READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all questions.
If working is needed for any question it must be shown below that question.
Essential working must be shown for full marks to be awarded.

Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to
three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
1 (a) Use set notation to describe the shaded region in the Venn diagram.

\[ P \cap Q \]

Answer .......................................... [1]

(b) \( \mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)
\( A = \{x : x \text{ is a factor of } 12\} \)
\( B = \{x : x \text{ is a multiple of } 2\} \)
\( C = \{x : x \text{ is a square number}\} \)

(i) Show this information on the Venn diagram below.

\[ \mathcal{E} \]

\[ A \]
\[ B \]
\[ C \]

(ii) Find \( n(A \cap B) \).

Answer .......................................... [1]

(iii) Find \( n(A \cap (B \cup C)) \).

Answer .......................................... [1]

(iv) One subset in the Venn diagram in part (b)(i) has no elements.

Use set notation to describe this subset.

Answer .......................................... [1]
(c)  (i) Write 540 as the product of its prime factors.

Answer ........................................... [2]

(ii) \( p \) is the smallest possible integer such that \( 540p \) is a square number.

Find \( \sqrt[4]{540p} \), giving your answer as the product of its prime factors.

Answer ........................................... [2]

2  (a) Sami invests $2000 in an account paying compound interest at a rate of 1.8% per year.

Calculate the total interest paid to Sami after 3 years.

Answer $ ....................................... [3]

(b) Theresa takes out a loan.
She repays the loan over one year at a rate of $54 per month.
The total she repays is 8% greater than the value of the original loan.

Work out the value of the original loan.

Answer $ ....................................... [3]
3 (a) Solve \( 4(p - 3) = 2p + 7 \).

Answer \( p = \cdots \) [2]

(b) Solve these simultaneous equations.

\[
\begin{align*}
2x - y &= 5 \\
7x + 2y &= 1
\end{align*}
\]

Show your working.

Answer \( x = \cdots \) [3]  

\[
\begin{align*}
y &= \cdots
\end{align*}
\]
(c) Simplify \[ \frac{m^2 + 3m}{2m^2 + 5m - 3}. \]

Answer ................................................................ [3]

(d) \( b \) is directly proportional to the cube of \( a \).

Given that \( b = 4 \) when \( a = 2 \), find \( b \) when \( a = 5 \).

Answer \( b = \) ........................................... [3]
Twelve lettered tiles spelling the word TRIGONOMETRY are placed inside a bag.

(a) A tile is taken at random from the bag.

Find the probability that the tile shows a letter R.
Give your answer as a fraction in its simplest form.

Answer .......................................... [1]

(b) All the tiles are placed back in the bag, a tile is then taken at random and placed on the table.
A second tile is taken at random and placed to the right of the first tile.
A third tile is taken at random and placed to the right of the second tile.

1st 2nd 3rd

Find the probability that, in the order the tiles were placed on the table, they spell GET.

Answer .......................................... [2]
(c) Vowels are the letters A, E, I, O and U. All other letters are consonants. All the twelve tiles are placed back in the bag and two tiles are taken at random, without replacement.

(i) Complete the tree diagram.

(ii) Find the probability that the tiles both show vowels.

\[ \text{Answer} \quad \frac{3}{11} \quad [1] \]

(iii) Find the probability that one tile shows a vowel and one tile shows a consonant.

\[ \text{Answer} \quad \frac{4}{12} \quad [2] \]
5 (a) 1, 7, 13, 19, 25, …

(i) Find an expression, in terms of \( n \), for the \( n \)th term of this sequence.

\[
\text{Answer} \quad \text{...........................................} \quad [2]
\]

(ii) Explain why 251 is not a term in this sequence.

\[
\text{Answer} \quad \text{....................................................................................................................................} \quad [1]
\]

(b) Here is another sequence.

5, 8, 13, 20, 29, …

The \( p \)th term of this sequence is \( p^2 + 4 \).

Write down an expression, in terms of \( p \), for the \( p \)th term of these sequences.

(i) –2, 1, 6, 13, 22, …

\[
\text{Answer} \quad \text{...........................................} \quad [1]
\]

(ii) 7, 12, 19, 28, 39, …

\[
\text{Answer} \quad \text{...........................................} \quad [1]
\]
(c) The diagrams below show the first three patterns in a sequence. The patterns are made from short diagonal lines.

Pattern 1

Pattern 2

Pattern 3

(i) Draw Pattern 4 on the dotty grid below.

(ii) Complete the table below for the number of short lines in Patterns 4 and 5.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of short lines</td>
<td>4</td>
<td>10</td>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(iii) Find an expression, in terms of \( t \), for the number of short lines in Pattern \( t \).

Answer ...........................................  [2]
6  (a) $ABC$ is a triangle with $AC = 6$ cm and $BC = 9$ cm. $AB$ has been drawn below.

(i) Using a ruler and a pair of compasses only, construct triangle $ABC$. [2]

(ii) Measure $BAC$.  

