



Cambridge International AS & A Level

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

1 Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

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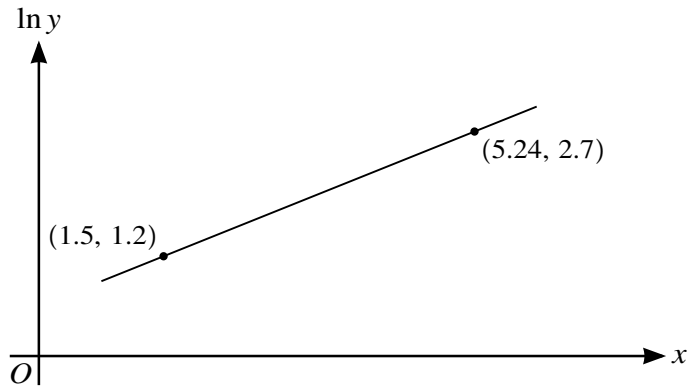
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The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points $(1.5, 1.2)$ and $(5.24, 2.7)$ as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

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3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$

[5]

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4 A curve has equation $y = \cos x \sin 2x$.

Find the x -coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

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- 5 (a) Express $\sqrt{2} \cos x - \sqrt{5} \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

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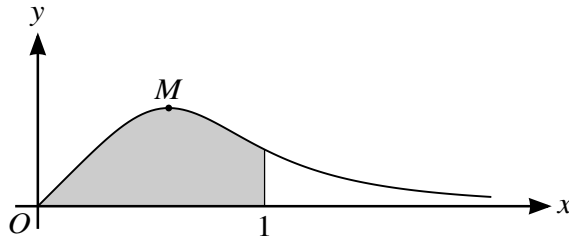
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The diagram shows the curve $y = \frac{x}{1 + 3x^4}$, for $x \geq 0$, and its maximum point M .

- (a) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [4]

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- (b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 1$. [5]

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7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y - 1}{(x + 1)(x + 3)}$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x . [9]

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A series of 20 horizontal dotted lines for writing.

- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leq 1$ and $\arg(z - 4i) \geq -\frac{1}{4}\pi$. [4]

- (ii) Find the least value of $\text{Im } z$ for points in this region, giving your answer in an exact form. [2]

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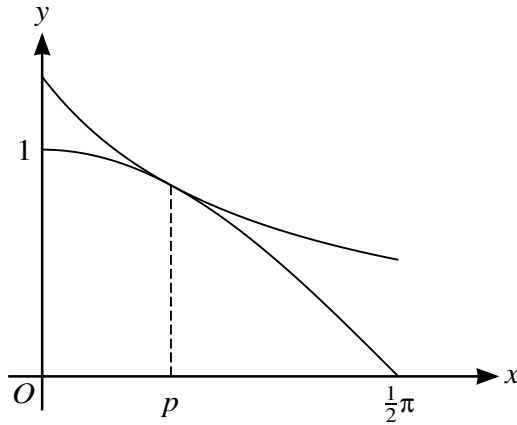
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The diagram shows the curves $y = \cos x$ and $y = \frac{k}{1+x}$, where k is a constant, for $0 \leq x \leq \frac{1}{2}\pi$. The curves touch at the point where $x = p$.

(a) Show that p satisfies the equation $\tan p = \frac{1}{1+p}$. [5]

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(b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\frac{1}{1+p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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(c) Hence find the value of k correct to 2 decimal places. [2]

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The line through M and N intersects the line through O and B at the point P .

(b) Find the position vector of P . [3]

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(c) Calculate angle OPM , giving your answer in degrees. [3]

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