $\frac{d\theta}{dt} = \frac{dr}{dt} \div \frac{dr}{d\theta}$ $\frac{d\theta}{dt} = -0.842$	B1 M1 A1	may be implied for use of $\frac{15}{\text{their (iii)}}$ allow –0.84 or –0.843