$Answer$  ..........................................  [1]

(b) A rectangular field has dimensions 220 m by 350 m, each correct to the nearest 10 metres.

Calculate the upper bound for the area of the field.

$Answer$  ..........................................  $m^2$ [2]
The points $P$, $Q$, $R$ and $S$ lie on the circumference of a circle. 
$PQRS$ is a trapezium with $PQ$ parallel to $SR$. 
$T$ is the point on $SR$ such that $\angle QPT = 66^\circ$, $\angle QTR = 35^\circ$ and $\angle TQR = 79^\circ$.

(i) Find $\angle PTS$, giving a reason for your answer.

Answer $\angle PTS = \ldots$. because $\ldots$ [2]

(ii) Find $\angle PTQ$.

Answer $\ldots$ [1]

(iii) Complete the statements below to show that triangle $PQT$ is congruent to triangle $RTQ$.

1. Angle $PTQ = \text{Angle } \ldots$ 
2. Angle $PQT = \text{Angle } \ldots$ 
3. $\ldots$

Triangle $PQT$ is congruent to triangle $RTQ$.

Congruency condition $\ldots$ [3]
The diagram shows the net of an open box of height 3 cm. The area of the base of the box is 15 cm$^2$. The length of the rectangular base is $x$ cm. The total area of the net is $A$ cm$^2$.

(a) Show that $A = 15 + 6x + \frac{90}{x}$.

(b) Graham has one of these open boxes. The total area of the net of his box is 65 cm$^2$.

Write down an equation in $x$ and solve it to find the length of the base of Graham’s box. Give your answer correct to 2 decimal places.

Answer .................................... cm [4]
(c) (i) Complete the table below for $A = 15 + 6x + \frac{90}{x}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>72</td>
<td>63</td>
<td>61.5</td>
<td>63</td>
<td>66</td>
<td>69.9</td>
<td></td>
</tr>
</tbody>
</table>

(ii) Draw the graph of $A = 15 + 6x + \frac{90}{x}$ for $2 \leq x \leq 8$.

(iii) Delilah has one of these open boxes. The area of the net of her box is 68 cm$^2$.

Use your graph to find the length and width of Delilah’s box.

Answer

<table>
<thead>
<tr>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>................. cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width</th>
</tr>
</thead>
</table>
| ................. cm | [2]
8 The grid shows triangles $A$ and $B$ and rectangle $R$.

(a) Triangle $A$ is mapped onto triangle $B$ by the single transformation $K$.

Find the matrix representing transformation $K$.

\[
\text{Answer} \quad \begin{pmatrix} \ & \ \ & 1 \ & \ & \ \\ & \ & & & & \ & \end{pmatrix}
\]

(b) Triangle $B$ is mapped onto triangle $C$ by a reflection in the $y$-axis.

On the diagram, draw triangle $C$. [1]
(c) Triangle $A$ is mapped onto triangle $C$ by the **single** transformation $L$.

Describe fully the **single** transformation $L$.

*Answer* ................................................................. [2]

(d) Rectangle $R$ is mapped onto rectangle $S$ by a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

On the diagram, draw rectangle $S$. [2]
The diagram shows a sector of a circle of radius 8 cm and angle 70°.

(a) Calculate the shaded area.

Answer ................................ cm² [4]
A piece of chocolate is in the shape of a prism with the shaded area from part (a) being its cross section.
The rectangular base of the chocolate is 16 cm by $x$ cm.
The piece of chocolate is to be placed in a box which is a cuboid of size 16 cm by $x$ cm by 1.5 cm.

(i) Show that the chocolate will fit inside the box.

(ii) These boxes are to be packed in cartons in the shape of a cuboid.
The size of each carton is 48 cm by 4$x$ cm by 24 cm.

Find the maximum number of boxes that can be packed inside one carton.

Answer ........................................... [2]
A boat leaves $A$ and travels 12 km to $B$.

(a) The boat leaves $A$ at 10 25 and travels at an average speed of 15 km/h.

At what time does the boat arrive at $B$?

Answer .................................. [2]

(b) The bearing of $B$ from $A$ is 056°.

$B$ is 2 km due west of $C$.

Calculate $AC$.

Answer .................................. km [4]
\( C \) is the base of a cliff.
The top of the cliff, \( D \), is vertically above \( C \).
\( DC \) is perpendicular to \( BC \) and \( DC = 105 \) m.

Calculate the angle of elevation of \( D \) from \( B \).

\[ \text{Answer} \] ........................................... [2]
11 (a) The grid shows the line \( 4y = x + 2 \).

By drawing appropriate lines, indicate the region \( R \) defined by all these inequalities.

\[
x \geq 1 \quad x + y \leq 5 \quad 4y \geq x + 2
\]

(b) \( A \) is the point \((-1, 3)\) and \( B \) is the point \((5, 5)\).

(i) Calculate the length \( AB \).

Answer .................................................. [2]

(ii) Find the equation of the line perpendicular to \( AB \) that passes through the midpoint of \( AB \).

Answer .................................................. [4]